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ENERGY BASED OPTIMUM DESIGN OF TUNED MASS DAMPERS

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Abstract. *Vibrations reduction in engineering systems, such as civil, mechanical and aeronautical, can be achieved by modifying rigidities, masses, damping or shape and by providing passive or active counter forces. Among the various control methods, the Tuned Mass Damper is one of the simplest and the most reliable control device. The present study focuses on optimum design of Tuned Mass Dampers based on an energy approach. For this aim, a simple one degree of freedom system subject to a seismic action and equipped with a Tuned Mass Damper is examined. The effectiveness of the control device in reducing the vibration level induced by seismic action is expressed in terms of reduction of the dissipated energy into the primary system with respect to the uncontrolled case. This index is assumed as the objective function to develop an optimum design to achieve Tuned Mass Damper optimum parameters, which give the best performances in vibration control. A comparison with the most common optimization based on displacement reduction is performed.*

1 INTRODUCTION

Passive seismic devices are extensively used to reduce damage induced in structures by earthquakes [1, 2, 3]. The Tuned Mass Damper TMD technology uses a weight, which oscillates at the same period as the main system, and an additional damper that connects two relatively moving points, when the system oscillates. Since the natural frequency of TMD is tuned in resonance with the fundamental mode of the primary system, a large amount of the main system vibrating energy is transferred to the TMD, and then dissipated by the damping, when the primary system is subjected to external disturbances. After the work of Ormondroyd and Den Hartog [4], different optimum design approaches of TMD have been developed to reduce vibration level induced in mechanical systems by various types of excitation sources [5, 6]. In [7] and [8], the TMD protection efficiency for a simple system excited at the support is investigated, considering also different environmental conditions. Performance of TMD in vibration reduction from an energy point of view was studied in [9]. In [10], the authors analyzed the seismic energy dissipation of inelastic structures with multiple TMDs. Some other studies have been developed for optimum design of TMD [11, 12]. This paper aims at evaluating the performance of TMD from the energy point of view. For this purpose, the performance of TMD is estimated the reduction of the energy dissipated by the main system. This index is used as objective function to perform an optimum design of TMD.

2 BASIC EQUATIONS AND COVARIANCE ANALYSIS

The TMD device is composed by a mass, a spring and a damper attached to the main system, as shown in Figure 1.

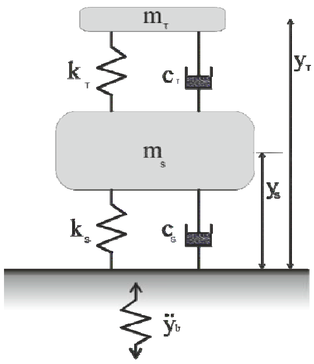

Figure 1: Mechanical model of a linear TMD system

The dynamic response of this system is governed by the dynamic equilibrium equation:

$$\mathbf{M}\ddot{\mathbf{y}}+\mathbf{C}\dot{\mathbf{y}}+\mathbf{K}\mathbf{y}=-\mathbf{M}\mathbf{r}\ddot{y}_g \tag{1}$$

where $\mathbf{y}=\left(y_T,y_S\right)^T$ is the relative base displacement vector, and **M**, **C** and **K** are the mass, the damping and the stiffness matrices.

$$\mathbf{M}=\begin{pmatrix} m_T & 0 \\ 0 & m_s \end{pmatrix};\ \mathbf{C}=\begin{pmatrix} c_T & -c_T \\ -c_T & c_T+c_s \end{pmatrix};\ \mathbf{K}=\begin{pmatrix} k_T & -k_T \\ -k_T & k_T+k_s \end{pmatrix};\ \mathbf{r}=\left\{1 \quad 1\right\}^T$$

Introducing the state space vector $\mathbf{z}_s=(y_T,y_S,\dot{y}_T,\dot{y}_S)^T$ the equation (1) becomes:

$$\dot{\mathbf{z}}_s=\mathbf{A}\mathbf{z}_s+\mathbf{r}_z\ddot{y}_g(t) \tag{2}$$

$$\mathbf{A}_s=\begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{H}_s^1 & \mathbf{H}_s^2 \end{pmatrix} \tag{3}$$

is the state matrix, $\mathbf{r}_z=\left(0,0,1,1\right)^T$, $\mathbf{I}$ and $\mathbf{0}$ are the unit and zero 2x2 matrices, and:

$$\mathbf{H}_s^{1} = -\mathbf{M}^{-1}\mathbf{K} = \begin{pmatrix} -\omega^2{}_T & +\omega^2{}_T \\ +\delta_T\omega^2{}_T & -\left(\delta_T\omega^2{}_T+\omega^2{}_s\right) \end{pmatrix} \qquad \mathbf{H}_s^{2} = -\mathbf{M}^{-1}\mathbf{C} = \begin{pmatrix} -2\xi_T\omega_T & +2\xi_T\omega_T \\ +\delta_T 2\xi_T\omega_T & -\left(\delta_T 2\xi_T\omega_T+2\xi_s\omega_s\right) \end{pmatrix} \quad (4)$$

The mechanical parameters are introduced:

$$\omega_T=\sqrt{\frac{k_T}{m_T}}\ ,\ \omega_s=\sqrt{\frac{k_S}{m_S}}\ ,\ \xi_T=\frac{c_T}{2\sqrt{m_T k_T}}\ \xi_s=\frac{c_S}{2\sqrt{m_S k_S}}\ \delta_T=\frac{m_T}{m_S}$$

In equation (1) $\ddot{y}_g(t)$ is the acceleration that excites the system at its support. In this study, in order to consider the intrinsic probabilistic nature of seismic motion, the Kanai-Tajimi model is adopted [12]. The space state covariance matrix $\mathbf{R_{zz}} = \mathrm{E}\left[\mathbf{zz}^T\right]$, where the symbol $\mathrm{E}[\ ]$ denotes the mathematical expectation, is obtained by the *Lyapunov equation* [19]:

$$\mathbf{AR_{zz}}+\mathbf{R_{zz}A}^T+\mathbf{B}=0 \quad (6)$$

where **A** is the state matrix. Matrix **B** has all null elements except $\left[\mathbf{B}\right]_{6,6}=2\pi S_0$.

The response of the unprotected system is given by the covariance matrix $\mathbf{R}_{\mathbf{z}_0\mathbf{z}_0}$, which can be evaluated by solving the following equation:

$$\mathbf{A}_0\mathbf{R}_{\mathbf{Z}_0\mathbf{Z}_0}+\mathbf{R}_{\mathbf{Z}_0\mathbf{Z}_0}\mathbf{A}_0{}^T+\mathbf{B}_0=\mathbf{0} \quad (7)$$

3 THE ENERGY EQUATIONS

For the system in Figure 1, the energy balance for unit of time, i.e. the power balance, is:

$$\dot{\mathbf{y}}^T\mathbf{M}\ddot{\mathbf{y}}+\dot{\mathbf{y}}^T\mathbf{C}\dot{\mathbf{y}}+\dot{\mathbf{y}}^T\mathbf{K}\mathbf{y}=-\dot{\mathbf{y}}^T\mathbf{M}\mathbf{r}\ddot{x}_g \quad (8)$$

Introducing the power terms, equation (30) becomes:

$$P_k(t) + P_D(t)+ P_S(t)=P_I(t) \quad (9)$$

$P_k(t)=\dot{\mathbf{y}}^T\mathbf{M}\ddot{\mathbf{y}}$ is the kinetic power at the time *t*, $P_D(t)=\dot{\mathbf{y}}^T\mathbf{C}\dot{\mathbf{y}}$ is the dissipated power at the time *t*, $P_S(t)=\dot{\mathbf{y}}^T\mathbf{K}\mathbf{y}$ is the elastic power at the time *t*, $P_I(t) -\dot{\mathbf{y}}^T\mathbf{Mr}\ddot{y}_g$ is the input power at the time *t*.

After some rearranging, from equation (31) one obtains:

$$m_T\dot{y}_T\ddot{y}_T+m_s\dot{y}_S\ddot{y}_S+c_S\dot{y}_S^2+c_T(\dot{y}_T-\dot{y}_S)\dot{y}_T+k_T(\dot{y}_T-\dot{y}_S)y_T+k_s\dot{y}_S y_S=-m_T\dot{y}_T\ddot{y}_g-m_s\dot{y}_S\ddot{y}_g \quad (10)$$

By dividing for m_s and by introducing the mass ratio $\delta_T=m_T/m_s$, one obtains:

$$\delta_T\dot{y}_T\ddot{y}_T + \dot{y}_S\ddot{y}_S+2\xi_S\omega_S\dot{y}_S^2+2\delta_T\xi_T\omega_T(\dot{y}_T-\dot{y}_S)\dot{y}_T+\delta_T\omega_T^2(\dot{y}_T-\dot{y}_S)y_T+\omega_S^2\dot{y}_S y_S=-\delta_T\dot{y}_T\ddot{y}_g-\dot{y}_S\ddot{y}_g \quad (11)$$

In stationary condition, the power balance is:

$$2\xi_s\omega_s\dot{y}_S^2+2\delta_T\xi_T\omega_T(\dot{y}_T-\dot{y}_S)\dot{y}_T-\delta_T\omega_T^2\dot{y}_S y_T=-\delta_T\dot{y}_T\ddot{y}_g-\dot{y}_S\ddot{y}_g \quad (12)$$

and, considering the power flow in mean value one obtains:

$$2\delta_T\xi_T\omega_T\sigma^2_{\dot{y}_T}+2\xi_s\omega_s\sigma^2_{\dot{y}_S}-2\delta_T\xi_T\omega_T\gamma_{\dot{y}_T\dot{y}_S}-\delta_T\omega_T^2\gamma_{\dot{y}_S y_T}=-\delta_T\gamma_{\dot{y}_T\ddot{y}_b}-\gamma_{\dot{y}_S\ddot{y}_b} \quad (13)$$

In equation (13) $\sigma^2_{\dot{y}_T}$ and $\sigma^2_{\dot{y}_S}$ are the variances of $\dot{y}_T$ and $\dot{y}_S$, respectively, whereas $\gamma_{\dot{y}_T\dot{y}_S}$ and $\gamma_{\dot{y}_S y_T}$ are the cross-correlations. In the following, the attention will be focused on the dissipated terms: P_{Dsyst} and P_{Ddevic} i.e. the power dissipated by the main system and the power dissipated by the TMD device. As well known, the dissipative term of the main system is strictly related to its damage, whereas the amount of the power dissipated by the device points out on its performance.

If the dissipative terms are evaluated in mean value, then they assume the form:

$$\mu_{P_{Dsyst}}=2\xi_s\omega_S\sigma^2_{\dot{y}_s} \quad (14)$$

$$\mu_{P_{Ddevict}}=2\delta_T\xi_T\omega_T\sigma^2_{\dot{y}_T}-2\delta_T\xi_T\omega_T\gamma_{\dot{y}_T\dot{y}_S} \quad (15)$$

4 OPTIMUM DESIGN OF TDM DEVICES

In this section the optimum design [13] of TMD is developed. The main goal of optimization is to define the TMD mechanical characteristics that reduce the amount of the energy dissipated by the primary system. The Factor Reduction of Dissipated Power (FRDP) is then introduced:

$$FRDP=\frac{\mu_{P_{Dsyst}}}{\mu_{P_{Dsyst0}}} \quad (16)$$

where $\mu_{P_{Dsyst}}$ is the mean of the power dissipated by the main system when it is equipped with the TMD, and $\mu_{P_{Dsyst0}}$ is the same quantity for the uncontrolled system. In addition, the power flow dissipated by the device $\mu_{P_{Ddevic}}$ will be considered in the following. After the stochastic index is introduced, the optimum TMD parameters are evaluated by performing the solution of the optimum problem. In detail, for the TMD the optimum design is formulated as the evaluation of Design Vector $\mathbf{b}=(\omega_T,\xi_T)$, which is able to minimize the Objective Function (OF), represent by the TMD performance index:
The following optimum design problem is then formulated:

Find $\mathbf{b}=(\omega_T,\xi_T)$ (45)

Which Minimizes $OF=FRDP=\frac{\mu_{P_{Dsyst}}}{\mu_{P_{Dsyst0}}}$ (17)

The optimization problem is formulated as an uncostrained minimization, so that an Evolutive Alghoritm has been used. In details, the standard Genetic Algorithm (GA) has been used to solve numerically the problem [14, 15, 16, 17, 18].

5 NUMERICAL ANALYSIS UNDER STOCHASTIC INPUT

In this section, the results of optimum design of TMD are presented and discussed. The system damping is $\xi_s=2\%$, the frequency $\omega_S=2\pi$(rad/sec) and the mass ratio is $\delta_T=0.01$. Input is characterized by the following parameters: $\omega_f=20$rad/sec and $\xi_f=0.5$.

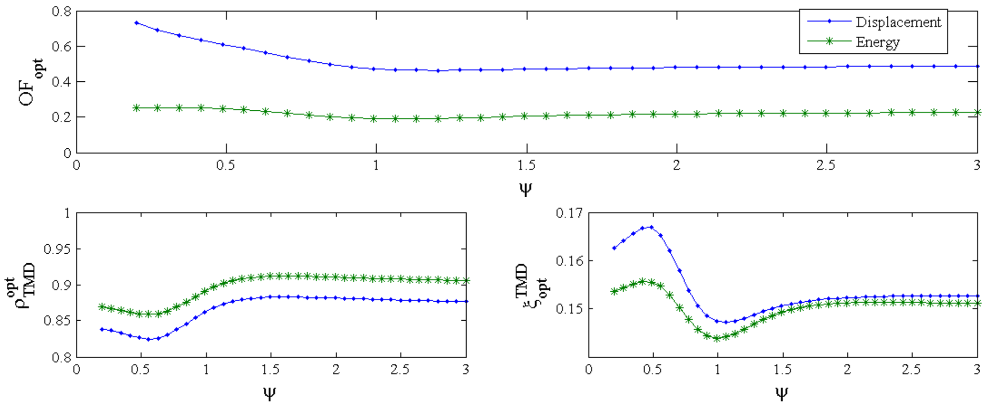

Figure 2: Comparison between the two optimization criteria. Plots represent minimized OFs and optimum design variables versus ψ. The mass ratio is δ_T is 0.01, and $\xi_S=2\%$.

Optimum design developed considering the energy criterion is compared with results obtained considering the approach based on displacement reduction [12]. In figure 2 performance indices (energy reduction ad displacement reduction) are plotted versus $\psi = {}^{\omega_f}\!/_{\omega_s}$, both in the cases of displacement criterion and energy criterion. On the y-axis the symbol OF appears, whereas the demarcation between the two criteria is pointed out by different line types. In addition, also the TMD optimum parameters ρ_{opt}^{TMD} - ξ_{opt}^{TMD} that give the optimum TMD efficiency in two performance criteria are shown. One observes that in case of optimum design developed by the energy approach the FRDP assumes approximately a constant value, equal to 0.2, over the entire range of frequency investigated. This means a reduction of the 80% of the energy dissipated by the main system with respect to the uncontrolled case.
This result is of great relevance, especially when a hysteretic behaviour takes place into the structure. As well known, the amount of the energy dissipated is strictly correlated to the global and local damage level of the vibrating system.
However, exactly a minimum of FRDP is observed around $\psi = 1$, i.e. if the dominant frequency of the excitation matches the resonance frequency of the system to which the TMD frequency is tuned. This means, obvious, that the maximum effectiveness of TMD is reached when the main system is in resonance with the ground motion frequency. An important outcome is that the efficiency of TMD from the energy point of view is earthquake independent if TMD parameters are suitably chosen by optimum design strategy.
If the displacement criterion is considered, the TMD performances are more manifestly earthquake dependant, and a variability of the versus ψ is noticeable. The minimum of the OF, equal almost to 0.45, is reached for $\psi = 1$; then, for $\psi > 1$ the performance is almost constant, whereas for $\psi < 1$ the TMD effectiveness decreases and becomes inefficient when ψ decreases. With regard to the optimum parameters which let the best TMD performance, the variability towards the frequency content of ground motion is similar in the two examined optimum criteria. However, one can notice that the energy criterion gives greater ρ_{opt}^{TMD} values. Moreover, ρ_{opt}^{TMD} approaches to unit near to the resonance condition, even if, in the analyzed case this happens not exactly at $\psi = 1$, because the excitation here considered is not a narrow band one.

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