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VIBRATION ANALYSIS OF HIGH-SPEED RAIL SYSTEM USING THE MOVING ELEMENT METHOD

Reda Mezeh[1,a], Marwan Sadek[1,b], Fadi Hage Chehade[2], Dalia ABDELMASSIH[3], Nisrine MAKHOUL[4] and Ahmad YOUNES[5]

[1] Laboratory of Civil Engineering and geoEnvironment (LGCgE), University of Lille 1- Polytech Lille, Villeneuve d'Ascq 59655, France.
[a] reda.mezeh@ed.univ-lille1.fr
[b] marwan.sadek@polytech-lille.fr

[2]Lebanese University, Doctoral School of Science & Technology- PRASE, Beirut, Lebanon.
fchehade@ul.edu.lb

[3]Lebanese University, Doctoral School of Science & Technology- PRASE, Beirut, Lebanon.
daliaabdelmassih@ul.edu.lb

[4]University of Balamand, PO Box 10, Triploi Lebanon
Nisrine.Makhoul@balamand.edu.lb

[5]Lebanese University, University Institute of Technology, Saida, Lebanon.
ayounes@ul.edu.lb

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Abstract. *The paper proposes to extend the modeling of the rolling vibration to that of the impact vibration caused by discrete irregularities on wheels and / or rail. It presents a time domain formulation of the vertical interaction between the wheel and the rail, for the prediction of the impact vibrations.*

The problem of interaction wheel / rail is solved by the moving element method. The input data of the interaction model, the contact model wheel / rail and vibrational pattern of the wheel are firstly formulated. It allows the definition of a relative vertical movement between the wheel and the rail, called relative roughness.

The proposed computational scheme is applied to investigate the dynamic response of high-speed rail system. A parametric study is carried out function of the main influential parameters mainly the traveling train speed and the severity of track irregularity. The results of numerical simulations show the efficiency of the proposed method and its main advantage for the analysis of the generated vibrations.

1 INTRODUCTION

Railway noise and the induced vibrations transmitted into the ground are considered a major problem, in view of their influence on the conformity of the neighboring population. Several authors have been interested in dealing with a problem of interaction between train and track to quantify the induced noise level and then compare it with the acceptable level that is mentioned in the technical regulation. In this context, in France the Noise Act 1992 imposes the acceptable noise level limits for residents; at the European scale, the maximum noise levels are limited at source by the SPI.

The finite element method FEM is a well-established numerical method for solving complex problems including the case of moving loads. For example, Thambiratnam et al. [1] presented a simple finite element method for the dynamic analysis of beams on elastic foundation subjected to a concentrated moving load.

In dealing with moving load problems, the classical FEM encounters difficulty when the moving load travels in high speed or/and the dynamic excitation has a small wavelength, these difficulties can be overcome by refinement of the mesh size, but at the expense of significant increase in computational time. Another problem will be encountered, when the moving load approaches the boundary of the finite domain.

In an attempt to overcome the complication encountered by FEM, Krenk et al. [2] proposed the use of the FEM in convected coordinates to obtain the response of an elastic half-space subject to a moving load. Koh et al. [14] adopted the idea of convected coordinates for solving train-track problems, and named the numerical algorithm as moving element method (MEM). This method was subsequently applied to the analysis of moving loads on a viscoelastic half-space [3, 4, 5]. Anderson [5] dealt with the problem of a beam loaded by a variable harmonic loading, adopting an analytical formulation.

In this paper, the particular convection problem of a moving load in infinite Euler beam resting on a Kelvin foundation which simulates a train-track interaction problem, has been treated in a new way. The main idea of this approach is to use the classic formulation of the FEM in a fixed reference. By adopting a suitable temporal mesh, the numerical system that results has been solved by an adaptive algorithm, ie the geometric configuration of the dynamic system at the beginning of a time step was found using the system configuration in the step which proceeds, by using a suitable interpolation functions. The disturbing effect of the negative numerical damping resulting from the use of the Galerkin formulation in the MEM which may be in the case of high velocity the cause of a numerical divergence will be treated by this approach, without the need to introducing an amount of physical damping. The performance and the tolerance of this method have been tested for the excitation in the form of relative roughness between rail and wheel.

2 FORMULATION AND METHODOLOGY

The High speed rail-HSR- system is comprised of a train traversing over a rail beam in the positive x- direction. The origin of the fixed axis x is located on the centre of the beam such that the train is at x=0 when t=0. The constant passing velocity of the train is denoted by V. The rolling surface of the rail is considered to have some imperfections due to rail irregularity. For simplicity, we assume no interaction between the two rails of the railway track, then the half of the lower structure subjected to half weight of the train was studied. In a typical analysis, the two parts are connected as a coupled system that accounts for interaction between the wheel and the rail.

2.1. Train model

The train is modeled as a moving spring-mass model, which is a system of three rigid components, namely the car body, bogie and wheel-set; as shown in figure 1. The car body and the bogie of mass m_1 and m_2 respectively, are connected through the secondary suspension system which is modeled as a spring-damper unit. The inter-connection between the bogie and the wheel set of mass m_3 occurs through the primary suspension system. The three vertical degrees of freedom of the car body, bogie and wheel set are denoted by u_1, u_2 and u_3 respectively. The contact between the wheel and the rail beam is modeled by the contact force F_c.

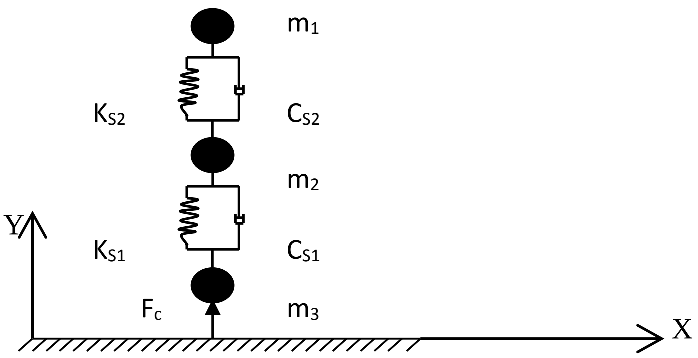

Figure 1 Moving sprung mass model

The governing equations for the train model may be written as:

$$
\begin{aligned}
&m_1\ddot{u}_1 + K_{s2}(u_1 - u_2) + C_{s2}(\dot{u}_1 - \dot{u}_2) = -m_1 g \\
&m_2\ddot{u}_2 + K_{s1}(u_2 - u_3) + C_{s1}(\dot{u}_2 - \dot{u}_3) - K_{s2}(u_1 - u_2) - C_{s2}(\dot{u}_1 - \dot{u}_2) = -m_2 g \\
&m_3\ddot{u}_3 - K_{s1}(u_2 - u_3) - C_{s1}(\dot{u}_2 - \dot{u}_3) = -m_3 g + F_c
\end{aligned}
\qquad (1)
$$

2.2. Equation of motion in the fixed co-ordinates

In this study we assume the Winkler hypothesis in which the foundation's reactive pressure is proportional to the deflection of the rail beam [6].

The railway track is modeled as an Euler-Bernoulli beam with constant bending stiffness EI and mass per unit length $\overline{m}$ supported by a viscoelastic layer comprising of vertical springs $\overline{K}$ and dashpots $\overline{C}$. The vertical displacement and rotation of the track are denoted by y and θ respectively. The governing equation of the rail beam subjected to a moving train load in the fixed co-ordinates system (x,y) is given by:

$$
EI\frac{\partial^4 y}{\partial x^4} + \overline{K}y + \overline{C}\frac{\partial y}{\partial t} + \overline{m}\frac{\partial^2 y}{\partial t^2} = -F_c\delta(x - s) \qquad (2)
$$

Where t is the time, s is the distance traveled by the train at instant t; δ is the Dirac-delta function.

The railway beam is now truncated and discretized into so-called moving elements as shown in figure 2. For a typical moving element, the governing differential equation is multiplied by an arbitrary weighting function and integrated over the element length. By adopting Galerkin's approach, the element mass, damping and stiffness matrices can be obtain as given below

$$M_e = \bar{m}\int_0^L N^T N dr$$
$$C_e = \bar{C}\int_0^L N^T N dr \quad (3)$$
$$K_e = EI\int_0^L N^T_{,rr} N_{,rr} dr + \bar{K}\int_0^L N^T N dr$$

Where N refers to the vector of shape functions, which are represented by Hermitian cubic polynomials and r, is the local element reference. The subscript r denotes partial derivative with respect to r.

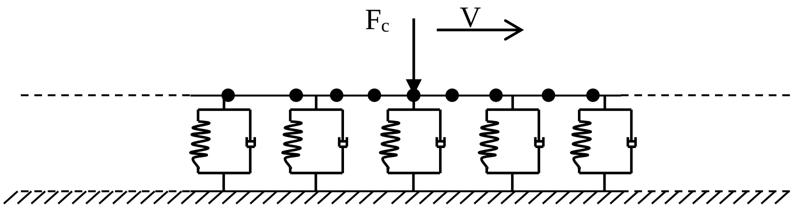

Figure 2 Discretization of the railway beam into moving elements

After assembling the individual element matrices with respect to their positions in the railway track and considering the train model as well as the wheel-rail interaction, the equation of motion for the combined train-track system can be written as

$$M\ddot{Z} + C\dot{Z} + KZ = P \quad (4)$$

Where Z is the global displacement vector of the train-track system; M, C, and K the global mass, damping and stiffness matrices, respectively; and P the global external load vector.

2.3. Wheel-rail contact model

In this study, we use the simplified approach based on a linearized Hertz contact model [4], in which it is assumed that the pseudo-static reaction force at the contact point equals the self-weight of the upper structure W [9]. This model allows us to study the phenomenon of wheel jumping, which takes place in the case of the absence of contact between wheel and rail.
The linearized contact force F_c is given by:

$$F_c = \begin{cases} K_L(y_{rc} + y_t - u_w), (y_{rc} + y_t - u_w) \geq 0 \\ 0, \quad\quad\quad\quad\quad\quad , (y_{rc} + y_t - u_w) < 0 \end{cases} \quad (5)$$

Where K_L is the linearized Hertzian spring constant [4] computed as follows:

$$K_L = \sqrt[3]{\frac{3E^2W\sqrt{R_w R_r}}{2(1-\nu^2)^2}} \quad (6)$$

Where y_{rc}, y_t and u_w denote respectively, the displacement of the track at contact point, track surface irregularity and the displacement of the wheel in contact with the track; R_w and R_r denote the radii of the wheel and railhead, respectively; E denotes the elastic modulus of the material; and ν the Poisson's ratio. Track irregularity is the major cause of dynamic loads generated by a moving train. In this paper, the vertical track irregularity profile y_t is assumed as a sinusoidal function [4, 7, 10] which can be written as:

$$y_t = a_t \sin(\frac{2\pi x}{\chi_t}) \quad (7)$$

Where a_t represent the amplitude and λ_t the wave length of the track irregularity.

2.4. Moving element method

A new approach of the moving element method MEM is proposed for solving this dynamic problem; the principle of this method is to solve the problem of a moving load, on a small time step by considering the problem as a problem with concentrated fixed load with variable amplitude over the studied time interval. Using the proposed interpolation functions, the initial condition of the next time step is deduced. The formulation of this numerical method is based on that of the finite element method, where the matrices are provided in the paragraph 2.2.This method is efficient since we can find the dynamic response of the system when moving a small distance, saying up to a portion of a millimeter; which is very expensive and sometimes can't be solved by the classical FEM, which requires the creation of a node in every point of application of a moving charge. In our model, we adopted a time-space coupled mesh chosen precisely to minimize on the one hand the boundary effect, and the disruption of the dynamic response due to a possible reflection of the induced waves and in the other hand the mesh size effect which is found to have a significant influence on the system dynamic response. Figure 3 shows a schematic drawing explaining briefly the proposed MEM algorithm.

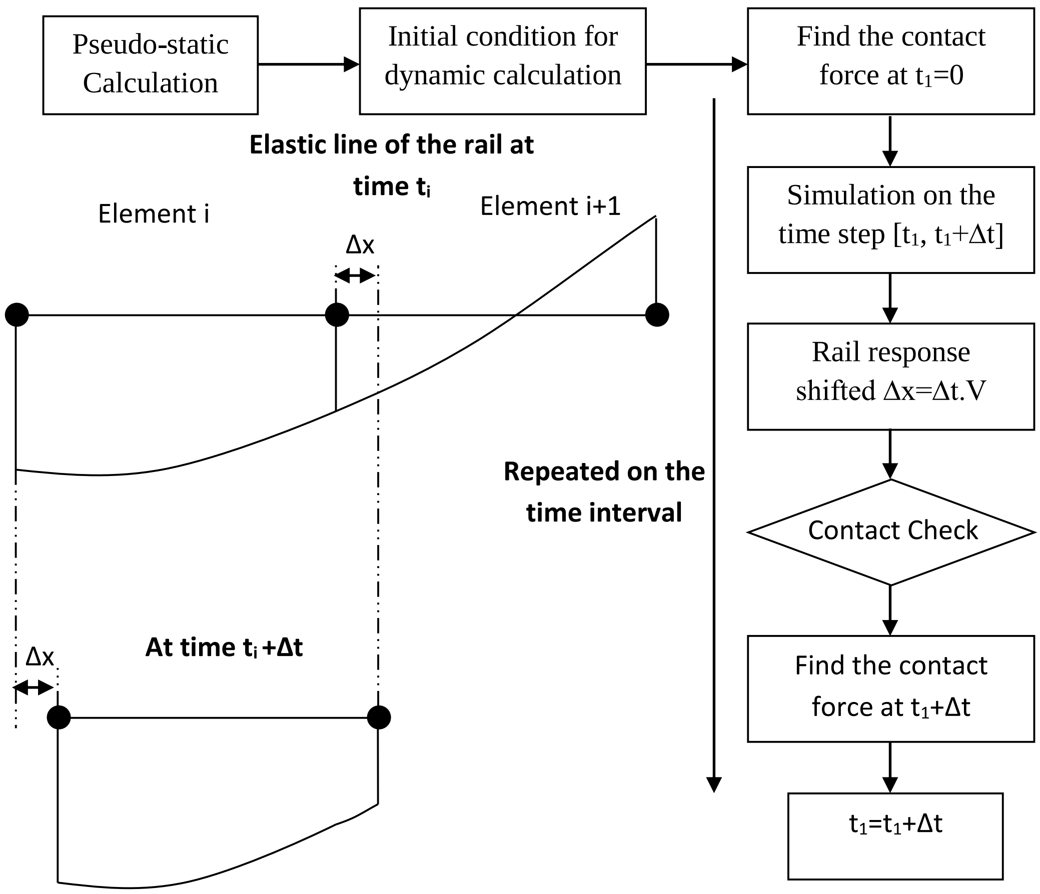

Figure 3 Schematic algorithm of the MEM

3 VALIDATION OF THE MEM

To investigate the usefulness and the tolerance of the proposed method, we will compare in this part the dynamic response of a beam resting on a viscoelastic foundation which is subjected to a concentrated moving harmonic force, which travels at velocity V as illustrated in figure 4, found analytically to the results provided by the numerical method.

Based on the analytical method adopted by Anderson et al. [5], the displacement field can be written in the moving coordinates (X,Y), where the origin is attached to the point of application of load; as follow:

$$U(X,t)=\begin{cases} Ae^{-\beta_1 X+i(\alpha_1 X-\omega t)}+Be^{-\beta_2 X+i(\alpha_2 X-\omega t)}, & X\geq 0 \\ Ce^{-\beta_3 X+i(\alpha_3 X-\omega t)}+De^{-\beta_4 X+i(\alpha_4 X-\omega t)}, & X<0 \end{cases} \tag{8}$$

Where A, B, C, and D are constants to be determined using continuity conditions of displacement field and solicitations, α_j and β_j are the real part and the imaginary part of the wavenumbers; α_j represents propagation and β_j represents attenuation of the jth wave component.

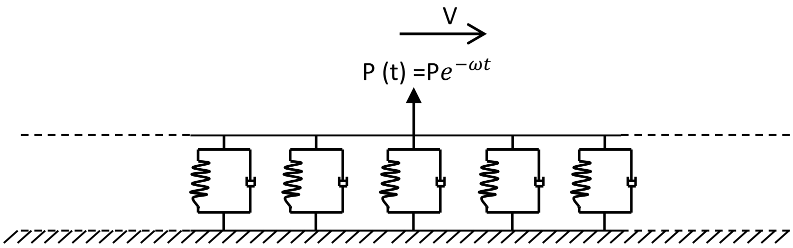

Figure 4 Rail-beam subjected to harmonic excitation

The validations are realized on linear structures for which the analytical solution may be found for all velocities and frequencies of load. We use here in the analysis a European high-speed rail which has the following properties: S=76.86 cm^2, I=3060 cm^4, ρ=7850 Kg/m^3, E=2×10^{11} N/m^2, m=60.34 Kg/m, EI=6.12×10^6 N.m^2, the stiffness of the Winkler foundation used is 1.6×10^7 N/m [16].

In order to choice the reasonable parameters in analysis; we define the critical values of velocity, of frequency and of viscous damping [16] which depends only on the structural characters:

$$V_0=\sqrt[4]{\frac{EI\bar{K}}{\bar{m}^2}} \qquad \omega_0=\sqrt{\frac{\bar{K}}{\bar{m}}} \qquad C_0=2\sqrt{\bar{m}\bar{K}} \tag{9}$$

To test the capability of the proposed approach of the MEM to solve a problem where the load exciting the beam is variable in both time and space, a finite beam section of the length L is modeled, and numerical integration is performed over the time interval T_I. The analysis is carried for single-frequency harmonic excitation at various combinations of loading frequency and convection velocity. The displacement time series on the central node and the beam displacement field after 15 periods are displayed. The numerical results (the dotted line) are plotted against the corresponding analytical solution (the continuous line) as shown in figures 5 and 6. Note that the analytical solution are stationary, whereas the response provided by the MEM gives rise to a transitional part, which will end when the loading excite the firsts eigen frequencies of the beam. In all cases, the numerical results after several periods are seen to be nearly identical to the analytical results. Note that the proposed method has no limitation on the convection velocity and load pulsation, in other words there is no negative numerical damping disrupting the dynamic response. Then a difference with the classical MEM appears that there is no need to insert an additional physical damping to compensate the negative numerical damping.

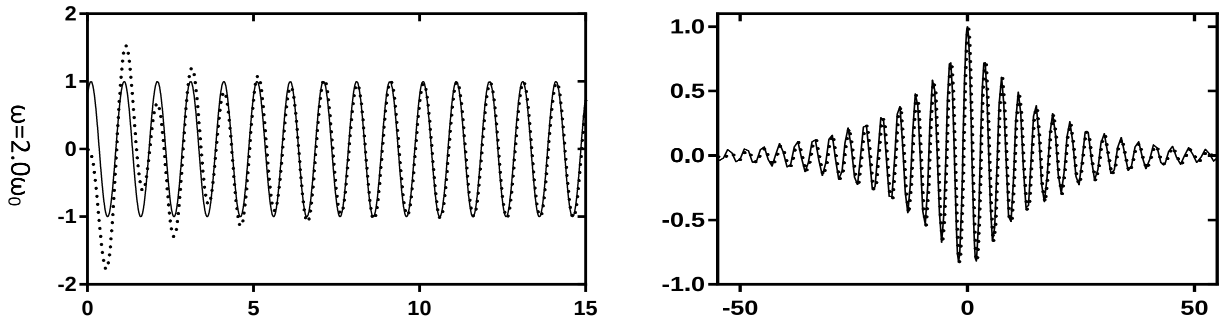

Figure 5 Numerical (dotted) versus analytical (continuous) results, V=0

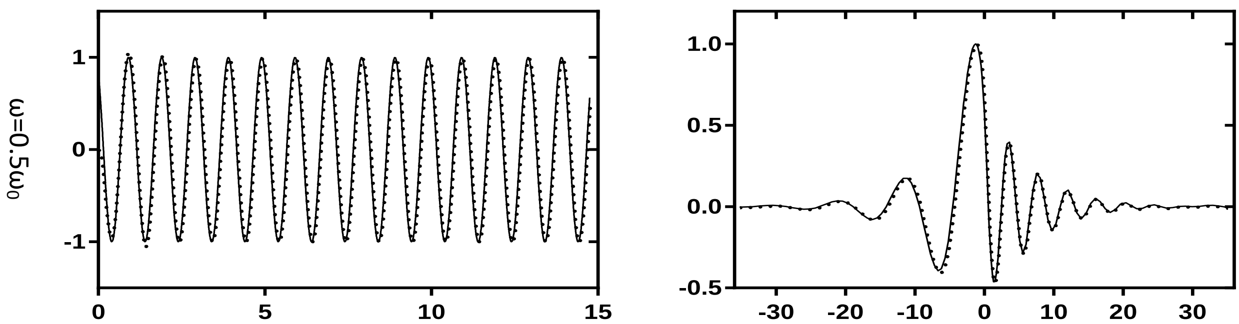

Figure 6 Numerical (dotted) versus analytical (continuous) results, $V=V_0$

m_1	3500 Kg	m_2	250 Kg	m_3	350 Kg
K_{s1}	$1.26*10^6$ Nm^{-1}	K_{s2}	$1.41*10^5$ Nm^{-1}		
C_{s1}	$7.1*10^3$ Nsm^{-1}	C_{s2}	$8.87*10^3$ Nsm^{-1}		

Table 1 Parameters for train model

4 NUMERICAL RESULTS

In this section, the dynamic behavior of a train traveling along a railway track is simulated using the moving sprung-mass model. The MEM model adopted in the study comprises of a truncated railway track of 70 m length discretized non-uniformly with elements ranging from a coarse 1 m to a more refined 0.2 m size. The equations of motion are solved using Runge-Kutta method employing a variable time step. Values of parameters related to the properties of train are summarized in table 1.

4.1. Effect of passing speed

As the response of high-speed rails system strongly depends on the severity of track irregularity, it would be useful to investigate the effects of irregularity wavelengths and train speeds on the response of the HSRs. The amplitude of all track irregularities considered in this investigation is taken to be 0.5 mm. Figure 7 shows the variation of dynamic amplification factor DAF in wheel-rail contact force against train-speed for various track irregularity wavelength.

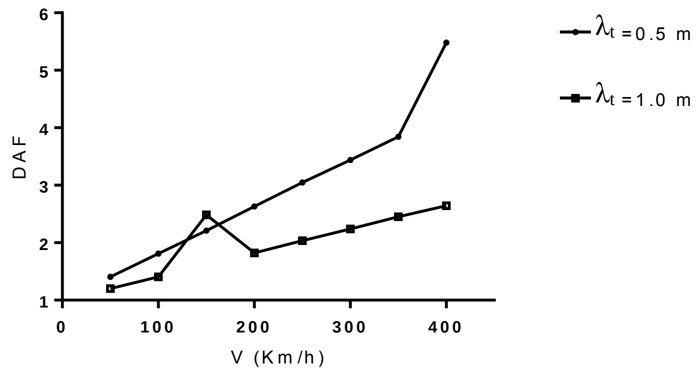

Figure 7 DAF against train speed V for various track irregularity wavelength

The general trend of the curves is increasing with respect to the passing speed, it can be seen that the DAF is generally close to 1.0 for small values of V. Conversely, when the wavelength is small resulting in a more severe track irregularity condition.

4.2. Effect of track irregularity amplitude

The effects of train speed and track irregularity amplitude are investigated in this part. The wavelength of all track irregularity is taken to be 0.5 m. Figure 8 shows the variation of dynamic amplification factor (DAF) in wheel-rail contact force against track irregularity amplitude for various train speeds.

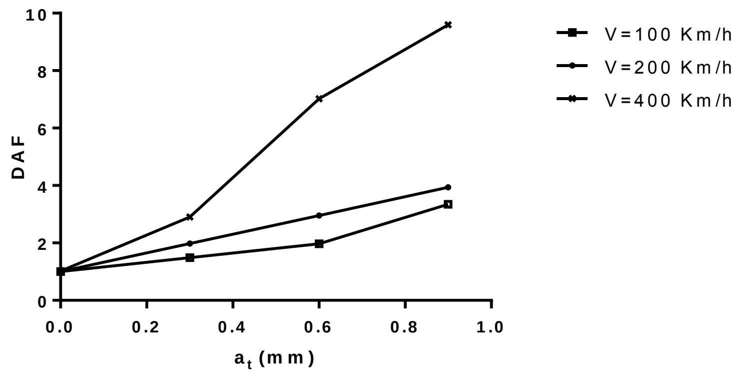

Figure 8 DAF against train irregularity amplitude for various passing speed

For the perfectly smooth (a_t=0 mm) track, the DAF is found be 1 as to be expected in view that there is no dynamic load. The results in Fig. 8 also show that when the amplitude of track irregularity and/or train speed increase, the DAF is increased.

5 CONCLUSION

The paper has presented a numerical model for the simulation of dynamic train and track interactions at the wheel/rail interface based on adaptive form of the FEM. The computational

model adopts the linearized contact theory of Hertz to account for the contact between the wheel and rail. Wheel/rail irregularities are the common source that creates dynamic disturbances to both the vehicle and the track system. It has been demonstrated that the model proposed in the paper is an efficient research tool to investigate the dynamic effects caused by rail defect, which are damaging to the vehicle and track components. The results obtained using the MEM with the proposed computational procedure to account for moving harmonic load are found to compare well with available analytical solutions in the literature.
In particular, the dynamic response of the track/train system has been shown to dramatically increase with an increase in the train speed. The relationship between the vehicle components, namely the unsprung mass and the dynamic wheel/rail load, can be quantified using the established model.

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