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CORRELATION BETWEEN THE PROBABILISTIC SEISMIC ASSSMENT OF FRAME STRUCTURES WITHOUT AND WITH ADDED DAMPERS

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Abstract. *This paper deals with the parameters which influence the probability of reaching the near collapse limit state of RC frame structures equipped with nonlinear fluid viscous dampers. The study can be divided into two steps. The first aims to assess how the median and the dispersion of seismic demand can vary in the RC frame structures equipped with nonlinear fluid viscous dampers, considering a wide set of ground motions; these parameters have been compared with those obtained for RC structures without dampers. The second step evaluates the expression in closed form given in the 2000 SAC/FEMA method to assess the annual probability of failure of RC structures. This calculation has been performed using the parameters obtained in the previous step. The annual probability of failure has been estimated for the RC frame structures with and without dampers, considering a wide set of ground motions and different method to approximate the hazard curve. The adopted case study is a frame configuration characterized by three bays and six floors. The analyses have allowed to evaluate the variability of the terms of the closed form expression proposed in the 2000 SAC/FEMA method, and their influence on the definition of the annual probability of failure.*

1 INTRODUCTION

Motivated by the recent seismic events, there has been an increase of concern towards seismic assessment and retrofit of existing buildings [1-3]. One of the innovative technique of seismic retrofit is the insertion of supplemental damping systems in existing buildings. They are activated by the movement of the main structural system and reduce the overall dynamic response of the structure. The dissipation of energy is diverted to these mechanical devices that can be inspected and even replaced after an earthquake [1]. Among them, those considered in this paper are passive energy dissipation systems, which dissipate a portion of the seismic energy input to a structure without external power source. An efficient passive energy dissipation system is made by nonlinear fluid viscous dampers, which have been also considered in this paper. They are rate dependent devices which provide a velocity dependent force. The typology considered is orificed fluid dampers, which dissipate energy by a flow of fluid inside a cylindrical closed container, which is forced to flow through the action of a piston. Nonlinear fluid viscous dampers have the characteristic of having a lower velocity exponent than the unity. In this way, in the event of a velocity spike, the force in the viscous damper is controlled to avoid overloading the dampers or the bracing system to which it is connected [1, 4]. Their advantages are the reduction of damper forces at high velocities, the supply of higher dampers forces at low speed and the dissipation of a larger amount of energy than the other dampers. Since the assessment of seismic response is considerably complex for the presence of a large number of uncertainties, it is better to adopt a probabilistic approach. For this reason, a probabilistic approach has been followed in this paper, in particular the 2000 SAC/FEMA method [5]. This approach provides a closed form expression to evaluate the annual probability of exceeding a specified performance level for a given structure. The variability of terms inside the closed form expression and their influence on the probability of exceeding a specified performance level, have been analyzed here, considering the near collapse limit state, a wide set of ground motions and different methods to approximate the hazard curve. The study has been performed without applying scaling factors to the earthquake records, but considering different records for increasing values of seismic intensity.

The considered case study is a RC frame, characterized by three bays and six floors, designed to resist only gravity loads; nonlinear fluid viscous dampers have been inserted for the seismic retrofit. The seismic demand parameters here considered are the maximum displacement at the top of the structure and the maximum interstorey drift. The probabilistic assessment on the basis of the results obtained from nonlinear dynamic analyses has been performed for those parameters. Nine return periods have been chosen to identify nine values of seismic intensity and twenty ground motions have been selected for each of them. The analyses have been reported considering two different models for the plastic hinges behaviour: the first model with post peak strength deterioration, the second model without it. In the first case the results have been obtained only for the records which converged for both structures, in the second case the results have been obtained for all the records considered, that is 180 for both structures.

2 PROBABILISTIC APPROACH

Among the probabilistic approaches there is the 2000 SAC/FEMA method, which thanks to its mathematical simplicity and comparatively light computational effort, provides directly a measure of the unconditional risk for a given structures located in a known seismic environment. The original formulation of the method was developed for steel moment resisting frames [5].

2.1 Description of the basis method

The 2000 SAC/FEMA method is based on the realization of a performance objective expressed as a specified performance level. Performance levels are quantified as expressions relating to generic structural variables "demand" and "capacity" [5].

2.1.1 Probability assessment formulation

The probability theory is applied through the representation of the three random elements of the problem:

1) the ground motion intensity, represented by the spectral acceleration S_a calculated approximately at the first natural period of the structure and for 5% or higher damping;
2) the displacement demand D;
3) the displacement capacity C.

Both displacement demand and capacity are measured in terms of maximum interstorey drift angle. The likelihood of various levels of future intense ground motions at the site is represented in the standard way by the function $H(s_a)$, which gives the annual probability that the (random) intensity S_a at the site will equal or exceed level s_a [5]. In order to obtain criteria based on required performance objectives, the probabilistic representation of S_a, D and C must be folded together. This folding together takes place in three phases; the first relates S_a and D to produce a drift hazard curve $H_D(d)$, which provides the annual probability that the drift demand D exceeds any specified value d. The second combines the curve obtained in the previous phase with the displacement capacity C, giving rise to P_{PL}, which is the annual probability that the performance level is not being met (e.g., the annual probability of collapse or the annual probability of exceeding the life safety level). Using the total probability theorem, $H_D(d)$ becomes, in discrete form:

$$H_D(d) = P[D \geq d] = \sum_{all x_i} P[D \geq d \mid S_a = x_i] P[S_a = x_i] \tag{1}$$

The second factor within the sum, that is the likelihood of a given level of spectral acceleration $P[S_a=x]$, can be easily obtained from the standard hazard curve $H(S_a)$. The first factor, that is the likelihood $P[D \geq d]$ that the drift exceeds d given that the value of S_a is known, can be obtained through structural response analysis [5]. In continuous, Eq. (1) becomes:

$$H_D(d) = \int P[D \geq d \mid S_a = x] |dH(x)| \tag{2}$$

where $|dH(x)|$ is the absolute value of the differential of the spectral acceleration hazard curve. Using the total probability theorem again P_{PL} becomes in discrete form:

$$P_{PL} = P[C \leq D] = \sum_{all d_i} P[C \leq D \mid D = d_i] P[D = d_i] \tag{3}$$

The second factor, the likelihood of given displacement demand level $P[D=d]$, can be determined by the drift hazard curve derived in Eq. (2). The first factor, the likelihood that the drift capacity is less than a specified value d given that the drift demand equals that value, $P[C \leq D | D=d]$ can be assumed, in a first approximation, independent of the information about the drift level itself, permitting this term to be simplified as below. The continuous form is:

$$P_{PL} = \int P[C \leq d] |dH_D(x)| \tag{4}$$

The second factor $|dH_D(d)|$ is defined as the absolute value of the differential of the drift demand hazard curve.

2.1.2 Probability assessment in closed form

In principle, Eqs. (2) and (4) can be solved numerically for any assumption about the form of the probabilistic representation of the three elements $H(s_a)$, D and C. Three approximations of the probabilistic representations of the three terms above have been proposed in order to obtain a closed form expression of Eq. (4).

The first assumes that the site hazard curve can be approximated in the region around P_{PLSa} (in the region of hazard levels close the limit state probability P_{PL}) by the form:

$$H(s_a) = P[S_a \geq s_a] = k_o s_a^{-k} \tag{5}$$

which implies that the hazard curve is linear on a log-log plot in the region of interest. The second assumes that the median drift demand $\hat{D}$ can be represented approximately in the region around P_{PLSa}, by the form:

$$\hat{D} = a(S_a)^b \tag{6}$$

Lastly, the third assumes that the drift demand D is lognormally distributed about the median with the standard deviation of the natural logarithm, $\beta_{D|Sa}$; this definition will be considered as dispersion. Also the drift capacity C is assumed to be lognormally distributed with dispersion β_C. Making use of Eq. (6) and of the lognormal distribution, the first factor in Eq. (2) can be assumed as:

$$P[D \geq d \mid S_a = x] = 1 - \Phi\left(\ln\left[d/ax^b\right]/\beta_{D|S_a}\right) \tag{7}$$

in which Φ is the widely tabulated standardized Gaussian distribution function. Using this result and Eq. (5), Eq. (2) becomes:

$$H_D(d) = P[D \geq d] = H\left(s_a^d\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\beta_{D|S_a}^2\right] \tag{8}$$

in which s_a^d is defined as the spectral acceleration corresponding to the drift level d, that is the inverse of Eq. (6):

$$s_a^d = (d/a)^{1/b} \tag{9}$$

Assuming that the drift capacity C has a median value $\hat{C}$ and that is lognormally distributed with dispersion β_C, the first factor of Eq. (2) becomes:

$$P[C \leq d] = \Phi\left(\ln\left[d/\hat{C}\right]/\beta_C\right) \tag{10}$$

Substituting and carrying out the integration of Eq. (2), it can be obtained the required relation, i.e. the expression in closed form of Eq. (4):

$$P_{PL} = H\left(s_a^{\hat{C}}\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{D|S_a}^2 + \beta_C^2\right)\right] \tag{11}$$

where s_a^C is the spectral acceleration associated to the attainment of the capacity. From Eq. (11) it is possible to note how the effect of the dispersion increases the P_{PL} through the exponential correction factor $\exp[(1/2)(k^2/b^2)\,(\beta_{D|Sa}^2+\beta_C^2)]$. It is so called because it corrects the total randomness, both in drift and in capacity.

3 THE CONSIDERED CASE STUDY

The considered case study is a RC frame characterized by three bays and six floors (Fig. 1). This frame has been designed to resist only gravity loads. Nonlinear fluid viscous dampers have been inserted so that the structure can sustain a high level of seismic action. Dampers of mechanical properties already known have been considered in this paper [6], since its aim is the study of the parameters which affect the property of reaching the near collapse limit state for RC structure equipped with given nonlinear fluid viscous dampers. With regard to the mechanical properties of dampers, the exponent of velocity is α=0.5, the supplemental damping provided by the damping system is equal to 24.5% and the damping coefficient corresponds to 556 kN $(s/m)^{0.5}$. As for the geometry of the structure, each bay is 6 m wide and each interstorey is 3.2 m high. The beams are 30 cm wide and 60 cm deep in all floors. On the ground floor the columns at the edges have a square cross section with a 40 cm side length, while the central ones with a 45 cm side length; on the first floor both columns have square cross section 40x40 cm; on the third floor 35x35 and on the last three floors 30x30 cm. A concrete with a cylinder strength equal to 28 MPa and a steel with a yield strength equal to 450 MPa have been assumed in this study. The seismic weights are 561.6 kN for the sixth floor, 833.4 kN for the fifth and fourth floor, 838.6 kN for the third floor, 849.8 kN for the second floor and 859.2 kN for the first floor. The structure is assumed to be located in Santa Sofia, Italy.

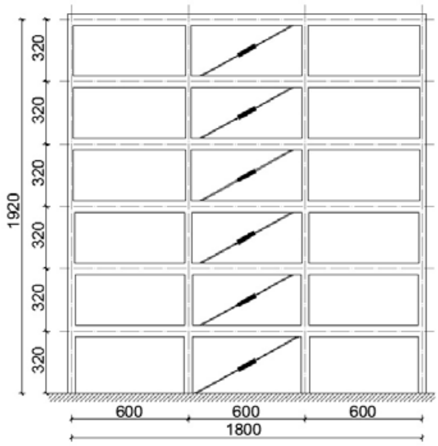


Figure 1: Geometrical characteristics of the considered RC frame (dimensions in cm).

Since the aim of this paper is the probabilistic assessment of the seismic response of RC structures equipped with nonlinear fluid viscous dampers, it is necessary to carry out a wide number of nonlinear dynamic analysis and consequently to select a high number of spectrum-compatible recorded ground motions. Nine return periods have been considered in this study (T_R=30, 50, 101, 201, 475, 664, 975, 1950 and 2475 years) and for each of them, 20 recorded ground motions characterized by an elastic acceleration response spectrum compatible with the code spectrum [7] have been selected through the software Rexel [8].

The code elastic response spectrum for the assumed site and for a soil type C has been determined for each considered return period [7]. Given the site and the soil type, the PGA for T_R=475 is equal to 0.29 g. Each record has been scaled to the code design value of PGA. Figure 2 shows the spectra [9] of the 20 selected records for T_R=475years and their average spectrum compared to the code spectrum.

The nonlinear dynamic analyses have been performed using a concentrated plasticity model implemented in a FE computer program (SAP2000) [10]. A moment-rotation curve has been assigned to the plastic hinges, located at the ends of each element. The moment-rotation curve has been identified by assigning the yielding and the ultimate bending moments and the corresponding chord rotations, which have been provided by empirical relations given in the Commentary to the National code and similar to those of Panagiotakos and Fardis [11].

In order to carry out assessments with a different number of results, two different moment-rotation curves have been considered. At first, a moment rotation curve with post peak strength deterioration has been used (bilinear moment rotation curve, Figure 3a); then a moment-rotation curve without post peak strength deterioration (trilinear moment rotation curve, Figure 3b) has been considered to ease the convergence of the large number of analyses and to obtain a greater number of results for the probabilistic assessment.

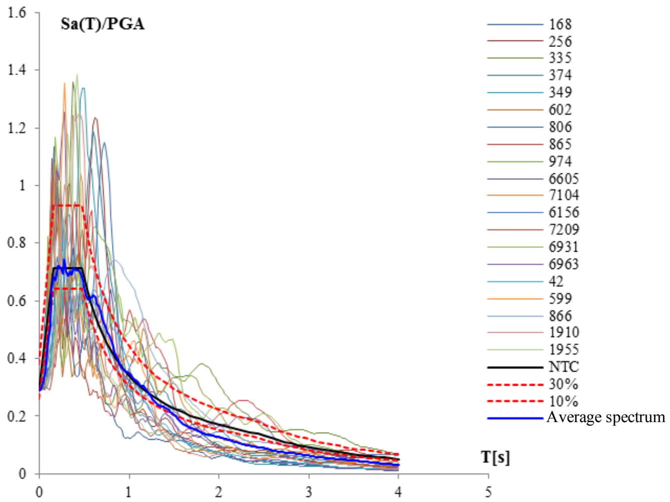


Figure 2: Spectra of the 20 selected compatible records for T_R=475 years, average spectrum of the selected records, code spectrum and tolerance limits (30 % and 10%).

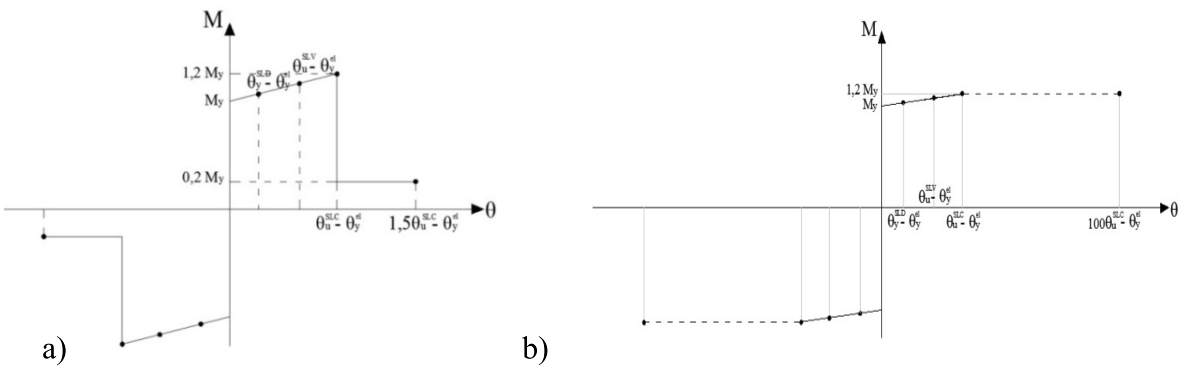


Figure 3: a) bilinear moment-rotation curve with post peak strength deterioration, assigned to plastic hinges ; b) trilinear moment-rotation curve without strength deterioration, assigned to plastic hinges.

The obtained results are grouped in three cases, as shown in Table 1. In the first case a bilinear moment rotation curve (Figure 3a) is considered and the probabilistic assessment is made on 170 and 104 records respectively for the structure with and without dampers. In the second case the same plastic hinge model of the first case is adopted, but the probabilistic assessment is made on the same number of records, i.e. 104, for both structures. In the third case a trilinear moment rotation curve (Figure 3b) is assumed and the probabilistic assessment is performed on 180 records for both structures.

Case 1)	Bilinear moment-rotation curve with post peak strength deterioration	
	Structure with dampers	Structure without dampers
	170 results	104 results
Case 2)	Bilinear moment-rotation curve with post peak strength deterioration	
	Structure with dampers	Structure without dampers
	104 results	104 results
Case 3)	Trilinear moment-rotation curve without post peak strength deterioration	
	Structure with dampers	Structure without dampers
	180 results	180 results

Table 1: The three cases in which the performed analyses have been grouped.

4 RESULTS AND COMMENTS

4.1 Probabilistic seismic demand analysis

The following parameters have been examined for each nonlinear dynamic analysis:

- profiles of maximum displacement, obtained by the envelope of the maximum displacements which occur on each floor during the seismic event;
- profiles of maximum interstorey drift, which represent the envelope of maximum interstorey drifts occurring during the seismic event.

Once obtained the maximum displacement and maximum interstorey drift profiles, the maximum displacement on the top of building (D_{roof}) and the maximum interstorey drift (δ_{max}) along the height have been determined for each record and each return period. The values thus obtained have been plotted in graphs, which have a parameter representing the seismic intensity ($S_a(T_1)$, spectral acceleration at the first natural period of the structure and for 5% damping) as abscissa and a parameter representing the seismic demand (D_{roof} or δ_{max}) as ordinate. In Figures 4 and 5 the points D_{roof} -$S_a(T_1)$ and δ_{max} -$S_a(T_1)$ obtained with each record have been represented for the structure with and without dampers, respectively.

In order to perform the probabilistic assessment, the median and the dispersion values have been determined. As for the median, it is assumed from the scientific literature [5] that the distribution of median values of the seismic demand parameters follows the relation:

$$MeD = a\left(S_a(T_1)\right)^b \tag{12}$$

where MeD is the median value of demand parameter D, $S_a(T_1)$ is the spectral acceleration and a, b are constants deriving from a regression analysis. These constants have been identified for D_{roof} and δ_{max} once the points MeD_{roof} -$S_a(T_1)$ and $Me\delta_{max}$ -$S_a(T_1)$ have been determined for each return period.

As for the dispersion, two different dispersion formulations have been considered. The first considers a variable dispersion with the seismic intensity. The dispersion for each return period is obtained through the formulation proposed from scientific literature: standard deviation of demand parameters of natural logarithm; it is indicated with the notation β_{regr} and it is obtained through the expression:

$$\beta_{D|S_a} = \sqrt{\frac{\sum_k \left(\ln x_k - \hat{\mu}\right)}{n}} \tag{13}$$

where n is the number of values and $\hat{\mu}$ is the median, determined as the mean of the natural logarithm of the results:

$$\hat{\mu} = \frac{\sum_k \ln x_k}{n} \tag{14}$$

A regression analysis has been carried out on the obtained dispersion values determining the parameters of the straight line which interpolates the dispersion values best:

$$\beta_{D|S_a} = a + bS_a(T_1) \tag{15}$$

In order to evaluate the accuracy of the obtained correlation, the parameters R^2 (determination index) and the errors (mean and standard deviation of residues) have been considered. The second formulation, denoted with β_{cost}, considers a parameter of constant dispersion with seismic intensity. This is obtained performing a regression analysis of ln$D$ on lnS_a on the totality of the results. The value thus obtained is the standard deviation of the residues.

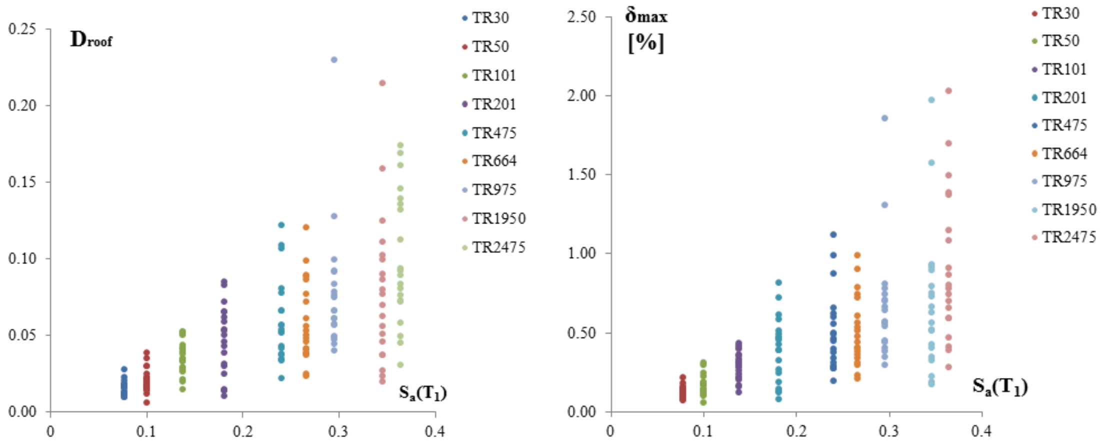


Figure 4: D_{roof} - $S_a(T_1)$ and δ_{max} - $S_a(T_1)$ points for the structure with dampers.

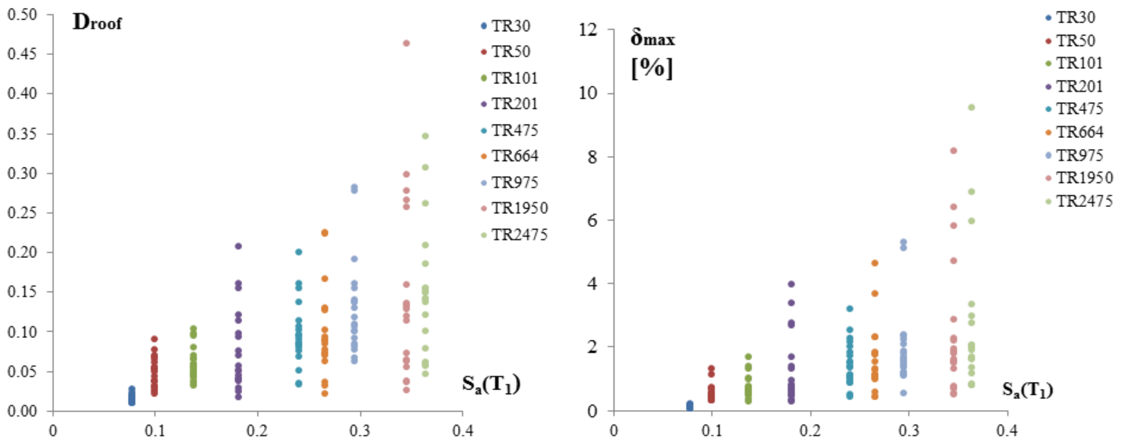
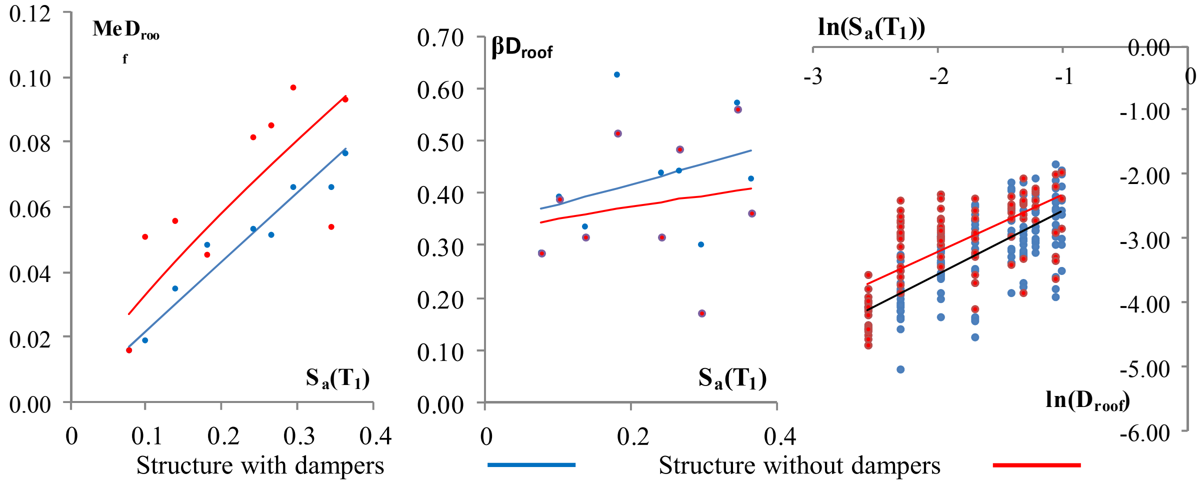


Figure 5: D_{roof} - $S_a(T_1)$ and δ_{max} - $S_a(T_1)$ points for the structure without dampers.

Table 2 shows the expressions of median and dispersion of demand parameters for the analyses of Case 1 (see Table 1). Figures 6 and 7 illustrates for the same case the graphs of median and dispersion of demand parameters as a function of seismic intensity. The same results are reported in Table 3, Figures 8 and 9 for the analyses of Case 2, and in Table 4 and Figures 10 and 11 for the analyses of Case 3. Figure 6 shows that the median roof displacements (D_{roof}) of the structure without dampers are greater than those of the structure with dampers, as expected. With regard to the dispersion values (β_{regr}, Figure 6b) both for the structure with dampers and without dampers, it increases with seismic intensity.

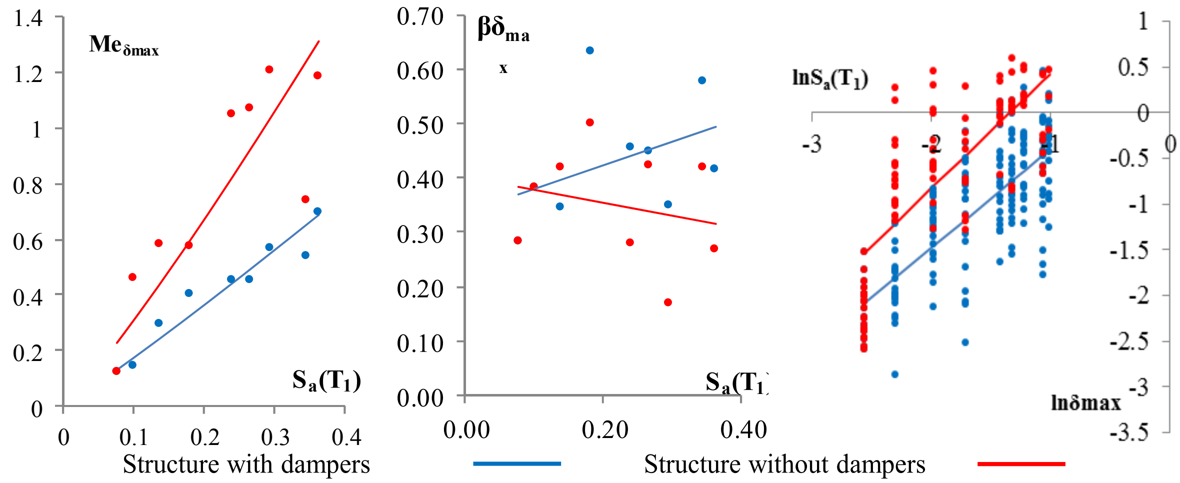
D_{roof}		δ_{max}	
Structures with dampers 170 records	Structures without dampers 104 records	Structures with dampers 170 records	Structures without dampers 104 records
$MeD_{roof}= 0.2114 \cdot S_a^{0.9872}$	$MeD_{roof}= 0.2129 \cdot S_a^{0.8059}$	$Me\delta_{max}= 2.041 \cdot S_a^{1.0755}$	$Me\delta_{max}= 4.1216 \cdot S_a^{1.1327}$
β_{regr}=0.3385+0.3894S_a	β_{regr}=0.3274+0.2272S_a	β_{regr}=0.3359+0.4341S_a	β_{regr}=0.4016-0.2375S_a
β_{cost} = 0.4523	β_{cost} = 0.5252	β_{cost} = 0.4595	β_{cost} = 0.56296

Table 2: Expressions of median and dispersion for D_{roof} and δ_{max}, Case 1.



(a) (b) (c)

Figure 6: Case 1: a) MeD_{roof}- $S_a(T_1)$; b) βD_{roof}- $S_a(T_1)$; c) lnD_{roof}-ln$S_a(T_1)$



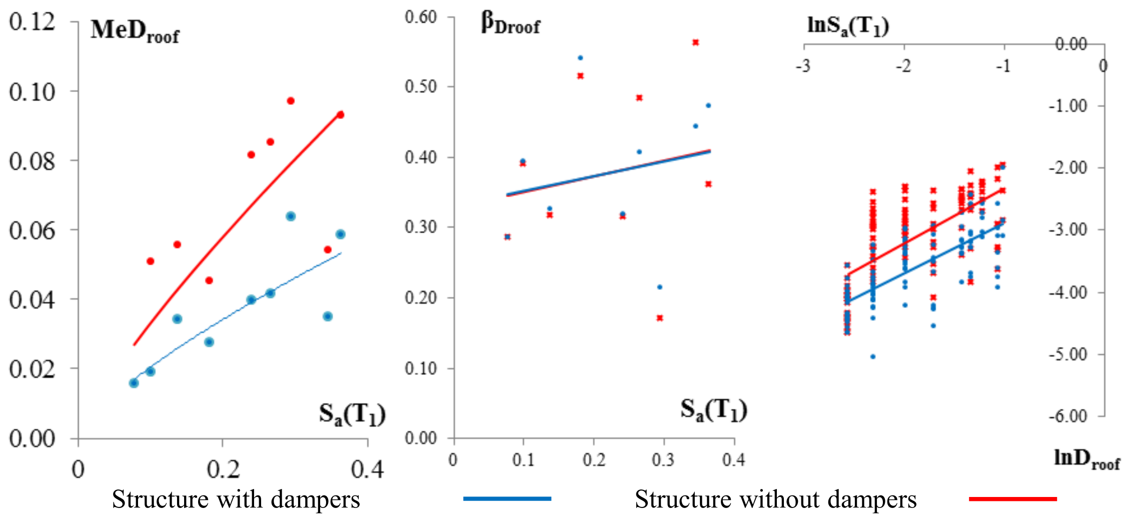
(a) (b) (c)

Figure 7: Case 1: a) $Me\delta_{max}$- $S_a(T_1)$; b) $\beta\delta_{max}$- $S_a(T_1)$; c) lnδ_{max}-ln$S_a(T_1)$

Moreover, the dispersion of the structure with dampers is greater than that without dampers; this is a trend not in line with the expectations. Lastly, Figure 6c illustrates that, in accordance with Figure 6a, the displacements of the structure with dampers are smaller than those obtained for the structure without dampers. Considering the parameter of constant dispersion (β_{cost}, Table 2), the dispersion of the structure without dampers is greater than that with dampers; therefore the trend of β_{cost} is in line with the expectations. As regards Fig. 7 the same remarks are made for the maximum interstorey drifts (δ_{max}); in addition it is possible to observe that: the variable dispersion (β_{regr}) decreases for the structure without dampers when seismic intensity increases and this trend is not in line with the expectations; as for the parameter of constant dispersion (β_{cost}), the values of dispersion are slightly higher than those obtained for the maximum displacement at the top of the building (D_{roof}).

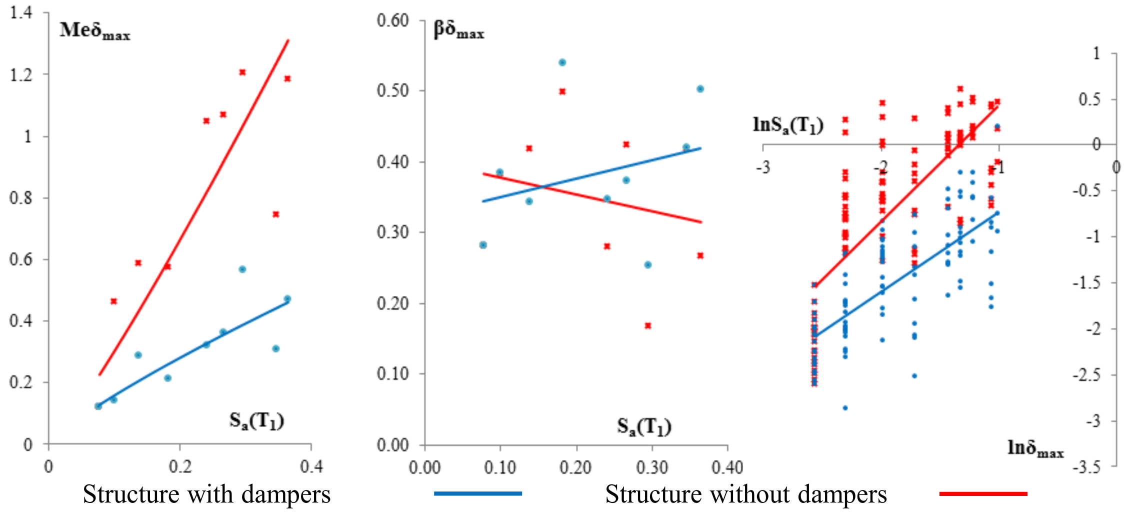
D_{roof}		δ_{max}	
Structures with dampers 104 records	Structures without dampers 104 records	Structures with dampers 104 records	Structures without dampers 104 records
$MeD_{roof}= 0.1127\cdot S_a^{0.7398}$	$MeD_{roof} = 0.2129\cdot S_a^{0.8059}$	$Me\delta_{max} = 1.0595\cdot S_a^{0.8221}$	$Me\delta_{max} = 4.1216\cdot S_a^{1.1327}$
β_{regr}=0.3306+0.2143S_a	β_{regr}=0.3274+0.2272 S_a	β_{regr}=0.3254+0.257 S_a	β_{regr}=0.4016-0.2375 S_a
β_{cost} = 0.42236	β_{cost} = 0.5252	β_{cost} = 0.42239	β_{cost} = 0.56296

Table 3: Expressions of median and dispersion for D_{roof} and δ_{max}, Case 2.



(a) (b) (c)

Figure 8: Case 2: a) MeD_{roof}- $S_a(T_1)$; b) βD_{roof}- $S_a(T_1)$; c) lnD_{roof}-ln$S_a(T_1)$

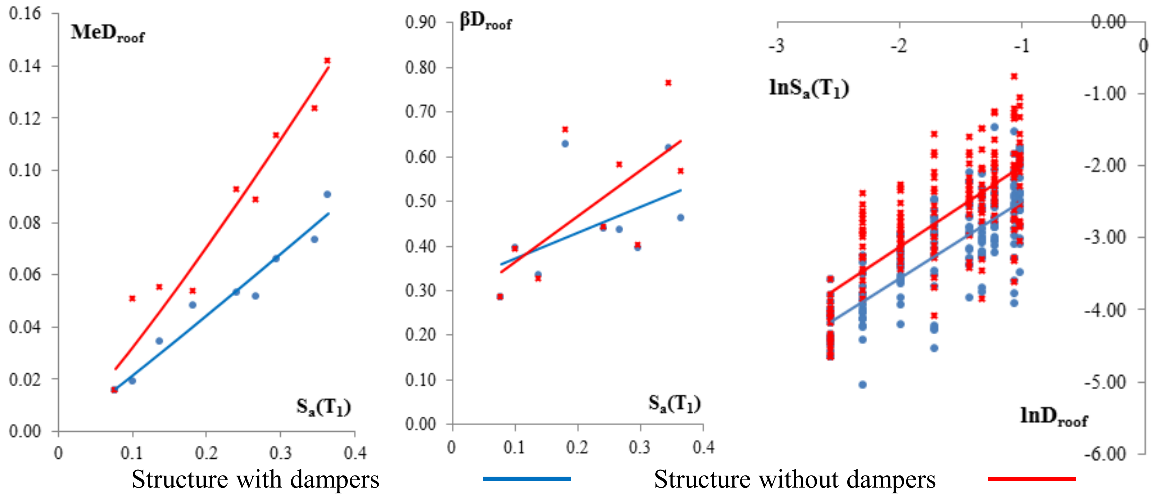


(a) (b) (c)

Figure 9: Case 2: a) $Me\delta_{max}$- $S_a(T_1)$; b) $\beta\delta_{max}$- $S_a(T_1)$; c) lnδ_{max}-ln$S_a(T_1)$

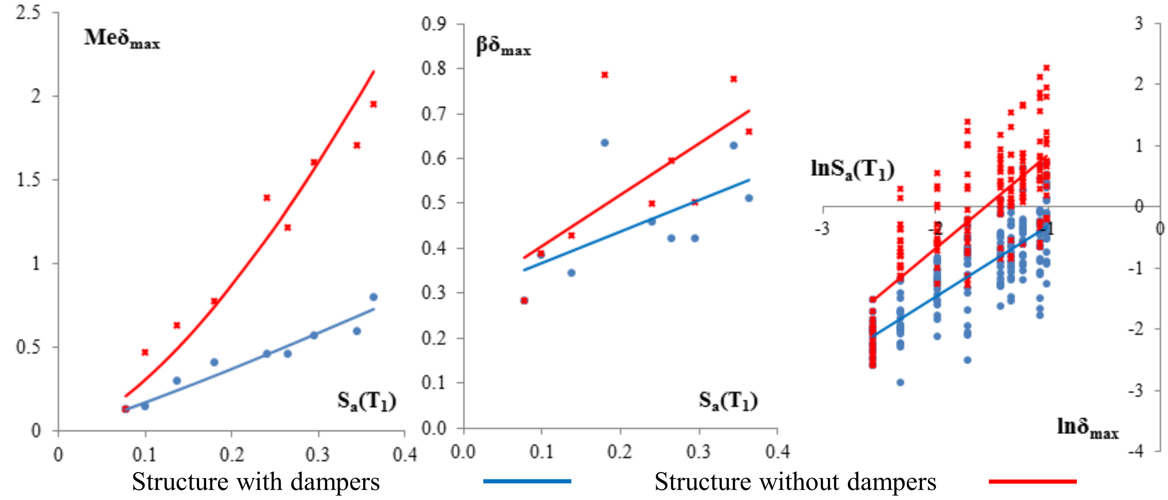
D_{roof}		δ_{max}	
Structures with dampers 180 records	Structures without dampers 180 records	Structures with dampers 180 records	Structures without dampers 180 records
$MeD_{roof}= 0.2421\cdot S_a^{1.0523}$	$MeD_{roof}= 0.44\cdot S_a^{1.1357}$	$Me\delta_{max}= 2.2724\cdot S_a^{1.1285}$	$Me\delta_{max}= 9.8605\cdot S_a^{1.5088}$
β_{regr}=0.3145+0.5809S_a	β_{regr}=0.2626+1.0257S_a	β_{regr}=0.2975+0.7001S_a	β_{regr}=0.2906+1.1428S_a
β_{cost} = 0.4696	β_{cost} = 0.5651	β_{cost} = 0.4803	β_{cost} = 0.6389

Table 4: Expressions of median and dispersion for D_{roof} and δ_{max}, Case 3.



(a) (b) (c)

Figure 10: Case 3: a) MeD_{roof}- $S_a(T_1)$; b) βD_{roof}- $S_a(T_1)$; c) lnD_{roof}-ln$S_a(T_1)$



(a) (b) (c)

Figure 11: Case 3: a) $Me\delta_{max}$- $S_a(T_1)$; b) $\beta\delta_{max}$- $S_a(T_1)$; c) lnδ_{max}-ln$S_a(T_1)$

Figure 8a shows smaller values of median D_{roof} than for case 1. Figure 8b confirms that the dispersion (β_{regr}) always increases for both the structure when the seismic intensity increases. Moreover, the values of dispersion (β_{regr}) decrease for the structure with dampers if we consider a smaller number of records. Lastly, Figure 8c and Table 4 illustrate that also a smaller value of dispersion (β_{cost}) is obtained for the structure with dampers if we consider a smaller number of records. Figure 9 shows that the same remarks made for the maximum roof displacement (D_{roof}) can be made for the maximum interstorey drift, i.e. both the values of median and dispersion decrease for the structure with dampers, considering a smaller number of records. However, passing from the structure with damper to the one without damper, a trend in lines with the expectations is not obtained yet for β_{regr}.

If we consider a greater number of records for both the structures (Case 3), a trend in line with the expectations is obtained. As for the maximum roof displacement (D_{roof}) one can note that (Figure 10):

- the median values for the structure without dampers are greater than those obtained for the structure with dampers (Figure 10a);
- the dispersion β_{regr} always increases for both the structures when seismic intensity increases and the expected trend is obtained, that is the dispersion of the structure without dampers is greater than that with dampers (Figure 10b);
- the dispersion β_{cost} is greater for both structures than the previous cases but it always maintains the same trend, that is the dispersion for the structure without dampers is greater than that for the structure with dampers (Figure 10c).

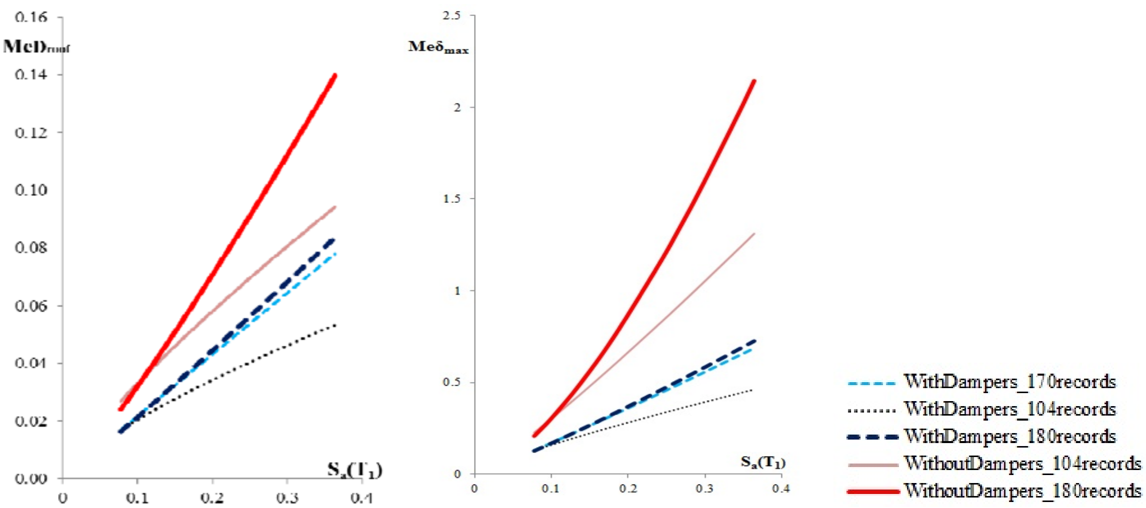
The same remarks can be made for the maximum interstorey drift (δ_{max}, Figure 11), with the additional observation that the decreasing trend disappears for β_{regr}, which always increases with seismic intensity.

Figure 12,13 and 14 compare, for the three cases of Table 1, the curves of median and dispersion of maximum roof displacement and interstorey drift as functions of the seismic intensity. Figure 12 shows that both for D_{roof} and δ_{max}:

- the median values of displacement for the structure without dampers are greater than those for the structure with dampers;
- the values of D_{roof} and δ_{max} increases when the number of records increases and the accuracy of the correlation increases.

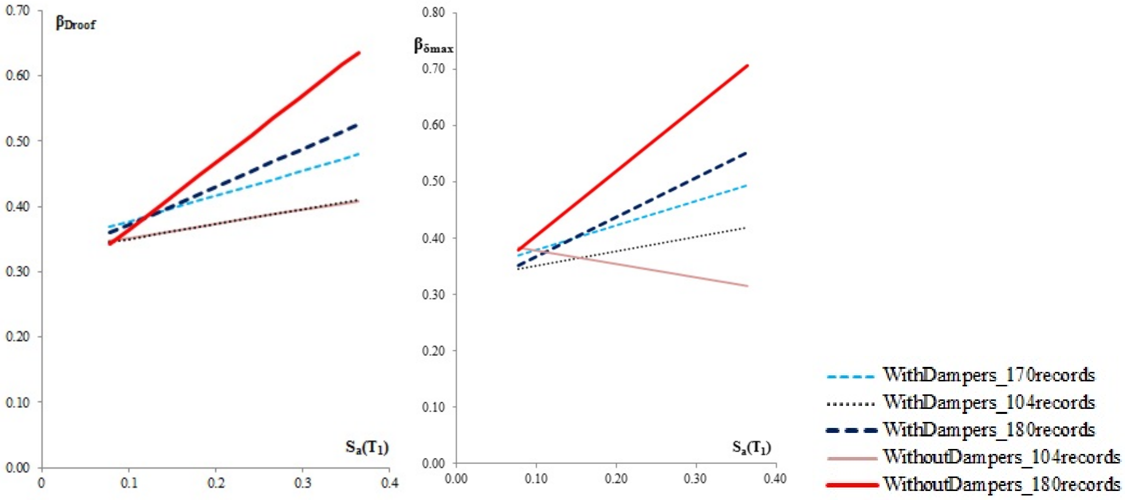
Figure 13 illustrates the trend of the dispersion variable with the seismic intensity. It is possible to observe that:

- the variable dispersion (β_{regr}) increases with the increasing of seismic intensity, with the exception of δ_{max} with 104 records;
- the variable dispersion (β_{regr}) increases for both structures and the accuracy of the correlation increases when the number of records increases. With 180 records for both structures the expected trend is obtained, that is the dispersion for the structure without damper is greater than that for the structure with dampers.



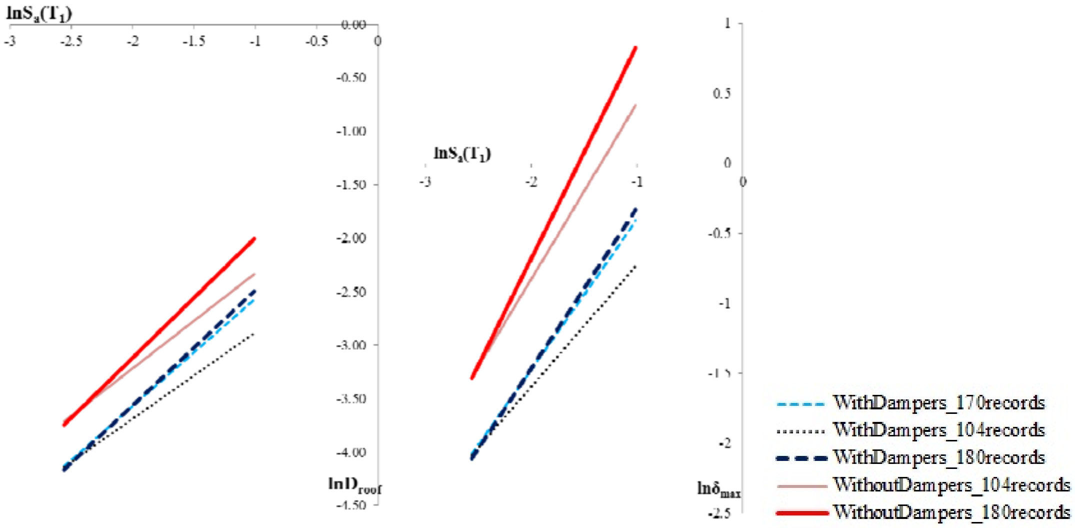
(a) (b)

Figure 12: Comparison of cases 1, 2 and 3: a) MeD_{roof}- $S_a(T_1)$; b) $Me\delta_{max}$ -$S_a(T_1)$



(a) (b)

Figure 13: Comparison of cases 1, 2 and 3: a) βD_{roof}- $S_a(T_1)$; b) $\beta\delta_{max}$- $S_a(T_1)$



(a) (b)

Figure 14: Comparison of cases 1, 2 and 3: a) $\ln D_{roof}$-$\ln S_a(T_1)$; b) $ln\delta_{max}$-$\ln S_a(T_1)$

	Structure with dampers			Structure without dampers	
	104 records	170 records	180 records	104 records	180 records
$\beta_{cost}D_{roof}$	0,42236	0,4523	0,4696	0,5252	0,5651
$\beta_{cost}\delta_{max}$	0,42239	0,4595	0,4803	0,5629	0,6389

Table 5: β_{cost} with the increasing of the number of records for D_{roof} and δ_{max}.

Table 5 illustrates that the parameter of constant dispersion (β_{cost}) increases when the number of records increases for both structures and for both demand parameters (D_{roof} and δ_{max}). Lastly, with regard to dispersion, it is useful to perform some evaluations. By calculating the ratio between the parameter of constant dispersion obtained for both structures with and without dampers in the three cases, it comes out that the dispersion of the structure with dampers is about 80% of the dispersion of the structure without dampers (Table 6). By calculating the ratio between the coefficients of the regression lines obtained for the structures with and without dampers in the three cases, one gets the relation which allows to obtain the dispersion of the structure with dampers, knowing that for the structure without dampers (Table 7). The most reliable correlations are all obtained for Case 3, characterized by a greater number of records (180).

		Case 1	Case 2	Case 3
D_{roof}	$\beta_{Dam}/\beta_{WithoutDam}$	0.8612	0.8042	0.831
δ_{max}	$\beta_{Dam}/\beta_{WithoutDam}$	0.8162	0.7503	0.7518

Table 6: $\beta_{cost,}$ ratio between the parameters of constant dispersion, obtained for both structures with and without dampers in the three cases.

	D_{roof}		δ_{max}	
$\beta=\alpha+\lambda S_a$; $\alpha = a_{Dam}/a_{WithoutDam}$; $\lambda = b_{Dam}/b_{WithoutDam}$				
	α	λ	α	λ
CASE 1	1.714	1.034	-1.828	0.836
CASE 2	0.943	1.009	-1.082	0.810
CASE 3	0.566	1.198	0.613	1.024

Table 7: $\beta_{regr,}$ ratio between the coefficients of the regression lines obtained for the structure with and without dampers in the three cases.

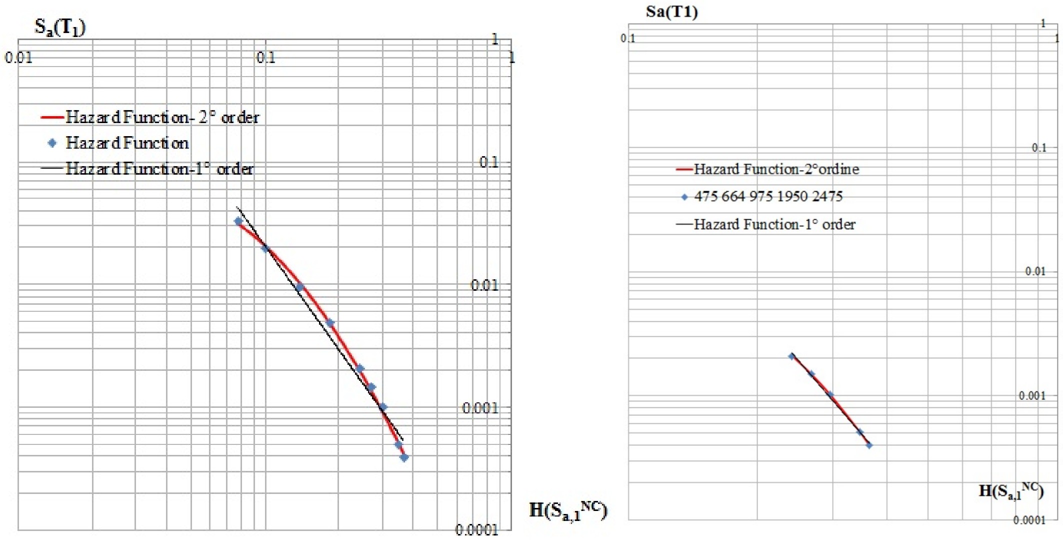
4.2 Evaluation of the annual failure probability

The second step of this research is the study of the simplified formula proposed by the 2000 SAC/FEMA method to assess the annual probability of exceeding a given performance level (Eq. (11)). The following assumptions have been considered in this study:

- consideration of the near collapse limit state;
- adoption of different criteria for approximating the hazard curve: interpolation on the whole range of examined return periods (criterion (a), T_R=30, 50, 101, 201, 475, 664, 975, 1950 and 2475 years); interpolation on a small range close to the return period of 975 years, which is associated in the code [7] to the near collapse limit state (criterion (b) T_R=475, 664, 975, 1950 and 2475 years). Moreover, interpolations of the first and second order have been performed for both the criteria identifying thus four hazard curves. The approximation of the second order has been determined according to the method proposed by Vamvatsikos [12]. The expressions of the obtained hazard curves are reported in Table 8, the graphs of such relations are illustrated in Figure 15.
- determination of the capacity values for the near collapse limit state and for the two considered demand parameters according to two criteria described in the following. In this way two collapse conditions are defined, one based on the ultimate roof displacement ($D_{roof,u}$), the other on the ultimate interstorey drift ($\delta_{max,u}$). $D_{roof,u}$ has been determined through a pushover analysis, under a modal pattern of lateral load, as the roof displacement when the first plastic hinge in a column reaches the collapse rotation. This value is $D_{roof,u}$=0.145 m. $\delta_{max,u}$, to be compared with the demand, evaluated in terms of maximum drift along the height, has been determined through a pushover analysis as the maximum drift along the height when the first plastic hinge in a column reaches the collapse rotation. Since in the pushover analysis a column sway mechanism at the fourth storey has been observed, the mentioned drift corresponds to the ultimate drift of the fourth storey, equal to 2.5375%. It should be noticed also that during the nonlinear dynamic analyses, in all the cases in which the collapse has been reached, a mechanism at the third or fourth storey has been observed.

		Hazard curve criterion a)
Hazard curve criterion (a)	Approximation of the first order	$H\left(S_a^{NC}\right)=3\cdot10^{-5}\left(S_{a,1}^{NC}\right)^{-2.827}$
	Approximation of the second order	$H\left(S_a^{NC}\right)=2.62\cdot10^{-6}\cdot e^{\left(-0.878\ln^2 S_a(T_1)-5.923\ln S_a(T_1)\right)}$
Hazard curve criterion (b)	Approximation of the first order	$H\left(S_a^{NC}\right)=7\cdot10^{-6}\left(S_{a,1}^{NC}\right)^{-4.03}$
	Approximation of the second order	$H\left(S_a^{NC}\right)=3.04\cdot10^{-7}\cdot e^{\left(-2.18\ln^2 S_a(T_1)-9.312\ln S_a(T_1)\right)}$

Table 8: Approximation of the first and second order of the hazard curve, obtained with criterion (a) and (b).



(a) (b)

Figure 15: Approximations of the first and second order for the hazard curve with criterion (a) and (b)

Firstly, the variability of the parameters of the expression of the annual probability of failure is examined, in particular the value $H(S_{a,1}^{NC})$ of the hazard function, the dispersion parameter variable (β_{regr}) with seismic intensity and the constant dispersion (β_{cost}). With regard to the first order approximation of the hazard curve, the results reported in Table 9 show that:

- the spectral acceleration at failure decreases when the number of records increases and consequently the corresponding mean annual frequency increases; moreover, the parameter of constant dispersion also increases.
- comparing the values obtained for the structure with and without dampers, the greatest difference between the spectral acceleration at failure occurs in the second case with a smaller number of records; this shows a great variability of parameters according to the number of considered records; there is also a great variability of the values of mean annual frequency.
- moreover, the dispersion which varies with seismic intensity (β_{regr}), is greater for the structure with dampers than for the structure without dampers; on the contrary the parameter of constant dispersion (β_{cost}) is greater for the structure without dampers.

	$D_{roof,u}$=0.145 m					$\delta_{max,u}$=2.5375%				
	Structure with dampers			Structure without dampers		Structure with dampers			Structure without dampers	
	104	170	180	104	180	104	170	180	104	180
$S_{a,1}{}^{NC}$	1.41	0.68	0.61	0.62	0.37	2.89	1.22	1.11	0.65	0.41
$H(S_{a,1}{}^{NC})$ Hazard curve (a)	$1.2\cdot10^{-5}$	$8.8\cdot10^{-5}$	$1.2\cdot10^{-4}$	$1.2\cdot10^{-4}$	$4.7\cdot10^{-4}$	$1.5\cdot10^{-6}$	$1.7\cdot10^{-5}$	$2.3\cdot10^{-5}$	$1.0\cdot10^{-4}$	$3.8\cdot10^{-4}$
$H(S_{a,1}{}^{NC})$ Hazard curve (b)	$1.8\cdot10^{-6}$	$3.3\cdot10^{-5}$	$4.9\cdot10^{-5}$	$4.7\cdot10^{-5}$	$3.6\cdot10^{-4}$	$9.7\cdot10^{-8}$	$3.1\cdot10^{-6}$	$4.7\cdot10^{-6}$	$3.9\cdot10^{-5}$	$2.6\cdot10^{-4}$
β_{regr}	0.63	0.60	0.67	0.47	0.65	1.07	0.87	1.07	0.25	0.75
β_{cost}	0.42	0.45	0.47	0.52	0.56	0.42	0.46	0.48	0.56	0.64

Table 9: D_{roof} and δ_{max}, variability of the parameters which define the annual probability of failure, varying the number of records for the approximation of the first order of the hazard curve.

Secondly, the variability of the annual probability of failure is examined. With regard to the first order approximation of the hazard curve, the results are reported in Tables 10 to 13, which show the values of probability together with the corresponding values of hazard and dispersion. Tables 10 and 11, relative to the collapse defined by $D_{roof,u}$, shows that:

- under the same conditions of structure and dispersion, the values of annual probability of failure vary considerably depending on the hazard curves (a) or (b);
- under the same hazard curves (a) or (b), a slightly higher value of mean annual frequency and a smaller value of dispersion determine higher values of annual probability of failure; this shows the great influence of the dispersion on the final value of the annual probability of failure.
- as for the parameter of constant dispersion, a much greater annual probability of failure has been obtained for the structure without dampers than for the structure with dampers; this does not always occur when the parameter of variable dispersion is used, as for the Case 2, hazard curve (b).

The same observations can be made for the collapse defined by $\delta_{max,u}$ (Tables 12 and 13) with the additional observation that, when the variable dispersion parameter (β_{regr}) is used, the annual probability of failure is much greater for the structure with dampers than for the structure without dampers. Consequently, the results are in line with the expectations only when the parameter of constant dispersion (β_{cost}) is used.

As for the second order approximation of the hazard curve, for both the demand parameters and for the same structure, there is a smaller difference among the values of $H(S_{a,1}{}^{NC})$ determined with different intervals for interpolating the hazard curve (criterion (a) or (b)) than with the first order approximation. This is shown in Tables 14 and 15 for the Case 3.

	$D_{roof,u}$=0.145 m					
	Case 1		Case 2		Case 3	
	Dampers 170	Without dampers 104	Dampers 104	Without dampers 104	Dampers 180	Without dampers 180
$H(S_{a,1}{}^{NC})$ Hazard curve (a)	$8.83\cdot10^{-5}$	$1.15\cdot10^{-4}$	$1.15\cdot10^{-5}$	$1.12\cdot10^{-4}$	$1.19\cdot10^{-4}$	$4.75\cdot10^{-4}$
β_{regr}	0.6043	0.4686	0.6319	0.4686	0.6714	0.6486
$P_{F,NC}$	$5.38\cdot10^{-4}$	$7.05\cdot10^{-4}$	$3.67\cdot10^{-4}$	$7.05\cdot10^{-4}$	$7.95\cdot10^{-4}$	$2.21\cdot10^{-3}$
β_{cost}	0.4523	0.5252	0.4224	0.5252	0.4696	0.5651
$P_{F,NC}$	$2.79\cdot10^{-4}$	$9.96\cdot10^{-4}$	$7.32\cdot10^{-5}$	$9.96\cdot10^{-4}$	$3.46\cdot10^{-4}$	$1.62\cdot10^{-3}$

Table 10: Annual failure probability for the first order approximation of the hazard curve with criterion (a) and for collapse defined by $D_{roof,u}$.

	$D_{roof,u}$=0.145 m					
	Case 1		Case 2		Case 3	
	Dampers 170	Without dampers 104	Dampers 104	Without dampers 104	Dampers 180	Without dampers 180
$H(S_{a,1}^{NC})$ Hazard curve (b)	$3.26\cdot10^{-5}$	$4.76\cdot10^{-5}$	$1.77\cdot10^{-6}$	$4.76\cdot10^{-5}$	$4.98\cdot10^{-5}$	$3.59\cdot10^{-4}$
β_{regr}	0.6043	0.4686	0.6319	0.4686	0.6714	0.6486
$P_{F,NC}$	$1.28\cdot10^{-3}$	$1.89\cdot10^{-3}$	$2.04\cdot10^{-3}$	$1.89\cdot10^{-3}$	$2.37\cdot10^{-3}$	$8.18\cdot10^{-3}$
β_{cost}	0.4523	0.5252	0.4224	0.5252	0.4696	0.5651
$P_{F,NC}$	$3.37\cdot10^{-4}$	$3.82\cdot10^{-3}$	$7.69\cdot10^{-5}$	$3.82\cdot10^{-3}$	$4.37\cdot10^{-4}$	$4.32\cdot10^{-3}$

Table 11: Annual failure probability for the first order approximation of the hazard curve with criterion (b) and for collapse defined by $D_{roof,u}$.

	$\delta_{max,u}$=2.5375%					
	Case 1		Case 2		Case 3	
	Dampers 170	Without dampers 104	Dampers 104	Without dampers 104	Dampers 180	Without dampers 180
$H(S_{a,1}^{NC})$ Hazard curve (a)	$1.69\cdot10^{-5}$	$1.01\cdot10^{-4}$	$1.48\cdot10^{-6}$	$1.01\cdot10^{-4}$	$2.27\cdot10^{-5}$	$3.82\cdot10^{-4}$
β_{regr}	0.867	0.2468	1.0689	0.2468	1.0695	0.7554
$P_{F,NC}$	$2.94\cdot10^{-4}$	$1.54\cdot10^{-4}$	$2.00\cdot10^{-3}$	$1.54\cdot10^{-4}$	$1.04\cdot10^{-3}$	$1.19\cdot10^{-3}$
β_{cost}	0.4595	0.5629	0.4224	0.5629	0.4803	0.6389
$P_{F,NC}$	$4.55\cdot10^{-5}$	$3.42\cdot10^{-4}$	$6.69\cdot10^{-6}$	$3.42\cdot10^{-4}$	$5.95\cdot10^{-5}$	$8.92\cdot10^{-4}$

Table 12: Annual failure probability for the first order approximation of the hazard curve with criterion (a) and for collapse defined by $\delta_{max,u}$.

	$\delta_{max,u}$=2.5375%					
	Case 1		Case 2		Case 3	
	Dampers 170	Without dampers 104	Dampers 104	Without dampers 104	Dampers 180	Without dampers 180
$H(S_{a,1}^{NC})$ Hazard curve (b)	$3.09\cdot10^{-6}$	$3.93\cdot10^{-5}$	$9.67\cdot10^{-8}$	$3.93\cdot10^{-5}$	$4.72\cdot10^{-6}$	$2.63\cdot10^{-4}$
β_{regr}	0.867	0.2468	1.0689	0.2468	1.0695	0.7554
$P_{F,NC}$	$1.02\cdot10^{-3}$	$9.33\cdot10^{-5}$	$2.21\cdot10^{-1}$	$9.33\cdot10^{-5}$	$1.12\cdot10^{-2}$	$2.64\cdot10^{-3}$
β_{cost}	0.4595	0.5629	0.4224	0.5629	0.4803	0.6389
$P_{F,NC}$	$2.31\cdot10^{-5}$	$4.72\cdot10^{-4}$	$2.05\cdot10^{-6}$	$4.72\cdot10^{-4}$	$3.33\cdot10^{-5}$	$1.48\cdot10^{-3}$

Table 13: Annual failure probability for the first order approximation of the hazard curve with criterion (b) and for collapse defined by $\delta_{max,u}$.

Tables 14 and 15 report the values of the annual probability of failure obtained as follows: with the approximation of the first order suggested by the 2000 SAC/FEMA method [5] (Eq.(11)); with the approximation of the second order suggested by Vamvatsikos [12]. The latter can be evaluated as follows:

$$H(S)=k_0\cdot e^{\left(-k_2\ln^2 S_a(T_1)-k_1\ln S_a(T_1)\right)} \tag{16}$$

where $k_1>0$ and $k_2\geq0$. Using the approximation of the second order for the hazard curve, the new closed-form expression of the annual probability of failure [12] becomes:

$$P_{F,NC} = \sqrt{p} \cdot k_0^{1-p} \cdot \left[H\left(S_{a,1}^{NC}\right)\right]^p \cdot e^{\left(\frac{1}{2} p k_1^2 \beta_{sc}^2\right)} \quad (17)$$

where:

$$p = \frac{1}{1 + 2k_2 \beta_{Sc}^2} \qquad 0<p<1 \quad (18)$$

The difference between Eqs. (11) and (17) is the insertion of the factor p. Substituting k_2=0 in Eq. (18), it can be noticed that p becomes equal to unity and that Eq. (17) simplifies as:

$$P_{F,NC} = \left[H\left(S_{a,1}^{NC}\right)\right]^p \cdot e^{\left(\frac{1}{2} p k_1^2 \beta_{sc}^2\right)} \quad (19)$$

It can be observed that the reduction of Eq.(17) to Eq.(19) does not coincide exactly with the original formulation of the SAC/ FEMA [2] , but it is a simplification.

From Tables 14 and 15 it is possible to notice that, considering the second order approximation of the hazard curve (Eq.(17)), the variation, for the same structure and dispersion, of the annual failure probability with the number of values of T_R (9 values of T_R corresponds to criterion (a), 5 values of T_R to criterion (b)) is reduced if compared with the same variation obtained with the first order approximation. This is a consequence of the above mentioned reduction of the variation of the hazard value $H(S_{a,1}^{NC})$.

Moreover, considering the second order approximation of the hazard curve, a lower influence of the dispersion on the annual failure probability has been obtained than with the first order approximation. Finally it is possible to observe that, considering the second order approximation and using both the dispersion parameters, the results are in line with the expectations, i.e. the annual failure probability for the structure without dampers is always much greater than that obtained for the structure without dampers.

$D_{roof,u}$ = 0.145 m With Dampers $S_{a,1}^{NC}$=0.6144, a=0.2421, b=1.0523, β_c=0.275						
	9 T_R I order (2000SAC/FEMA method)	9 T_R I order (Vamvatsikos)	9 T_R II order (Vamvatsikos)	5 T_R I order (2000SAC/FEMA method)	5 T_R I order (Vamvatsikos)	5 T_R II order (Vamvatsikos)
$H(S_{a,1}^{NC})$	$1.189{\cdot}10^{-4}$	$1.189{\cdot}10^{-4}$	$3.81{\cdot}10^{-5}$	$4.985{\cdot}10^{-5}$	$4.985{\cdot}10^{-5}$	$1.69{\cdot}10^{-5}$
			β_{regr} = 0.6717			
$P_{F,NC}$	$7.95{\cdot}10^{-4}$	$7.21{\cdot}10^{-4}$	$7.19{\cdot}10^{-4}$	$2.37{\cdot}10^{-3}$	$1.94{\cdot}10^{-3}$	$4.99{\cdot}10^{-4}$
			β_{cost}=0.4696			
$P_{F,NC}$	$3.46{\cdot}10^{-4}$	$2.87{\cdot}10^{-4}$	$2.49{\cdot}10^{-4}$	$4.37{\cdot}10^{-4}$	$2.98{\cdot}10^{-4}$	$2.20{\cdot}10^{-4}$
Without dampers $S_{a,1}^{NC}$=0.3763, a=0.44, b=1.1357, β_c=0.275						
	9 T_R I order (2000SAC/FEMA method)	9 T_R I order (Vamvatsikos)	9 T_R II order (Vamvatsikos)	5 T_R I order (2000SAC/FEMA method)	5 T_R I order (Vamvatsikos)	5 T_R II order (Vamvatsikos)
$H(S_{a,1}^{NC})$	$4.75{\cdot}10^{-4}$	$4.75{\cdot}10^{-4}$	$3.70{\cdot}10^{-4}$	$3.6{\cdot}10^{-4}$	$3.6{\cdot}10^{-4}$	$3.4{\cdot}10^{-4}$
			β_{regr} = 0.6486			
$P_{F,NC}$	$2.21{\cdot}10^{-3}$	$2.55{\cdot}10^{-3}$	$2.38{\cdot}10^{-3}$	$8.18{\cdot}10^{-3}$	$1.09{\cdot}10^{-2}$	$1.34{\cdot}10^{-3}$
			β_{cost}=0.5651			
$P_{F,NC}$	$1.62{\cdot}10^{-3}$	$1.70{\cdot}10^{-3}$	$1.81{\cdot}10^{-3}$	$4.32{\cdot}10^{-3}$	$4.80{\cdot}10^{-3}$	$1.21{\cdot}10^{-3}$

Table 14: Annual failure probability for different hazard curve approximations and for collapse defined by $D_{roof,u}$.

$\delta_{max,u}$ = 2.5375%						
With Dampers						
$S_{a,1}^{NC}$=1.1027, a=2.27, b=1.1285, β_c=0.275						
	9 T_R I order (2000SAC/FEMA method)	9 T_R I order (Vamvatsikos)	9 T_R II order (Vamvatsikos)	5 T_R I order (2000SAC/FEMA method)	5 T_R I order (Vamvatsikos)	5 T_R II order (Vamvatsikos)
$H(S_{a,1}^{NC})$	$2.27\cdot10^{-5}$	$2.27\cdot10^{-5}$	$1.46\cdot10^{-6}$	$4.72\cdot10^{-6}$	$4.72\cdot10^{-6}$	$1.20\cdot10^{-7}$
			β_{regr}=1.0695			
$P_{F,NC}$	$1.04\cdot10^{-3}$	$2.20\cdot10^{-3}$	$9.76\cdot10^{-4}$	$1.12\cdot10^{-2}$	$5.10\cdot10^{-2}$	$4.21\cdot10^{-4}$
			β_{cost}=0.4803			
$P_{F,NC}$	$5.95\cdot10^{-5}$	$5.72\cdot10^{-5}$	$2.586\cdot10^{-5}$	$3.33\cdot10^{-5}$	$3.073\cdot10^{-5}$	$1.98\cdot10^{-5}$
Without dampers						
$S_{a,1}^{NC}$=0.4067, a=9.86, b=1.1509, β_c=0.275						
	9 T_R I order (2000SAC/FEMA method)	9 T_R I order (Vamvatsikos)	9 T_R II order (Vamvatsikos)	5 T_R I order (2000SAC/FEMA method)	5 T_R I order (Vamvatsikos)	5 T_R II order (Vamvatsikos)
$H(S_{a,1}^{NC})$	$3.82\cdot10^{-4}$	$3.82\cdot10^{-4}$	$2.66\cdot10^{-4}$	$2.628\cdot10^{-4}$	$2.628\cdot10^{-4}$	$2.27\cdot10^{-4}$
			β_{regr}=0.7554			
$P_{F,NC}$	$1.19\cdot10^{-3}$	$3.73\cdot10^{-3}$	$2.75\cdot10^{-3}$	$2.64\cdot10^{-3}$	$2.70\cdot10^{-2}$	$1.30\cdot10^{-3}$
			β_{cost}=0.6389			
$P_{F,NC}$	$8.92\cdot10^{-4}$	$1.95\cdot10^{-3}$	$1.90\cdot10^{-3}$	$1.48\cdot10^{-3}$	$7.23\cdot10^{-3}$	$1.15\cdot10^{-3}$

Table 15: Annual failure probability for different hazard curve approximations and for collapse defined by $\delta_{max,u}$.

5. CONCLUSIONS

In conclusion, as for the probabilistic assessment we can note that it depends on the number of records considered. In particular, reliable trends have been obtained only for 180 records for both structures with and without dampers. As for the median, the median values for the structure without dampers are greater than those obtained for the structure with dampers regardless of the number of record considered. With regard to β_{regr} it is possible to notice that: it increases with seismic intensity; it depends on the results of the dynamic nonlinear analyses, and in particular on the number of records; the expected trend, i.e. greater dispersion for the structure without dampers than for the structure with dampers, has been determined only for the higher number of results (180 records). As for β_{cost}, we can note that: the expected trend has been obtained also for the lower number of records; the parameter of constant dispersion increases with the number of seismic events both for the structure with and without dampers. Moreover, relations between the expressions of β_{cost} and β_{regr} for the structure without and with dampers have been derived with the purpose to obtain the dispersion for the structure with dampers by knowing that for the structure without dampers.

It is possible to observe that the simplified formula to determine the annual probability of failure $P_{F,NC}$ suggested by the 2000 SAC/FEMA method is strongly sensitive to variations of the hazard curve and of the dispersion. It is possible to note that for the hazard curves determined with the first order approximations, different values of $P_{F,NC}$ have been obtained by changing the interval in which the hazard curve is interpolated. Moreover, the simplified formula for $P_{F,NC}$ is particularly affected by the dispersion. $P_{F,NC}$ values always in line with the expectations have been obtained only using the parameter of dispersion β_{cost}. Considering the second order approximation of the hazard curve, the variation of the annual failure probability when changing the interval for interpolating the hazard curve is reduced if compared with the same variation obtained with the first order approximation. With the mentioned second order approximation it is also reduced the influence of the values of dispersion and the $P_{F,NC}$ is always greater for the structure without dampers than for the structure with dampers.

Therefore, it is possible to deduce that it is better to use the parameter of constant dispersion β_{cost} when we consider the interpolation of the first order for the hazard curve. Otherwise, using the second order approximations allows to obtain values of probability less sensitive to the interval for interpolating the hazard curve and to the type of dispersion β_{regr} or β_{cost}.

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