

A MODEL FOR CARBON AND STAINLESS STEEL REINFORCING BARS INCLUDING INELASTIC BUCKLING FOR EVALUATION OF CAPACITY OF EXISTING STRUCTURES

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Abstract. *Proper steel model for reinforcing bar incorporating inelastic buckling is essential for precise seismic analysis of reinforced concrete structures. Steel model without considering inelastic buckling will obviously overestimate the corresponding stress if the rebar is under cyclic loading and the inelastic buckling emerges in compressive loading. In this paper, a modified version of the Monti-Nuti steel model that includes the inelastic buckling for reinforcing bar is proposed. This modified model considers the effect of the yielding stress on the critical slenderness and the behavioral anisotropy in tension and compression of some type of stainless steel rebars. A parameter identification is carried out to calibrate the main parameters of the modified model. The identified parameters are robust. The effectiveness of the modified model is verified comparing the numerical stress-strain curves of carbon steel and stainless steel rebar with the experimental ones.*

1 INTRODUCTION

There are plenty of existing reinforced concrete (r.c.) structures built more than thirty years ago when the effects of the seismic actions on the r.c. structures were not sufficiently considered. The stirrup spacing in the column of those existing structures exceeds the maximum stirrup spacing limit in current seismic design codes. According to the investigations on damaged r.c. structures during the severe earthquake, the longitudinal rebars in the column showed lateral deformations due to the insufficient confinement of the transversal rebar, and this phenomenon is called as inelastic buckling. In order to estimate the seismic performance of existing r.c. structures, proper steel rebar model is essential. All the models could be divided into three classes: the implicit algebraic model, the explicit algebraic model and the differential model, and among which the algebraic models are widely used in the numerical analysis with the Finite Element Method (FEM). In the implicit model, stress is the independent variable, and strain is the corresponding dependent variable. (i.e. Ramberg-Osgood [1]). On the contrary, in the explicit model, the strain is the independent variable and the stress is the corresponding dependent variable. Different explicit models were proposed to describe the behavior of the steel rebars. Menegotto and Pinto [2] built up a widely applied model to describe the cyclic stress-strain relationship. Filippou, Bertero and Popov [3] put forward one model including the isotropic hardening base on the Menegotto-Pinto Model. Monti and Nuti [4] proposed the first model which could consider the buckling of the reinforcement based on a series of experimental tests on carbon steel rebar. Gomes and Appleton [5] built up one model describing the monotonic compressive skeleton curve based on the equilibrium of plastic mechanism of a buckled rebar. Dhakal and Maekawa [6] suggested the monotonic skeleton curves of reinforcement in tension and compression respectively. Also they presented formulas to update the tangent modulus at the reversal points in tension and compression, respectively. Then the parameter b (hardening ratio) in the Menegotto-Pinto model is modified and the skeleton curves are applied to describe the stress-strain relationship when the strain exceeds the attained maximum or minimum strain of previous half cycles. The Gomes-Appleton model and Dhakal-Maekawa model have been adopted in OpenSees which was developed by Kunnath [7]. Zong [8] proposed the “beam on spring” model which considers the stiffness of the confinement from the transverse stirrups. In this paper, the modified Monti-Nuti model incorporating the inelastic buckling for steel rebar with or without anisotropy is proposed. Through comparing with the experimental stress-strain curves, the effectiveness of this model is validated.

2 ORIGINAL MONTI-NUTI MODEL FOR CARBON STEEL REBAR

Consisting of four hardening rules, named as kinematic rule, isotropic rule, memory rule and saturation rule, the Monti-Nuti steel model was based on a series of experimental observations. Different rebars specimens (old steel Italian type FeB44k) with yield stress of 450 MPa, diameter $D=16, 20, 24$ mm, were tested by monotonic tensile, monotonic compressive and cyclic tests applying different deformations histories.

It is shown that the slenderness ratio L/D (L is the length between two subsequent transversal rebars and D is the rebar diameter) determines the stress-strain relationship. When the slenderness ratio $L/D=5$, the monotonic compressive curve coincides with the monotonic tensile curve. While the monotonic compressive curve diverges from the tensile ones as soon as it reaches the yield point when the slenderness ratio value is $L/D=11$. When the slenderness ratio L/D varies between 5 and 11, the compressive curve coincides with the tensile one until a given deformation value and then the compressive curve diverges from the tensile one. The rules to update the model parameters are described below.

2.1 In absence of buckling

In absence of buckling, the yielding stress is updated by Eq. (1):

$$\sigma_y^{n+1} = \sigma_y^1 \text{sign}(-\xi_p^n) + P\Delta\sigma_K^n + (1-P)\Delta\sigma_{IM}^n \quad (1)$$

Where $\Delta\sigma_K^n$ is the Kinematic Rule, $\Delta\sigma_{IM}^n$ is the Isotropic Rule mixed with Memory Rule.

The branch after yielding could be defined by the hardening ratio b . Moreover, it is found that b varies according to the summation of plastic excursions of previous branches. b is defined by Eq. (2) named as Saturation Rule.

$$b^{n+1} = b_0 e^{b_0 E \sum \gamma / (\sigma_y - \sigma_\infty)} \quad (2)$$

2.2 In presence of buckling

In case of buckling of reinforcement, the yielding stress is updated by Eq. (3).

$$\sigma_y^{n+1} = \sigma_y^1 \text{sign}(-\xi_p^n) + P\Delta\sigma_{KM,b}^n + (1-P)\Delta\sigma_I^n \quad (3)$$

Where $\Delta\sigma_{KM,b}^n$ is the Kinematic Rule combined with Memory Rule, $\Delta\sigma_I^n$ is the Isotropic Rule.

Concerning the tensile branch, b is defined by Eq. (2), while b is defined by Eq. (4) in compressive branch, named as Saturation Rule.

$$b^- = \begin{cases} b_0^+ e^{b_0^+ E \sum \gamma / (\sigma_y - \sigma_\infty)}, & \sum \gamma \leq \gamma_s \\ b_0^- e^{b_0^- E \sum \gamma / (\sigma_y - \sigma_\infty)}, & \sum \gamma > \gamma_s \end{cases} \quad (4)$$

3 IMPROVEMENT OF ORIGINAL MONTI-NUTI MODEL

The original Monti-Nuti steel model is updated by the authors to consider the effects of the yield stress value on the critical slenderness ratio and the anisotropic behavior of some stainless steel types in tension and compression.

3.1 Effect of yielding stress on critical slenderness

According to the experiments on the carbon steel rebars with yield stress equal to 450 MPa, Monti and Nuti suggested that the critical slenderness ratio (the length between two subsequent transversal rebars divided by diameter of the reinforcing bar) is 5. Dhakal [6] demonstrated that both the yield strength and the slenderness have effect on the critical slenderness and proved that the combined parameter $L/D\sqrt{f_y}$ determines the cyclic behaviors. Through a series of numerical experiments, it is illustrated that the rebars with different slenderness ratio L/D and different yield strengths could generate the identical cyclic stress-strain curves if the combined parameter $L/D\sqrt{f_y}$ is equal. This conclusion is verified by Zong [8] who built three dimensional finite element model of individual reinforcing bar and verified that identical stress-strain curves could be obtained for the same value of the combined parameter $\sqrt{f_y/420}(L/D)$ where 420 representing the average yield stress of commonly used steel rebar in structure and the unit is MPa.

Based on the experimental data of Monti and Nuti [4], the slenderness ratio of longitudinal rebar should be redefined as $\sqrt{f_y/450}(L/D)$. If $\sqrt{f_y/450}(L/D)$ exceeds 5, the buckling of longitu-

dinal rebar will emerge. It could be concluded that the greater the yield strength, the smaller the critical slenderness ratio L/D when the buckling turns up.

3.2 Anisotropy of rebar

For the stainless steel reinforcing bar produced according to earlier production specifications, the yield stresses in tension and in compression are different, and this phenomenon is named as anisotropy of stainless steel. Tests on the stainless steel rebar specimens AISI304 or 1.4301 [9] provided by Valbruna Italy, with slenderness ratio equaling 5 and 11, the monotonic tensile and compressive curves are shown in Fig. 1. It could be observed that the yield stress in tension is around 800 MPa, but the yield stress in compression is 700 MPa.

In order to improve the versatility of the model for reinforcing bar with different yield stresses in tension and in compression which is different from the original carbon steel rebar, three parameters were named as anisotropy coefficient α , critical slenderness coefficient β and yield stress strengthening coefficient γ , defined in Eq. (5).

$$\alpha = f_{yt} / f_{yc}, \quad \beta = f_{yc} / f_Y, \quad \gamma = f_{yt} / f_Y \quad (5)$$

where f_Y is the yield strength of the carbon steel rebar FeB44k tested by Monti and Nuti [4], f_{yt} is the yield strength of stainless steel rebar in tension, f_{yc} is the yield strength in compression.

For the stainless steel with different yield stresses in tension and in compression, the critical slenderness should depend on the yield stress in compression because the buckling of reinforcement emerges under compressive load. Thus, the combined parameter should be modified as $L/D\sqrt{f_{yc}}$, where f_{yc} is the yield stress of the reinforcement in compression.

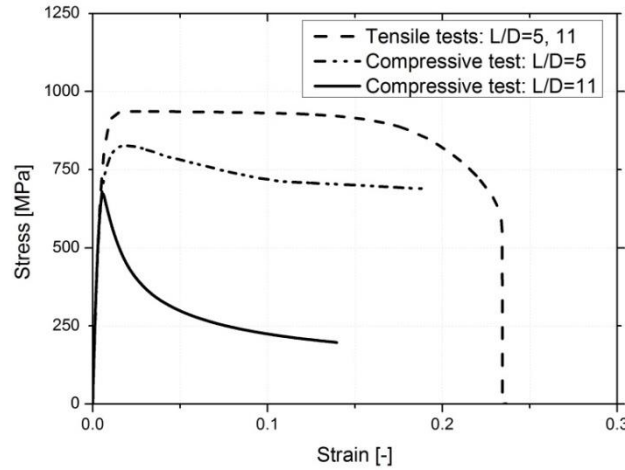


Fig. 1 Monotonic tests on stainless steel rebar

4 PARAMETER IDENTIFICATION

In order to do the parameter identification [10], the fitness function, as generally defined in Eq. (6), has the form:

$$f = \frac{\Delta Y}{Y} \quad (6)$$

Where ΔY represents the difference between the numerical curve and the experimental curve and is defined as: $\Delta Y = \sum_{i=1}^n y_{E,i}^2 - y_{N,i}^2$, and $y_{E,i}$ and $y_{N,i}$ are the stress on the experimental curve and numerical curve corresponding to the same x value; Y is the sum of the square of the experimental stresses, and is defined as: $Y = \sum_{i=1}^n y_{E,i}^2$.

If there are more than one experimental test, define the weight coefficient w_k to decide the contribution of each test, thus the fitness function could be defined as:

$$f = \sum_{k=1}^m w_k f_k \quad (7)$$

$$f_k = \frac{\Delta Y_k}{Y_k} \quad (8)$$

4.1 Framework of parameter identification

The general flowchart of the parameter identification method is shown in Fig. 2. It summarizes the main steps of the method used in this study by means of a MATLAB script that is not described here.

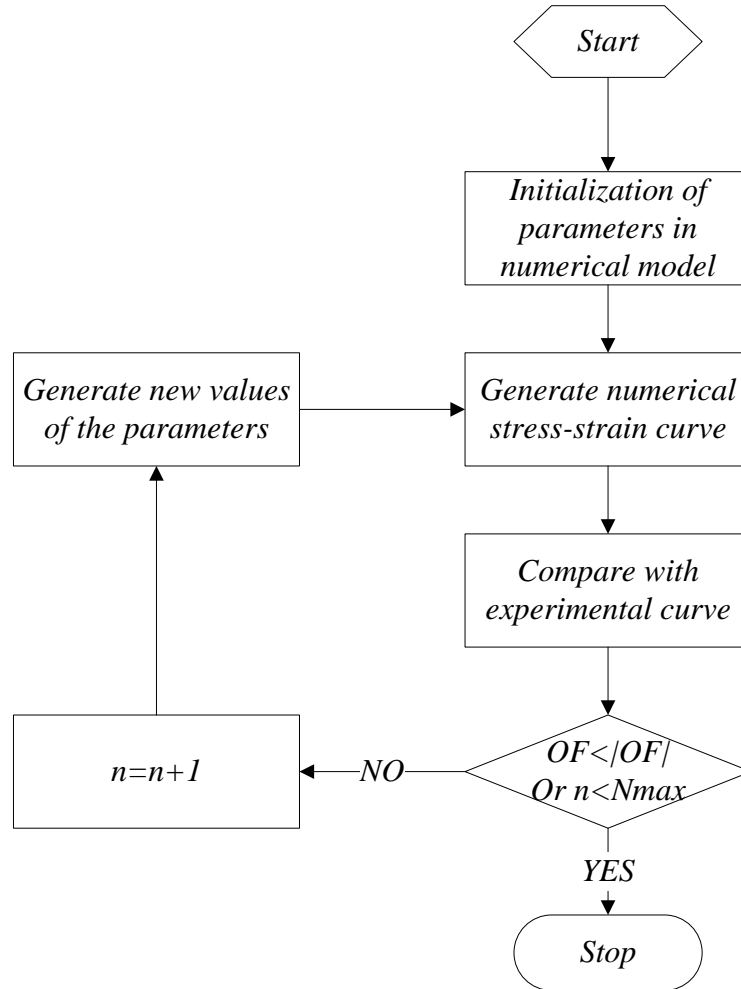


Fig. 2 General flowchart of parameter identification method

4.2 Proposed formulas to improve Monti-Nuti model

The anisotropy of the reinforcing bar results in different curve transitions in tension and in compression and then different parameters definitions to describe tensile and compressive curve branches. The parameters in the formula related to R were redefined in Eq. (9). The values of parameters R_0 , A_1 and A_2 are defined by Eqs. (10-14), considering the effects of anisotropy and the yield value. The anisotropy is considered by three parameters: anisotropy coefficient α , critical slenderness coefficient β and yield stress strengthening coefficient γ .

The formula for R in the original model is modified:

$$R = \begin{cases} R_0 - \frac{A_1^t \xi_{max}}{A_2^t + \xi_{max}}, & \text{in tensile branch} \\ R_0 - \frac{A_1^c \xi_{max}}{A_2^c + \xi_{max}}, & \text{in compressive branch} \end{cases} \quad (9)$$

where R_0 is the initial value of R , defined in Eq. (10); A_1^t , A_2^t are the coefficients under tensile loading defined in Eq. (11) and A_1^c , A_2^c are the coefficients under compressive loading defined in Eq. (12), which are related to the mechanical properties of reinforcing bar; in presence of buckling R_1^b is the lower bound, thus $R \geq R_1^b$, is defined in Eq. (13).

$$R_0 = \begin{cases} r_t R_1^b & (\text{in tension}) \\ r_c R_1^b & (\text{in compression}) \end{cases} \quad (10)$$

where r_t and r_c are the coefficients to calculate the initial value of R_0 for the tensile curve and compressive curve respectively.

$$A_1^t = \frac{1}{100} [\lambda_c \gamma^2 (\lambda - \lambda_c)] + \alpha, \quad A_2^t = -\frac{8\gamma}{10^4} (\lambda - 4\lambda_c) \quad (11)$$

$$A_1^c = \alpha A_1^t, \quad A_2^c = \frac{6\beta}{10^3} (\lambda - 2\beta) \quad (12)$$

$$R_1^b = 10 [L/D - \lambda_c] b^+, \quad \lambda_c = 5/\sqrt{\beta} \quad (13)$$

$$r_t = \frac{\gamma}{2}, \quad r_c = \frac{r_t}{\alpha} = \frac{\gamma}{2\alpha} \quad (14)$$

4.3 Robustness of Parameter Identification Method

The bounds of the parameters and the identified values for the experimental tests are shown in table 1 and table 2.

	XA1	XA2	XA3	XA	Mean	Standard Deviation	Difference (%) Standard Deviation/ Mean
A_1^t	2.524	2.765	2.473	2.759	2.587	0.127	4.9
A_2^t	0.007	0.020	0.004	0.010	0.010	0.007	67.2
A_1^c	2.949	2.254	2.911	2.403	2.705	0.319	11.8
A_2^c	0.002	0.001	0.012	0.001	0.005	0.005	99.3

Note: XA means the optimized values for XA1, XA2 and XA3, with $L/D=5$.

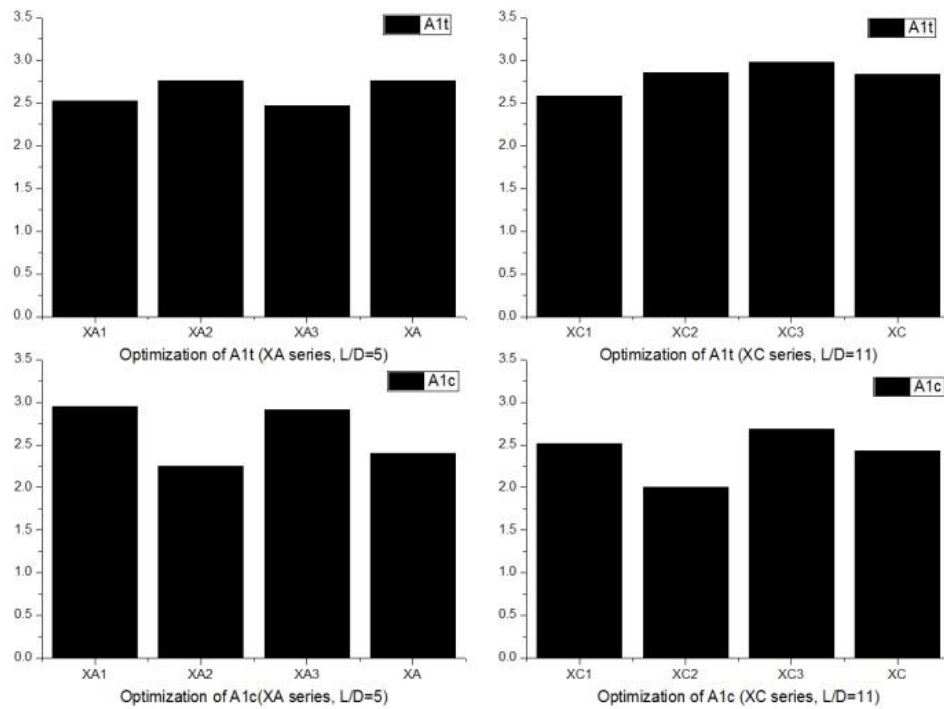
Table 1: Results of parameter identification for different experimental tests on rebar with $L/D=5$

	XC1	XC2	XC3	XC	Mean	Standard Deviation	Difference (%) Standard Deviation/ Mean
A_1^t	2.584	2.858	2.985	2.842	2.809	0.167	6.0
A_2^t	0.004	0.003	0.002	0.002	0.003	0.001	27.2
A_1^c	2.509	2.004	2.688	2.429	2.400	0.290	12.1
A_2^c	0.004	0.002	0.009	0.002	0.005	0.003	58.9

Note: XC means the optimized values for XC1, XC2 and XC3, with $L/D=11$.

Table 2: Results of parameter identification for different experimental tests on rebar with $L/D=11$

The robustness of the optimized could be observed from table 1 and table 2. A_1^t and A_1^c are sensitive to the numerical model, and the identified values of the parameters vary in a little range. But A_2^t and A_2^c are not sensitive to the model, thus the derivation of values have little effect on the numerical results. The column diagram of A_1^t and A_1^c is shown in Fig. 3, the variations of the parameter A_1^t and A_1^c for the stainless steel rebar with different slenderness in different loading cases are small.

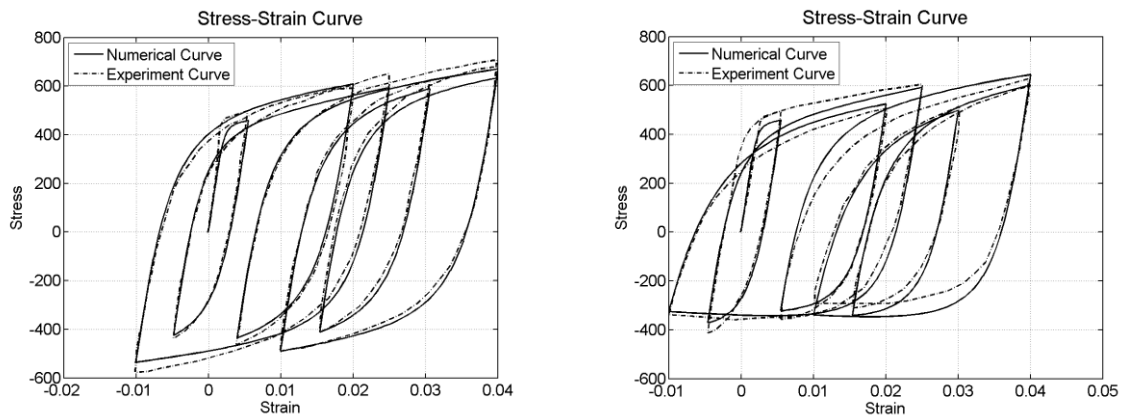
Fig. 3 Results of the optimization of the steel model parameters A_{1t} and A_{1c} for rebar with $L/D=5$ or rebar with $L/D=11$

5 VALIDATION OF THE MODIFIED MONTI-NUTI MODEL

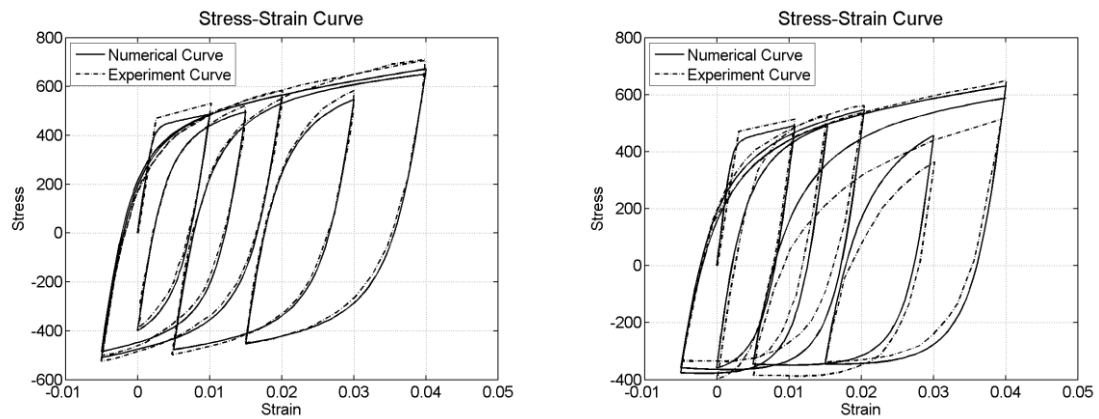
The proposed modified Monti-Nuti model was calibrated and validated by comparisons with experimental tests on carbon steel rebars and stainless steel rebars.

5.1 Steel Model for Carbon steel rebar

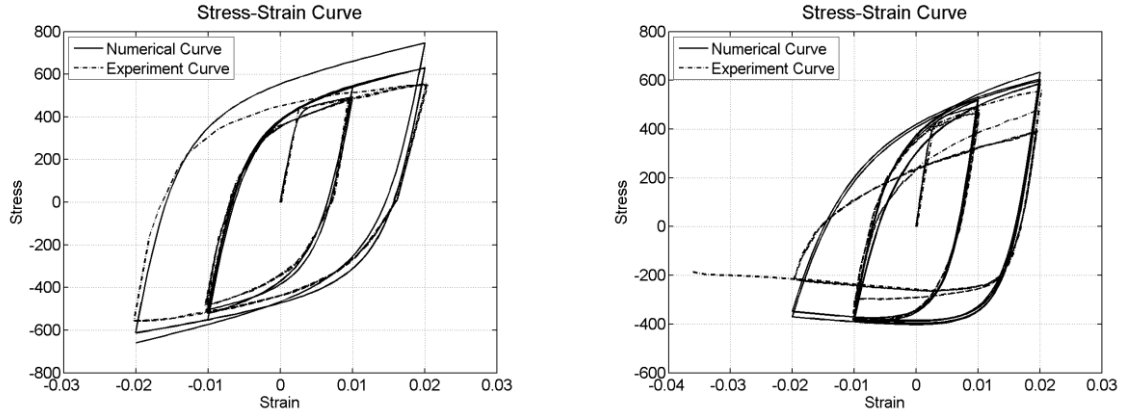
For the carbon steel rebar, the specimen steel type is FeB44k (old Italian steel type), with yield stress equaling 450 MPa, diameter 12mm, 16mm and 20mm. For loading cases A1, A2 and A3, the slenderness $L/D=5$, which means no buckling occurs during test; while C1, C2 and C3 with slenderness $L/D=11$, and the buckling emerges during test.



(a) in absence of buckling-A1 ($L/D=5$) (b) in presence of buckling-C1 ($L/D=11$)
Fig. 4 Comparison between numerical and experimental curves for carbon steel rebars with different slenderness and the same deformation history



(a) in absence of buckling-A2 ($L/D=5$) (b) in presence of buckling-C2 ($L/D=11$)
Fig. 5 Comparison between numerical and experimental curves for carbon steel rebars with different slenderness and the same deformation history

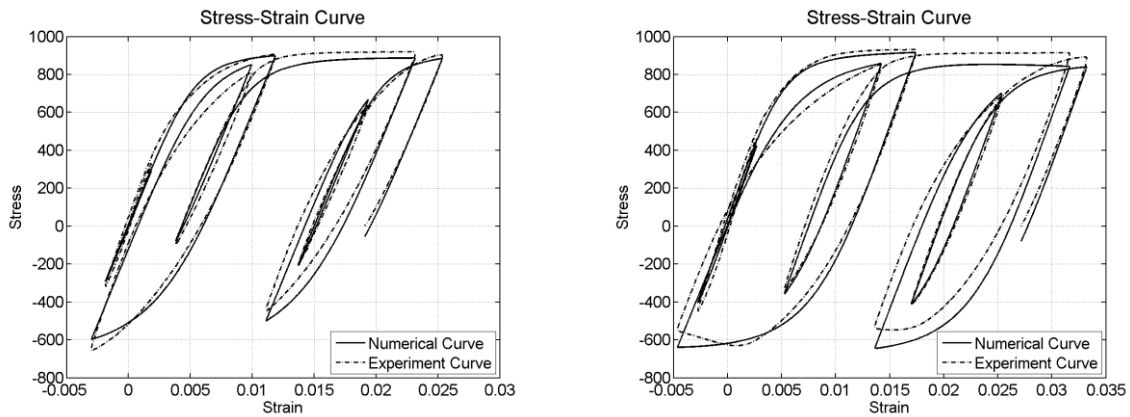


(a) in absence of buckling-A3 ($L/D=5$) (b) in presence of buckling-C3 ($L/D=11$)
 Fig. 6 Comparison between numerical and experimental curves for carbon steel rebars with different slenderness and the same deformation history

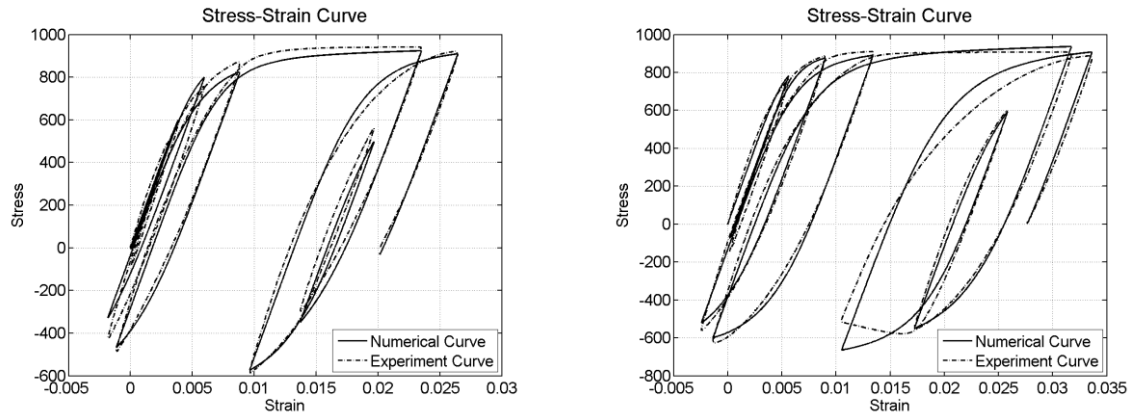
From the above comparison in Figs.4, 5 and 6, it is shown that the numerical curves could coincide with the experimental curves very well. There are some disagreements if the absolute compressive strain exceeds 0.01. However, the real absolute compressive strain of the rebar in r.c. structures could not exceed 0.01 due to the confinement of concrete, thus this model is still capable to simulate well the cyclic behaviors of rebar.

5.2 Steel Model for Stainless steel rebar

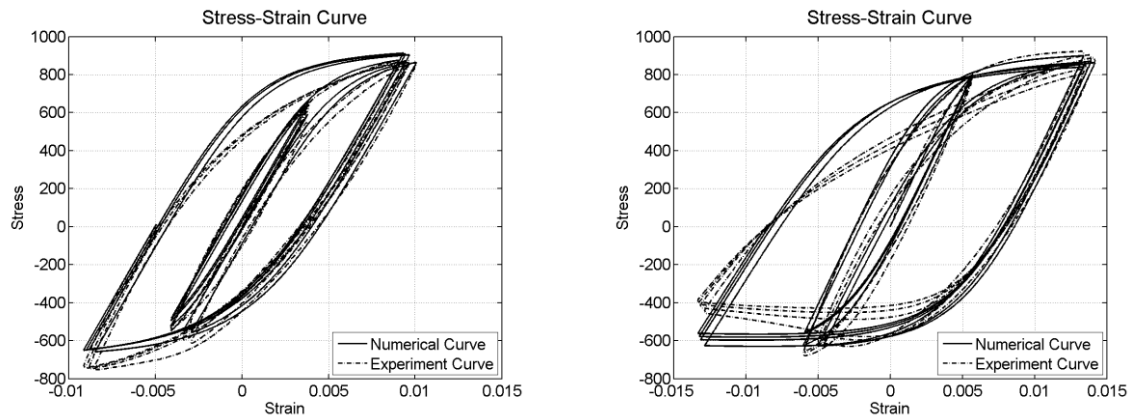
The specimen of the stainless steel rebar is AISI304 or 1.4301. The yield stress is equal to 780MPa in tension while it is equal to 680 MPa in compression. As for the loading cases XA1, XA2 and XA3, the slenderness $L/D=5$, and the diameter $D=12$ mm; while for XC1, XC2 and XC3, the slenderness $L/D=11$, and the diameter $D=12$ mm.



(a) in presence of buckling-XA1 ($L/D=5$) (b) in presence of buckling-XC1 ($L/D=11$)
 Fig. 7 Comparison of numerical curves and experimental ones for stainless steel rebars with different slenderness



(a) in presence of buckling-XA2 ($L/D=5$) (b) in presence of buckling-XC2 ($L/D=11$)
Fig. 8 Comparison of numerical curves and experimental ones for stainless steel rebars with different slenderness



(a) in presence of buckling-XA3 ($L/D=5$) (b) in presence of buckling-XC3 ($L/D=11$)
Fig. 9 Comparison of numerical curves and experimental ones for stainless steel rebars with different slenderness

From the comparisons in Figs. 7, 8 and 9, the effectiveness of the modified model could be verified.

6 CONCLUSIONS

This paper proposes a modified Monti-Nuti cyclic model including the inelastic buckling for the reinforcing bar. The inelastic buckling of the steel rebars has important effects on the seismic behavior of the existing r.c. structures with poor seismic details and drop of resistance and ductility are observed. Following conclusions are obtained:

- The inelastic buckling occurs when rebar slenderness is greater than the critical one. It is important to define the correct value of the critical slenderness. The yield strength has effects on the critical slenderness value. The proposed modified version of the steel model simulates properly the inelastic buckling including a correct estimation of the critical slenderness considering also the yield strength of the rebar.
- Some stainless steel rebar types show an anisotropic behavior in tension and compression (i.e. different yielding strength). This behavior is described well in the proposed modified

version of the steel model including different parameters for tension and compression model branches description in case of cyclic deformation history.

- The parameter identification method is adopted in this paper to identify properly the main parameters of the steel model equations for a better description of the cyclic behavior of the steel on the base of different experimental curves.
- Through comparing with the experimental stress-strain curves of carbon steel rebars and stainless steel rebars, the effectiveness of the modified Monti-Nuti is verified.

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