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SEISMIC VULNERABILITY OF THE PIERO DELLA FRANCESCA'S FRESCO CALLED "RESURRECTION"

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Abstract. *The structural analysis of monumental buildings requires to consider safety and conservation objectives, including also the possible presence of artistic assets. In order to face these issues, the paper presents an account of the results of diagnostic analyses conducted by the authors on a specific monumental masonry building: the Civic Museum of Sansepolcro, a small town near Arezzo; in fact, besides to be one of the most renowned civic structures built by Italian communes of Central Italy during the High Middle Ages to house their city governments, it is also characterized by the presence of Piero Della Francesca's fresco, known as "Resurrection", that is widely hailed as one of the masterpieces of late 15th-century Italian art. Within this context, the integrate use of three different modelling strategies of different complexity is discussed: the equivalent frame approach, the macro-block model and the finite element model. In the first part, a full 3D pushover analyses and a simplified approach based on the kinematic theorem of limit analysis are used in order to understand the large-scale structural performance of the building. Next, the results of the finite element method (FEM) tests conducted on a detailed 3D model of the wall containing the Piero Della Francesca's fresco are employed in order to investigate the origin of the surveyed crack pattern on this important artistic asset.*

1 INTRODUCTION

The preservation of cultural heritage assets must guarantee their capacity of lasting over time against decay, natural hazards and accidental events, without losing, as much as possible, their authenticity. Moreover, there is a need to ensure the safety of occupants, related to the use of the building (private or public). To this end it is necessary to make reference to the principle of “minimum intervention”, under the constraint of an “acceptable safety level”, a concept that still represents an open issue for monumental buildings. Furthermore, besides the preservation of the architectural value of the building, also immovable artistic assets should be considered in the assessment, such as: frescoes, stucco-works, pinnacles, battlements, banisters, balconies etc.

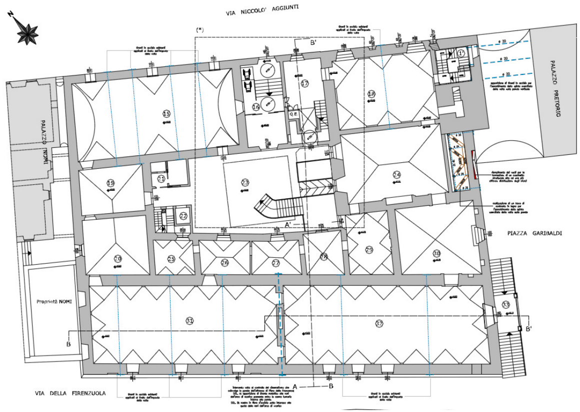
The seismic assessment of existing buildings, irrespective of whether they are historical, is a complex task, basically for two different reasons: a) the difficulty of interpreting and modeling the seismic response, because they have been designed without provisions for earthquake resistance and, in the case of ancient masonry structures, by an empirical approach; b) the difficulty of acquiring as-built information on material parameters and structural details, due to their spatial variability in the buildings and the need of avoiding invasive investigations [1]. To complicate the problem, every monumental building is characterized by its own history (the result of fusions, additions and replacements of structural elements that have led to its present configuration) so that each ancient building requires, though the typological similarities, a specific approach [2][3][4].

Within this context, a common denominator among the best known international guideline documents [5][6][7], on the assessment of cultural heritage assets, is the importance of the qualitative approach, based on historical analysis, the accurate investigation of structural details and the interpretation of seismic behavior, on the basis of observed damage in the building (due to previous events, if any) or on similar structures.

Within this field of research the present paper presents an account of the results of diagnostic analyses conducted by the authors on a illustrative case study: the Civic Museum of Sansepolcro. To analyze the structural behavior of the building and of its greatest artistic asset, the Piero Della Francesca’s fresco, a performance based multi-scale approach is followed [1][8]. In such a context, according to the [9][10], a global analysis of the Museum is made by using the equivalent frame approach. In particular, a full 3D pushover analysis of the entire building is performed in order to predict the large-scale structural performance of the building. Afterwards, a macro-element approach based on the kinematic theorem of limit analysis is used to take into account the structure response at architectonic elements scale (damage in macroelements, usually related to out-of-plane displacements). After completing the knowledge phase at macro-scale level, it is crucial to have, at the meso-scale level, a precise evaluation of the seismic vulnerability of the wall containing the fresco. To this end the results of the finite element method (FEM) tests conducted on a detailed 3D model of the wall are employed in order to provide an interpretation of the manifested crack pattern on this important artistic asset.

2 STRUCTURAL ANALYSIS METHODOLOGY

As already discussed in the introduction, the analysis of the seismic behavior of historic masonry buildings is a difficult task due to several aspects: the difficult numerical modelling of the nonlinear behavior of masonry material, with almost no tensile strength; the incomplete experimental characterization of the mechanical properties of masonry structural elements; the complexity of the geometrical configuration [12]. The scientific literature has proposed mechanical models which accurately predict the behavior of masonry material and that adopt different strategies to take into account the highly nonlinear behavior of the material both in tension (low tensile capacity and consequent cracking phenomena) and in compression. However, although these mathematical formulations can accurately describe the nonlinear phenomena that occur in the masonry, they are still hardly applicable to a complete 3D analysis of complex structural systems due to the large number of degrees of freedom required for meshing the structure. Within this context, the paper approaches the numerical problem using a performance based multi-scale analysis of structural system (Figure 1a) up to the masonry wall containing the most important artistic asset of the Museum: the Piero Della Francesca's fresco called "Resurrection" (Figure 1b).



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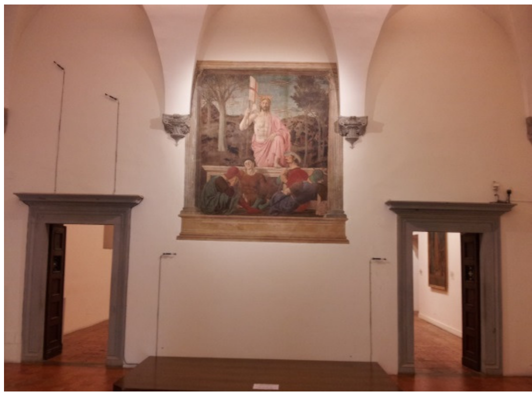
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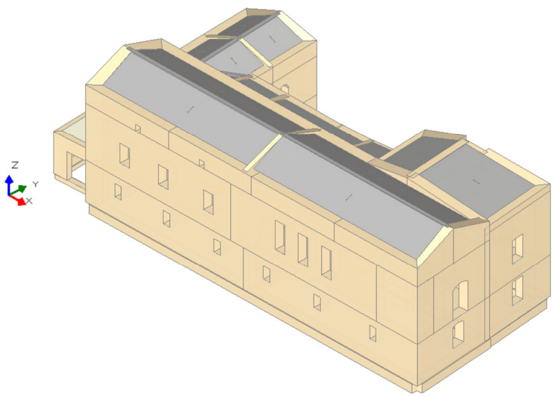
Figure 1: a) plan layout of the Civic Museum; b) Piero Della Francesca's fresco known as "Resurrezione".

2.1 Global scale: pushover analysis

The analysis of the seismic behavior was analyzed by using a nonlinear static analysis method: under conditions of constant gravity loads a monotonically increasing horizontal load was applied using a pushover analysis [13].

An equivalent frame model (Figure 2) was set up by considering each wall pier as a beam element in which the in-plane behavior was modelled as elastic–perfectly plastic with a limited plastic strain at failure. Due to the presence of deformable floors, similar consideration were used for spandrel walls (the reader is referred to [10] for further details): i.e. the horizontal masonry elements of a wall were modelled in the same way as the piers. Finally, in agreement with Italian Code NTC 2008 [9] three possible failure mechanisms are taken into account for the elements, namely a sliding failure, a diagonal shear cracking and a rocking failure due to the bending moment. Obviously, rocking failure is active for slender elements.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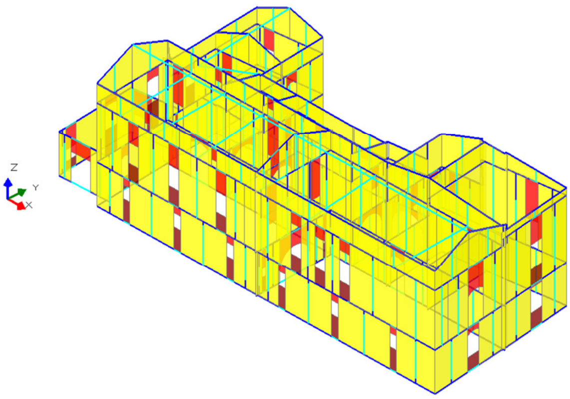
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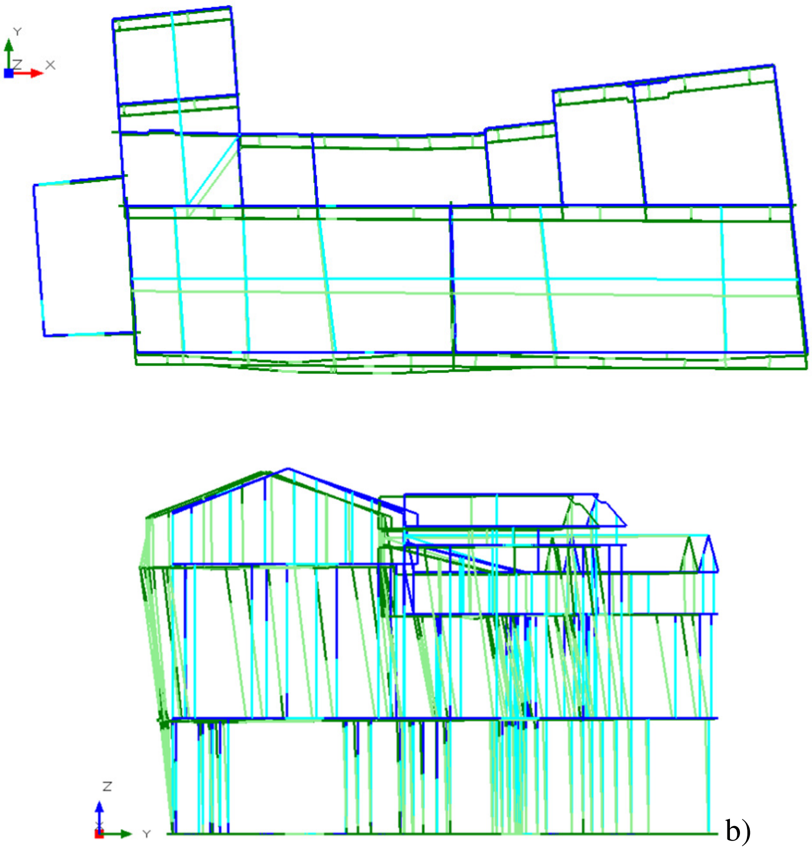
Figure 2: a) Geometrical model; b) Equivalent frame model.

Based on this analysis method, the effects of the seismic loads were evaluated by applying of two systems of horizontal forces perpendicular to one another. These forces, not acting simultaneously, were evaluated taking into account two load distributions. The first load distribution was directly proportional to the building's masses (uniform); the second load distribution was proportional to the product of the masses for the displacements of the corresponding first modal shape. These two load distributions could be considered as two limit states for the building's capacity. The first distributions assume that the horizontal loads representative of inertia seismic forces are constant with respect to the building's height. This means that the displacements on the lower level of the building are overestimated, while the opposite happens on the displacement on the top level. On the contrary the second distributions overestimate the displacements on the top level.

It is noteworthy to point out that a conventional pushover was assumed in the study, i.e. loads applied on the building did not change with the progressive degradation of the building that occurs during loading. This means that the conventional pushover did not account for the progressive changes in modal frequencies due to yielding and cracking on the structure during loading. This is a critical point for the application of conventional pushover to the analysis of historic masonry buildings, because it is predictable that the progressive damage of the building may also lead to period elongation, and therefore to different spectral amplifications [14]. Hence the hypothesis of invariance of static loads could cause an overestimation in the analysis of the masonry building seismic capacity especially when a non-uniform damage on the buildings or a high level of cracking are expected. However, also in its conventional form, the pushover provides an efficient alternative to expensive computational inelastic dynamic analyses and can nevertheless offer useful and effective information on the damage state that the building can develop under seismic loads.



a)

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Figure 3: Deformed shape at collapse: a) plain view; b) front view.

The critical load distribution for the Museum was the case of load acting on the y-direction (direction perpendicular to main building direction) and results (obtained by using the commercial software Aedes Pcm 2014, Figure 3) were mainly detailed next with respect to this case. Figure 4 points out that the 3D strength of the building is insufficient and this lack of resistance is mainly related to out-of-plane collapses of orthogonal walls which are at the same time the longest and the most loaded ones.

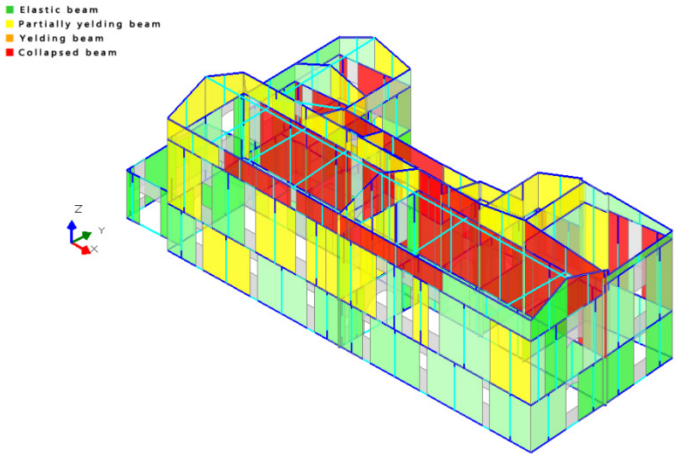


Figure 4: Damage pattern due to the rocking failure mechanism.

The displacement of a control point (center of mass of the roof level) versus the corresponding total shear is reported in Figure 5. Such a diagram can be named as the capacity curve of the palace in the y-direction as it furnishes, among other elements, the maximum seismic force which is borne by the structure, i.e. the level of the seismic action which activates the first local failure mechanism.

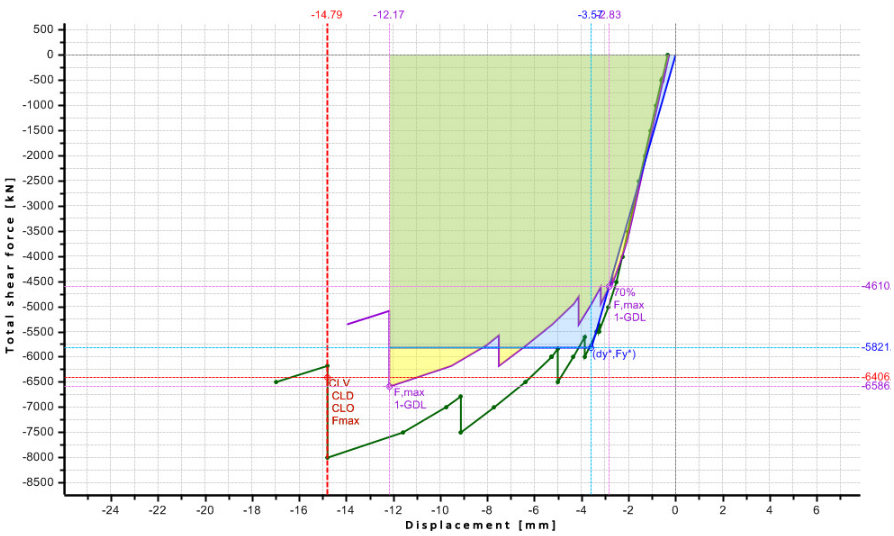


Figure 5: Structural response (maximum displacement vs total shear at the base) of the equivalent frame model: capacity curve of the actual structure (green line), the equivalent capacity curve (purple line) and the equivalent bilinear diagram (blue line).

The intensity measure (IM) adopted for the seismic assessment is the peak ground acceleration (PGA). On the basis of the computed values, seismic vulnerability is evaluated through a comparison between the maximum value of the PGA (obtained according to the procedure illustrated in [1], IM_{kn} = 0.165g) compatible with the fulfillment of the assumed performance levels (Life Safety) and the reference target value ($\overline{IM_{kn}}$ = 0.260g) of the seismic demand (in terms of PGA) obtained by means of the Italian code [9]. Since the safety factor ratio IM_{kn} / $\overline{IM_{kn}}$ is equal to 0.478 and it is below the minimum level of safety proposed, for instance, by [15], the structure is clearly not verified.

2.2 Architectonic elements scale: limit analysis

Previous results obtained by the pushover analysis refer to the assessment of the building as a whole, the total seismic mass of the structure being involved. However, it is worth pointing out that an exhaustive seismic assessment would require also the verification of possible local mechanisms (mainly out-of-plane ones). The attribute "local" refers to mechanisms that involve only a portion of the building and may not be accurately analyzed by the global model; for this reason they were studied by a different specific local model, by considering only a fraction of the total mass.

The examination of the collapse and the damage consequent to past earthquakes allowed to predict the occurrence of recurrent modes and collapse mechanisms in historical masonry buildings [16][17]. The structure was so considered as the assemblage of a certain number of components depending on structural compound geometry and shape (e.g. the whole façade) and on the details (e.g. quality of existing connections) whose behavior is similar to the behavior of analogous macro-elements in other buildings, where the collapse mechanisms are identified. To this end, several elementary macro-elements were identified in the building and modelled as a system of rigid bodies, articulated by hinges, whose geometry and distribution were defined according to potential failure mechanism.

After the selection of the elementary macro-elements, the collapse multiplier (α_0) was evaluated by applying the Theorem of Virtual Work (TVW) on the rigid-body system, where the seismic load was assumed as an overturning force while the gravity load was assumed as a stabilization load [7]:

$$\alpha_0\left(\sum_{i=1}^{n} P_i\delta_{x,i} + \sum_{j=n+1}^{n+m} P_j\delta_{x,j}\right) - \sum_{i=1}^{n} P_i\delta_{y,i} - \sum_{h=1}^{l} F_h\delta_h = W_{fe} \quad (1)$$

where P_i denotes the own weight of each element composing the kinematic chain; P_j (or F_h) is the weigh (or the load) transmitted at the selected macro-element by confining wall or roof; $\delta_{x,i}$ and $\delta_{x,j}$ are the virtual displacements of the elementary blocks centroids; $\delta_{y,i}$ and δ_h are the virtual displacements of the point of application of P_j and F_h respectively.

After the evaluation of the collapse multiplier (α_0), the corresponding seismic spectral acceleration α_0^* is evaluated by:

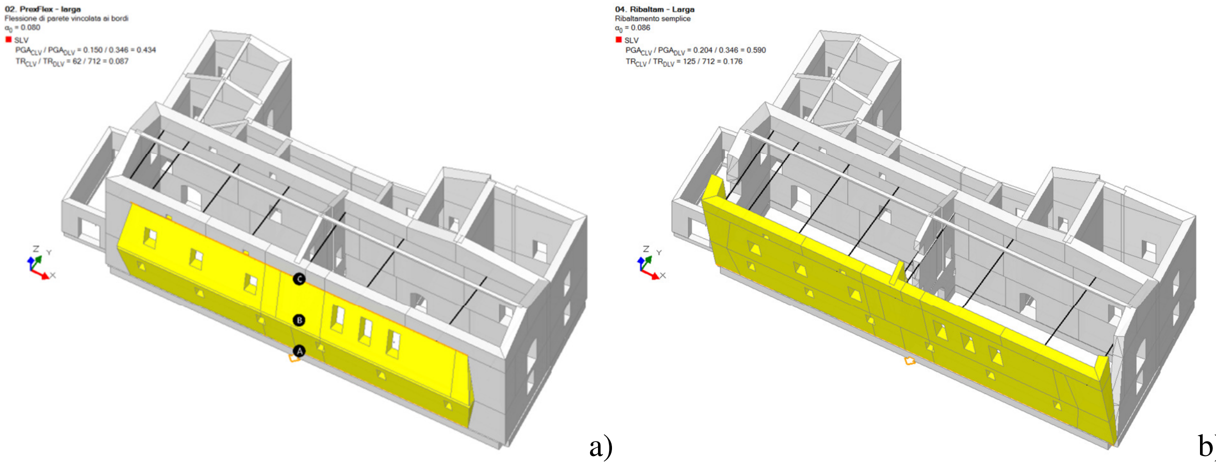
$$a_0^* = \frac{\alpha_0 \sum_{i=1}^{n+m} P_i}{M^* CF} \quad (2)$$

where M^* denotes the effective mass and CF is the confidence factor (assumed equal to 1.12 [7]). The respect of the analyzed limit state is assured by the respect of the following inequality:

$$a_0^* \geq \frac{a_g S}{q} \quad (3)$$

where $a_g S$ is the design elastic acceleration spectral ordinate, q is the behavior factor (assumed equal to 2.0 [9]); Z is the height, with respect to the ground, of the centroid of all inertial masses involved in the mechanism and H is the total height of the building.

Several possible out-of-plane mechanisms (first type mechanisms) which best represent the surveyed damage patterns were taken into account. In particular the paper reports the results on the mechanisms that could be activated on the lateral bearing walls (Figure 6) and on the main façade (Figure 7).



a) b)

Figure 6: Lateral wall overturning: a) free-standing wall (Mechanism 01); b) wall restrained at the top (Mechanism 02).

Figure 7: Main façade overturning: free-standing wall (Mechanism 03).

The evaluation of the vulnerability indexes upon Eq. (1)-(3) was been made by using an own code. Results from the macro-element approach, summarized in Table 1, agree with the results obtained with the pushover analysis via equivalent frame approach.

Title 1	Collapse multiplier (α_0)	Seismic spectral acceleration (a_0^*)	Seismic demand	Check
Mechanism 01 (lateral wall)	0.080	0.075g	0.173g	NO
Mechanism 02 (lateral wall)	0.086	0.102g	0.173g	NO
Mechanism 03 (main façade)	0.052	0.062g	0.173g	NO

Table 1: Vulnerability indexes, kinematic analysis results.

The critical behavior of the building is activated when the seismic load is acting on the transversal direction (y-direction) of the building. This is mainly due to the absence of a buttress capable to counteract the seismic action. Despite the good quality of the connection between the walls and the roof, this does not produce an effort in this direction. Other local collapses could arise in local areas of the Museum (main façade overturning) that however do not involve the entire structure.

2.3 Structural elements scale: FEM based analysis

A comprehensive reliable risk assessment for historical monuments should regard not only their architectural and structural components, but also movable (paintings, statues, libraries) and unmovable (frescos, stucco-works, pinnacles, battlements, banisters, balconies) artistic assets. This is an unexplored and critical issue, mainly owing to the evidence that the seismic damage of artistic assets is often related to small damage levels of the load bearing structure. The seismic protection of artistic assets thus requires the identification of the main damage mechanisms and, consequently, the development of suitable modelling strategies to evaluate their vulnerability and, if needed, design reliable and effective interventions.

Within this context, after completing the knowledge phase at macro-scale level, it was considered crucial to have, at the meso-scale level, a precise evaluation of the seismic vulnerability of the wall containing the most important artistic asset of the Museum: the Piero Della Francesca's fresco called "Resurrection". To this end the results of the finite element method (FEM) tests conducted on a detailed 3D model of the wall were employed in order to provide an interpretation of the manifested crack pattern on this important artistic asset.

A complete three-dimensional FE model of the wall was thus created using the commercial software ANSYS. The geometry of the wall was defined as accurately as possible, according to available drawings, photographs, and the photogrammetric and video-endoscopic survey. The numerical model has been built to accurately reproduce the geometry of the structure, focusing on the variations in the wall thickness, in the geometrical and structural irregularities and on the wall connections. In order to perform this operation the geometry of the wall was firstly reconstructed by means of CAD tools, next the volumes were imported and modelled by means of Solid65 elements (three-dimensional eight noded hexaedron isoparametric elements). The complete finite element model (FEM) is shown in Figure 8: it consists of 163952 nodes and 112128 elements, corresponding to 394992 degrees of freedom.

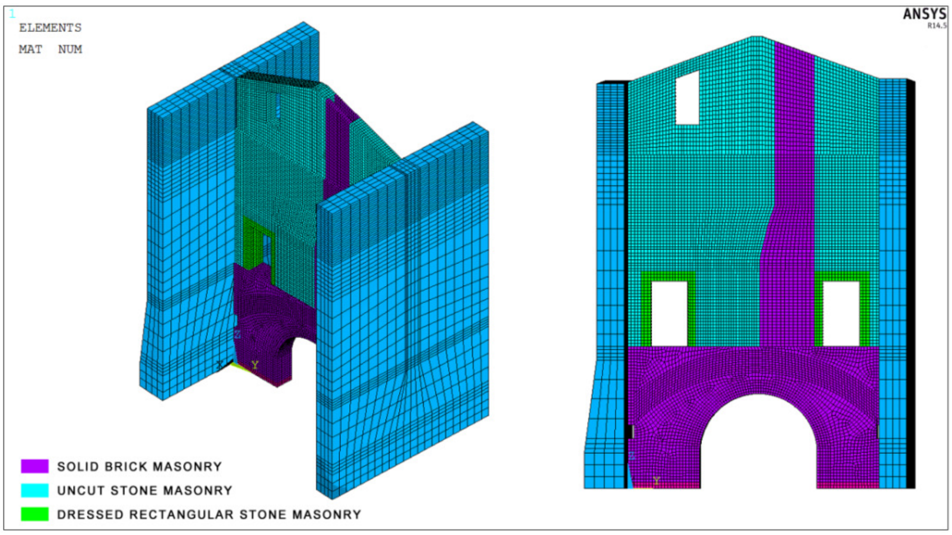


Figure 8: FEM model: mesh discretization and material map.

A physically non-linear model was adopted for masonry. For the sake of simplicity, both elastic and perfectly plastic behavior were adopted (a) with infinite ductility and (b) with isotropic behavior hypotheses for the material. It must be underlined that such assumptions do not take into account two important aspects of masonry at failure, the first related to the damaging behavior of joints, the latter related to the well-known higher masonry horizontal in-plane strength with respect to the vertical one, essentially due to brick staggering. Nevertheless, these aspects cannot be taken into account easily. On the other hand, the assumption of a zero tensile strength with frictional behavior is widely accepted [18] for the analysis of historical constructions. Within this approach a Mohr–Coulomb type failure criterion with tension cut-off type behavior was assumed for masonry. This failure criterion, initially adopted for concrete, accounts for both cracking and crushing failure modes through a smeared model. In particular, the masonry brittle behavior was here defined by means of only two constants: f_t (uniaxial tensile strength) and f_c (uniaxial compressive strength). Also, to increase the reliability of the proposed FEM, unilateral contact interfaces were used for the simulation of the joints between elements belonging to different periods in the history of the structure. The modeling of these joints necessitates the use of specific flexible/flexible contact elements. Specifically, for this application, an unilateral contact law was applied in the normal direction of each interface, indicating that no tension forces can be transmitted in this direction and a gap may appear if the stresses become zero. For the behavior in the tangential direction it was taken into account that sliding may or may not occur, by the usage of the Coulomb friction model with a friction coefficient equal to 0.5 [19].

Because of the artistic and historical value of the wall and in order to reduce the impact on conservation, in the present case in-situ tests could not be used in order to estimate the mechanical properties of the different types of brickwork forming the structure. Therefore, assumptions on the masonry material properties were made taking into account literature data for historic masonry (i.e. buildings with similar masonry texture [20]) and normative reference [10] deriving conservative values from these experiences. Table 2 report the selected values needed for the definition of the nonlinear model parameters with respect to the wall's masonry typologies.

Title 1	Solid brick masonry	Uncut stone masonry	Dressed rectangular stone masonry
Elastic modulus (N/mm^2)	1500	1230	2800
Poisson's ratio	0.25	0.25	0.25
Specific weight (kg/m^3)	1800	2000	2200
Uniaxial compressive strength (N/mm^2)	2.40	2.00	7.00
Uniaxial tensile strength (N/mm^2)	0.09	0.05	0.16

Table 2: Constitutive parameters of the materials adopted for nonlinear FEM analyses.

At this scale level, an equivalent static analysis was conducted, in which the structure was subjected to both vertical actions, i.e. self-weight and live loads, and seismic horizontal actions directly applied at the floor levels. More in details, the seismic action is modelled by a set of horizontal forces (acting along the x direction) which are applied with the distribution shown in Figure 9, where F_i represents the ultimate shear at the ith floor (with i = 1 to 3, being 3 the total number of floors) obtained from the pushover analyses at global scale. As for the boundary conditions, the FEM is perfectly constrained to the ground; no kinematic constraint is imposed between the wall and the surrounding structure.

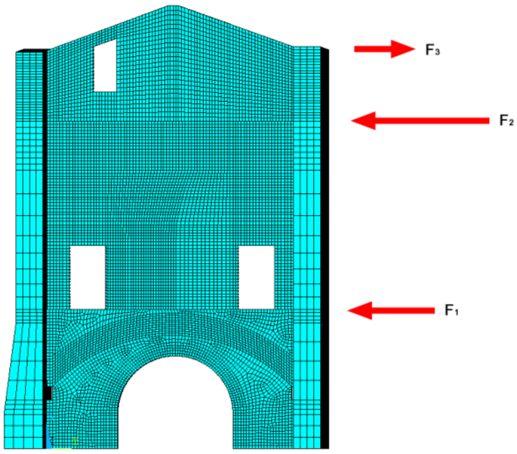


Figure 9: Application of the seismic action at every level.

Figure 10 shows the cracking pattern assessed at the end of the finite element analysis on the bearing wall. Cracking does not affect the structure as a whole, but mainly the southern (with diagonal and vertical cracks) and upper part of the wall close to the chimney (with vertical cracks between the inner and the outer leaf of the wall).

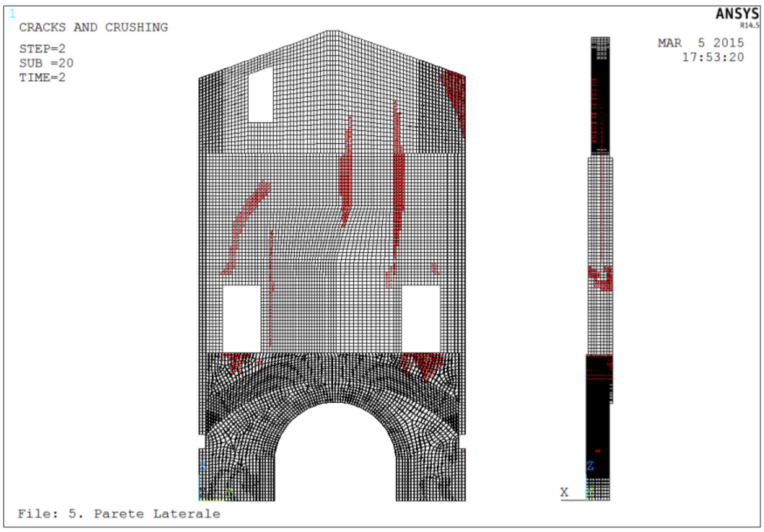


Figure 10: FEM model: cracking pattern.

As a general remark, it is possible to observe how the calculated cracking pattern matches the real crack distribution observed in the wall quite well. Cracks are in fact located in the neighborhood of the of the surveyed crack pattern, indicating that the performed analysis, at least from a qualitatively point of view, seems to explain most of the damage of the structure.

3 CONCLUSIONS

The paper approaches an account of the results of diagnostic analyses conducted by the authors on a specific monumental masonry building: the Civic Museum of Sansepolcro.

Assuming that preservation of monumental structures requires a detailed knowledge of the seismic behavior in order to minimize the interventions, for the sake of conservation, a performance based multi-scale approach has been adopted in the assessment procedure. In particular, the use of a 3D pushover analysis (equivalent frame approach) allowed, on the one hand, to improve the evaluation carried out through the nonlinear kinematic analysis (rigid macro-block model) and, on the other one, to calibrate the use of a refined 3D detailed model (finite element analysis with nonlinear constitutive laws for masonry) for catching the response of the wall containing the most important artistic asset of the Museum: the Piero Della Francesca's fresco called "Resurrection".

Results highlight some deficiencies related to the capability of the structural complex to fulfill the performance levels related to the conservation of the fresco and the safety of people who daily use the Museum. As is possible to notice, the simplified scheme of limit analysis (macro-element approach), in agreement with the results obtained with the pushover analysis (equivalent frame approach), points out that the strength of the building under transversal direction (y-direction) is insufficient and this lack of resistance is related to out-of-plane collapses of single panels. Such partial collapses are supposed to coincide with the failure of the entire complex, even if the inner structure of the building remains standing, since the building will be left in an irrecoverable state. On the other hand, the finite element analysis allow to obtain a screening of the most vulnerable parts of the wall containing the fresco, at the maximum seismic force which is borne by the structure, i.e. at the level of the seismic action which activates the first local failure mechanism. However, to refine the model other aspects remain to be investigated. In this perspective, the answer to the question, "What strength earthquake should be assumed as a reference for the fresco?" should be defined. To this end, numerical analyses are scheduled, to suggest that measures should be taken sooner or later to provide adequate guarantees of protection and safety. Without interventions aimed at eliminating the causes, the cracking situation observed is, in fact, inevitably destined to worsen with the occurrence of future seismic events.

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