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CREATION OF A NEW EQUATION OF MOTION TO CALCULATE DISSIPATED ENERGY BETWEEN TWO ADJACENT BUILDINGS (COMPDYN 2015)

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. Building pounding has been recently studied by different researchers around the world to evaluate impact between two adjacent buildings. Numerical studies are an important part of impact, in which several researchers have tried to simulate the impact by using different formulas. Estimation of the impact force and the dissipated energy depends significantly on some parameters of impact. Mass of bodies, stiffness of spring, coefficient of restitution, damping ratio of dashpot and impact velocity are some known and unknown parameters to simulate the impact and to measure dissipated energy during collision.

In this paper, a mathematic part of steady state response of single degree of freedom system due to the external force has been evaluated. Impact damping ratio due to viscose damping was calculated to simulate nonlinear viscoelastic model and the results of impact were compared with other similar impact damping ratios, suggested in previous studies. Here, a new equation of motion is presented to get the best available estimation of impact damping ratio and is also simulated to compute impact force during seismic excitation.

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INTRODUCTION

In the introduction to the paper, the author(s) will specify the present stage of the branch researches (by quoting the adequate bibliography) and will specify the purpose of the paper. Building pounding has been a stimulating research topic during the last few decades in earthquake engineering. This phenomenon describes impacts under two adjacent buildings under seismic excitations. If adjacent buildings were not separated suitably from each other, impact forces could cause damage to buildings even if structures were well designed. Unfortunately, enough attention is not being paid to the building pounding effects in the codes. However, several researchers have tried to investigate the effects of such collisions worldwide. Anagnostopolos [1] was among the first researchers, who explained the possible dangers due to building pounding. The 1985 Mexico City earthquake was one of the source of excitation which caused severe damages to the buildings. Several reports showed that in at least 15% of buildings damages were due to impact of adjacent buildings [2]. Investigation of the building pounding has been divided into two parts, experimental tests and numerical analyses. In this regard, Papadrakakis and Mouzakis [3] conducted shaking table experiments on pounding between two-story reinforced concrete buildings without separation distance under the earthquake records. Two steel buildings, having three and eight stories have been tested by shaking table (Filiatrault [4]). The tests were carried out with two different gaps, zero and 15 mm. The experimental results were compared with analytical predictions based on linear elastic spring theory. The comparisons showed that acceleration at the contact level was not well predicted. Watanaba and Kawashima [5] have investigated pounding of distributed masses to model colliding bridge decks. They showed that five elements were used per deck; the collision element stiffness would be five times greater than that of the diagram stiffness. Cole et al. [6] have indicated that building pounding and its impact would depend upon the structural properties and collision velocity of both buildings. A theoretical maximum collision force has been determined by them, for a system with two distributed masses. Velocity, mass and stiffness at the time of impact have an equation, and the number and magnitude of the impacts depend on these three options mentioned. Barros and Khatami [7] addressed some common misrepresentations in building codes on the issue of separation distance required between two adjacent concrete buildings under near-fault ground motions. In numerical analyses, link elements are located between two buildings investigated. Komodromos et al also took advantage of link elements extensively in his research [8]. Barros and Vasconcelos [9] investigated building pounding between two adjacent concrete buildings by numerical analyses. They presented the results of analyses using different stiffness and damping ratios. Two concrete buildings with 8 and 10 stories have been modeled by Raj Pant and Wijeyewickrema [10]. Different types of springs with different stiffness or various dampers with different damping ratio have also been used in recent numerical studies at FEUP by Cordeiro [11] and by Vasconcelos [12], in their parametric studies of pounding between adjacent buildings. Barros and Khatami [13] estimate the effect of damping ratio on the numerical study of impact forces between two adjacent concrete buildings subjected to pounding. In yet another study, Barros and Khatami [14] compare results of two SDOF frames with different link elements based on mathematic equations. In some of their analyses, structures were modeled as single degree freedom systems and collision was simulated using linear viscoelastic models of impact force.

In this study, a new equation of motion is suggested to calculate the impact force in the nonlinear viscoelastic model. By using a mathematic equation and developing the mentioned equation, damping impact ratio is simulated and the behavior of nonlinear viscoelastic model is compared with other suggested formula.

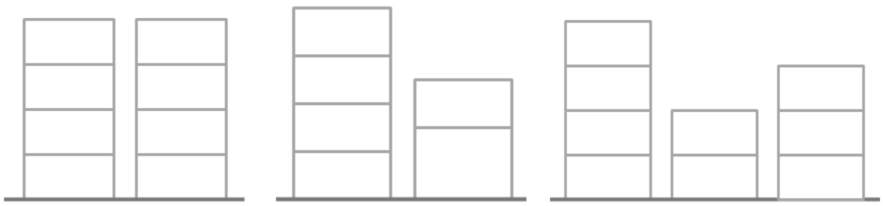
Building Pounding

Investigation of previous earthquake around the world shows specific characteristics related to building pounding. Pounding is one of the primary reasons of many building damages in earthquake. This phenomenon has been reported in many earthquakes between two adjacent buildings. Seismic pounding occurs during earthquake, in building with different dynamic characteristics, adjacent buildings vibrating out of phase and in buildings by having insufficient separation. Past studies and reports about building pounding have shown three cases of impact between adjacent buildings. The classification mentioned collisions are:

2,A- Adjacent buildings with the same heights and the same floor levels.

2,B- Adjacent buildings with different total height and with different levels.

2,C- Row of buildings.



2,A 2,B 2,C

Fig 1. Different types of pounding between buildings

Previous investigations have shown that the pounding hazard depends significantly upon four factors:

-building construction type based on relative height, period and story mass;

-ground acceleration;

-soil condition;

-distance between buildings.

The most important parameters are buildings height and distance between buildings. Since different buildings have various behaviors during seismic excitation, insufficient gap size between structures can provide damages for collided buildings. In order to evaluate pounding between two buildings and measure impact force and dissipated energy, an un-real link element between two bodies is assumed. Mentioned link element is included spring and dashpot, which are located parallel to each other. Dashpot is used to calculate the value of dissipated energy during impact.

Dissipated Energy in Viscous Damping and Rate Independent Damping

Pounding occurs when buildings have different dynamic characteristics, height, mass, number of stories and buildings materials. This phenomenon between adjacent buildings could cause severe damage and even collapse during seismic excitation.

To measure energy absorption due to impact, an un-real link element is assumed between adjacent structures. This link element can be obtained by spring and dashpot, which are located parallel with each other. Dashpot absorbs energy when the relative displacements exceed the available seismic gap. So, determination of impact force depends upon two different terms, which are mentioned as:

$$E_D = \int f_D d\delta = \int (c\dot{\delta})\dot{\delta}dt = \int c\dot{\delta}^2 dt \qquad (1)$$

$$E_S = \int f_S d\delta = \int (k\delta)\dot{\delta}dt \qquad (2)$$

It means, Impact force is calculated by stiffness of spring and damping of dashpot. It is simplified to be:

$f_S = k.\delta$ and $f_D = c.\dot{\delta}$. (3)

Where k is stiffness of spring and c denotes damping coefficient. δ and $\dot{\delta}$ are lateral displacement and velocity, respectively.

When there is collision, both terms should be added with each other to measure the impact force. The impact force is considered to be:

$$f = f_S + f_D \qquad (4)$$

It means, the impact force could be written as:

$$f = k\delta + c\dot{\delta} \qquad (5)$$

It is assumed that the lateral displacement becomes $\delta = \delta_0 \sin(wt - \phi)$, then impact force by using dashpot would be defined as:

$$f_D = c\dot{\delta}(t) = cw\delta_0 \cos(wt - \phi) \qquad (6)$$

$$f_D = cw\sqrt{\delta_0^2 - \delta_0^2 \sin^2(wt - \phi)} \qquad (7)$$

$$f_D = cw\sqrt{\delta_0^2 - [\delta(t)]^2} \qquad (8)$$

It could be an estimation that new equation would be represented as:

$$\left(\frac{\delta}{\delta_0}\right)^2 + \left(\frac{f_D}{cw\delta_0}\right)^2 = 1 \qquad (9)$$

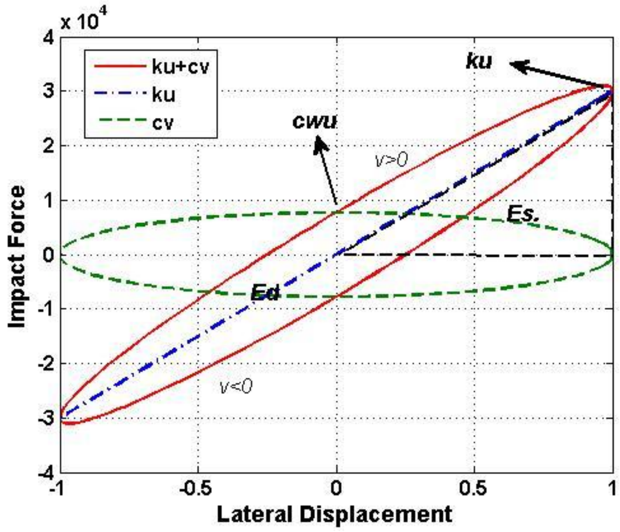


Fig 2. Hystresis loop by spring and dashpot

Three available terms of impact are depicted. Blue line shows a linear behavior by f_S, which is included a spring. It is obviously shown that impact force by using spring is $f_s = k\delta$, and dissipated energy is zero. Green line is an important curve in this figure. In fact, the curve is an ellipse, where energy is dissipated in this area and the value of dissipated energy is $\pi c w \delta_0^2$. When two different terms of impact joint with each other, ellipse rotates and red line is provided and is demonstrated a complete impact. Red ellipse is divided by two parts, E_{s0} and E_D.

Two different damping coefficients are explained by viscous damping and rate independent damping. Both mentioned damping are developed as:

$$E_D = \pi c w \delta_0^2 \tag{10}$$

$$\pi c w \delta_0^2 = 2\pi\zeta \frac{w}{w_n} k\delta_0^2 \tag{11}$$

In terms of viscose damping, it is recommended to use c for impact damping factor as:

$$c = \frac{2k\zeta}{w_n} \tag{12}$$

This equation of motion shows that impact damping coefficient depends significantly on damping ratio, frequency and stiffness of system.

In order to solve equation (12) to get impact damping coefficient with rate independent damping, it could be written as:

$$w_n = w\sqrt{1-\zeta} \tag{13}$$

$$c = \frac{2k\zeta}{w\sqrt{1-\zeta}} \tag{14}$$

Defining c from equation (14)

$$c = \frac{2\zeta\sqrt{km}}{\sqrt{1-\zeta}} \qquad (15)$$

The calculated equation shows a relation between damping coefficient system and rate independent damping.

Nonlinear Impact Models

Impact model between two bodies during earthquake has been shown by a spring and damper, which are located parallel with each other. The explanation of the impact model focuses on two structures next to each other when they collide during seismic excitation.

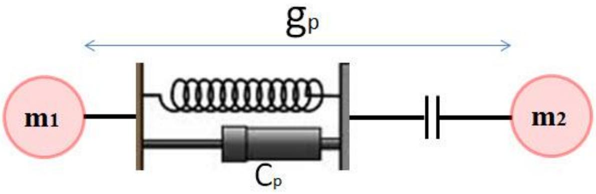


Fig 3. Hertz Damped Model

The impact force between two bodies depends significantly on collided masses, velocity and acceleration. In this part, some available models are demonstrated to get better image from contact.

In order to calculate value of the energy dissipation, some researchers have suggested different equations to simulate the damping ratio. They have tried to get the most appropriate assumption for evaluating dissipated energy during collisions. A summary of previous studies about impact damping ratio of pounding shows that the suggested formula by Anagnostopolus [1], and two represented formulas by Jankowski [15] are based on coefficient of restitution (CR) and calculate the impact damping ratio by mathematic equations. Coefficient of restitution is a factor, which simulate an equation between velocity before and after collision. This equation could be written as:

$$0 < CR = \frac{v_{before}}{v_{after}} < 1 \qquad (16)$$

As it is shown, coefficient of restitution is calculated to be in the range of 0 and 1. If CR becomes equal to 0, collision is perfectly plastic and if CR becomes equal to 1, collision shows an elastic behavior.

Non-Linear Viscoelastic Model

In order to improve the impact model, Jankowski [15] presented an idea that a nonlinear viscose damper located parallel to the spring for absorbing the energy. The equation of nonlinear viscoelastic model can be written as:

$$F_c(t) = k_h.\delta(t)^{1.5} + c_h\dot{\delta}(t) \tag{17}$$

$$c_h = 2\zeta\sqrt{k_h\sqrt{\delta(t)}\,\frac{m_1 m_2}{m_1+m_2}} \tag{18}$$

$$\zeta = \frac{9\sqrt{5}}{2}\,\frac{1-CR^2}{CR(CR(9\pi-16)+16)} \tag{19}$$

Another formula in terms of non-linear viscoelastic model was developed by Seyed Mahmoud [16]. He suggested a new equation to determine impact damping ratio, which absorbs the energy less than that simulated by Jankowski model. The equation could be written as:

$$\zeta = \frac{1-CR^2}{CR(CR(\pi-2)+2)} \tag{20}$$

Proposed Non-Linear Viscoelastic Model

To calculate impact damping ratio and develop the linear and nonlinear viscoelastic model using a mathematic process [15], a steady state response of single degree of freedom system due to the external force, $p(t) = p_0\sin(wt)$, was evaluated. Based on mentioned force, dissipated energy due to viscose damper in a harmonic excitation can be written by the following the equation:

$$\Delta E = \int f_D\, d\delta = \int_0^{2\pi/w}(c_k\dot{\delta})\dot{\delta}dt \tag{21}$$

And also by focusing strongly on damping term of equation of motion:

$$\Delta E = \int_0^{\delta_{\max}} c_k\dot{\delta}d\delta \tag{22}$$

Based on equations on dynamics of structure [17]:

$$\Delta E = 2\pi\zeta\frac{w}{w_n}k\delta^2 \tag{23}$$

The equation between w and w_n could be assumed to be 1. Considering final velocity and masses, it could be written as:

$$\int f_D d\delta = \int_0^{2\pi/w} (c_k \dot{\delta})\dot{\delta} dt = 0.5 m \dot{\delta}_{final}^2 \tag{24}$$

Where $\dot{\delta}_{final}$ denotes the final velocity and m is an equivalent of two masses ($m = \frac{m_1 m_2}{m_1 + m_2}$)

Solving the equation of 24 for δ:

$$\delta_{\max} = \sqrt{\frac{m}{4\pi\zeta k}}.\dot{\delta}_{final} \tag{25}$$

For each value of deformation during impact, as the energy transfers from elastic strain energy to kinetic energy, the relative velocity can be calculated as follows:

$$2\pi\zeta k \delta^2 + 0.5 m \dot{\delta}^2 = 0.5 m \dot{\delta}_{final}^2 \tag{26}$$

Solving equation 26,

$$0.5 m \dot{\delta}^2 = 0.5 m \dot{\delta}_{final}^2 - 2\pi\zeta k \delta^2 \tag{27}$$

Consequently, velocity would be:

$$\dot{\delta} = \sqrt{\dot{\delta}_{final}^2 - \frac{4k\xi\pi\delta^2}{m}} \tag{28}$$

Defining the equation 17 and considering displacement of zero ($\delta = 0$), it is an assumption that equation 28 can be changed increasing velocity to determine the velocity during collision for $\dot{\delta} > 0$ and $\dot{\delta} < 0$.

$$\dot{\delta} = -\sqrt{\dot{\delta}_{final}^2 - \frac{4k\xi\pi\delta^2}{m}} \quad \text{for } \dot{\delta} < 0$$

$$\dot{\delta} = \sqrt{\dot{\delta}_{final}^2 - \frac{4k\xi\pi\delta^2}{m}} + \frac{(\dot{\delta}_0 - |\dot{\delta}_{final}|)(\delta_{\max} - \delta)}{\delta_{\max}} \quad \text{for } \dot{\delta} > 0 \tag{29}$$

By using dissipated energy from damper and using equation 22, it could be calculated by:

$$\Delta E = 2\zeta \sqrt{k.m} \int \dot{\delta} d\delta \tag{30}$$

Considering above equations and substituting the equation 29 into equation 30:

$$\Delta E = 2\zeta\sqrt{k.m}(\int\sqrt{\dot{\delta}_{final}^{2} - \frac{4\zeta\pi k\delta^{2}}{m}}d\delta + \frac{(\dot{\delta}_{0} - |\dot{\delta}_{final}|)}{\delta_{\max}}\int_{0}^{\delta_{\max}}(\delta_{\max} - \delta)d\delta) \tag{31}$$

Considering the equation 25 into equation 31:

$$\Delta E = 2\zeta\sqrt{k.m}(\int_{0}^{\delta_{\max}}\sqrt{\frac{4\pi\zeta k\delta_{\max}^{2}}{m} - \frac{4\zeta\pi k\delta^{2}}{m}}d\delta) + \frac{\dot{\delta}_{0} - |\dot{\delta}_{final}|}{2}.\delta_{\max} \tag{32}$$

The equation is simplified to equation 32:

$$\Delta E = 2\zeta\sqrt{k.m}[(\sqrt{\frac{4\zeta\pi k}{m}})\int_{0}^{\delta_{\max}}\sqrt{\delta_{\max}^{2} - \delta^{2}}d\delta + \frac{\dot{\delta}_{0} - |\dot{\delta}_{final}|}{2}.\delta_{\max}] \tag{33}$$

$$\Delta E = 4k\sqrt{\pi}\zeta^{1.5}\int_{0}^{\delta_{\max}}\sqrt{\delta_{\max}^{2} - \delta^{2}}d\delta + 2\zeta\sqrt{k.m}\frac{\dot{\delta}_{0} - |\dot{\delta}_{final}|}{2}.\delta_{\max} \tag{34}$$

Solving the equation 34, it can be concluded that dissipated energy will be:

$$\Delta E = 4k\sqrt{\pi}\zeta^{1.5}(0.25\pi\delta_{\max}^{2}) + 2\zeta\sqrt{k.m}\frac{\dot{\delta}_{0} - |\dot{\delta}_{final}|}{2}.\delta_{\max} \tag{35}$$

As kinetic energy loss was noted in equation 32, it can be equal to dissipated energy from damper during impact as:

$$\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1 - CR^2)(\dot{\delta}_{imp})^2 = (4k\sqrt{\pi}\zeta^{1.5}(0.25\pi\delta_{\max}^{2})) + (2\zeta\sqrt{k.m}\frac{\dot{\delta}_{0} - |\dot{\delta}_{final}|}{2}.\sqrt{\frac{m}{4\pi\zeta k}}.\dot{\delta}_{final}) \tag{36}$$

Modifying $\delta_{\max}^{2}$ from equation 25 into equation 36:

$$\frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(1 - CR^2)(\dot{\delta}_{imp})^2 = (4k\sqrt{\pi}\zeta^{1.5}(0.25\pi\frac{m}{4\pi\zeta k}.\dot{\delta}_{final}^{2}) + (\zeta\sqrt{k.m}(\dot{\delta}_{0} - |\dot{\delta}_{final}|)\sqrt{\frac{m}{4\pi\zeta k}}.\dot{\delta}_{final}) \tag{37}$$

The equation 38 could be written as:

$$\frac{1}{2}(1 - CR^2)(\dot{\delta}_{imp})^2 = 0.25\sqrt{\pi}\zeta^{0.5}\dot{\delta}_{final}^{2} + \zeta^{0.5}\frac{\sqrt{\pi}}{\pi}(\dot{\delta}_{0} - |\dot{\delta}_{final}|).\dot{\delta}_{final}) \tag{38}$$

And finally, the equation for giving ζ :

$$\zeta = \left(\frac{(1-CR^2)}{CR(\sqrt{\pi}(0.5CR + \frac{1}{\pi}) - CR)}\right)^2 \tag{39}$$

Proposed non-linear impact model is considered based on coefficient of restitution, (CR). Selecting different CRs and calculating various results in terms of impact damping ratio, impact forces and dissipated energies is changed. Proposed impact damping ratio shows a curve based on different CRs, from zero to 320 using CR=0.9 and CR=0.1, respectively.

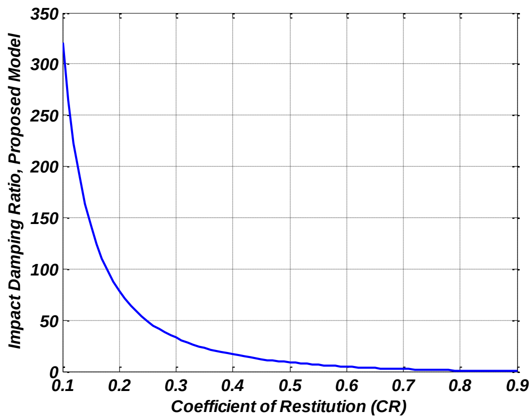


Fig 4. Comparison of different coefficient of restitution to calculate impact damping ratio

The value of ζ, calculated by coefficient of restitution is listed in the Table 1, as:

CR	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ζ	320.71	78.59	32.74	16.39	8.74	4.62	2.26	0.90	0.21

Table 1: Value of impact damping ratio using different coefficient of restitution

Conclusion

In this paper, building pounding between two adjacent structures was discussed and effect of pounding was studied. During earthquake, two buildings collide with each other and it could provide damage for structural elements. To investigate the pounding and impact between buildings, a non-real element is considered to calculate the value of impact and energy absorption. Mathematic equation of motion of impact force needs to define some parameters such as stiffness of spring, damping of dashpot, lateral displacement and velocity of bodies. Impact damping coefficient of dashpot is determined by using coefficient of restitution. Three previous models of this equation were evaluated by using linear viscoelastic model of impact and a new equation was suggested to calculate the damping of dashpot. A steady state response of single degree of freedom system due to the external force was considered and a new equation of motion was presented. To investigate the accuracy of suggested formula, the results of an experiential analysis were used and the results of formula were calibrated. Comparison of results showed good accuracy between experimental and numerical results. Finally, by using a SDOF system and an earthquake record and considering the proposed equation of motion, impact force of three investigated models were calculated. By focusing on suggested formula, impact force could be determined and dissipated energy shown by hysteresis loop.

REFERENCES

Anagnostopouls, S.A. "*Pounding of Building in Series during Earthquakes*". Earthquake Engineering and Structural Dynamic. 16(3).443-456.1998.

Bertero, V. "*Observation of Structural Pounding, Proc. ASCE International Conference on the 1985 Mexico City Earthquake*". 264-278.ASCE, NY, NY, USA.1986.

Papadrakakis, M and Mouzakis, H. "*A Lagrange multiplier solution method for pounding of building during earthquakes* ". Journal of Earthquake Engineering and Structural Dynamic, 20, 981-998.1991.

Filiatrault, A and Wagner, P. "*Analytical Prediction of Experimental Building Pounding*". Journal of Earthquake Engineering and Structural Dynamic, 24, 1131-1154.1995.

Kavashima C.G and Watanaba, K. "*Earthquake Induced Interaction between Adjacent Reinforced Concrete Structural with Non Equal Heights*" Earthquake Engineering and Structural Dynamics. 34(1).1-20. 2005.

Cole.G,L and Dhakal, R.P. " *The Effect of Diaphragm Wave Propagation on the Analysis of Pounding Structures*". Proc. 2 nd Int. Conference on computational method in structural dynamic an earthquake engineering (COMPADYN) Paper CD 200.Rhodes. Greece.2009.

Barros, R and Khatami, S.M. "*Importance of Separation Distance on Building Pounding under Near-Fault Ground Motion, using the Iranian Earthquake Code*" 9th International Congress on Civil Engineering, Isfahan University of Technology (IUT), Isfahan, Iran. 2012.

Komodromos, P and Polycarpou, P. "*On the Numerical Simulation of Impact for the Investigation of Earthquake-Induced Pounding of Building*". Tenth International Conference on Computational Structures Technology, Civil-Comp Press, Stirlingshire, Scotland.2010.

Vasconcelos, H. "*Study about Collision between Reinforced Concrete Building and Comparison of Their Resistance Envelope by Push over Analysis*" MS.c Thesis, FEUP. Portugal. 2011.

Raj, P and Wijeyewickrema, C. " *Seismic Pounding between Reinforced Concrete Buildings: A Study using two recently proposed Contact Element Models*". 14ECEE, Ohrid, Macedonia. 2010.