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# DISTRIBUTED AND CONCENTRATED INELASTICITY BEAM-COLUMN ELEMENTS: APPLICATION TO REINFORCED CONCRETE FRAMES AND VERIFICATION

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**Keywords:** Inelasticity, beam-column, displacement-based element, force-based element, discretization, earthquake engineering.

**Abstract:** *Inelastic beam elements are widely used in the analysis of structures subjected to seismic actions. These elements are capable of describing the linear and nonlinear force-displacement and moment-rotation response of beams, columns and shear walls. However, there are numerical inaccuracies associated with these element formulations, as well as differences in computational effort. In this paper, the fundamentals of displacement-based and force-based elements are briefly presented, with emphasis on localization issues due to strain softening, and regularization procedures necessary to achieve convergence to single solutions. The modelling criteria regarding the number of integration points, the number of elements in each member and the length of the prescribed plastic hinges are also discussed. A one-story reinforced concrete frame, representing industrial buildings, has been modelled in a finite element software with the objective of studying the efficiency of three element types: displacement- and force-based distributed plasticity elements and concentrated plasticity ones. The running times of the analyses confirm that the computational demand of models with displacement-based elements is larger than that of models with force-based elements. Analysis results have then been compared to the experimental values obtained from a series of pseudo-dynamic tests performed with increasing seismic intensity. All element formulations showed very good approximation of the global response. Overall, force-based and plastic hinge models performed slightly better than the displacement-based model.*

## 1 INTRODUCTION

Distributed and concentrated inelasticity beam-column elements are used for the assessment of structural response to strong ground motions. These elements are usually based on one of two formulations; the displacement-based stiffness method (DB), or the force-based flexibility method (FB). The DB formulation is the familiar textbook finite element method, with approximated interpolation functions for axial and transverse displacements. In contrast, the FB formulation is based on interpolation functions for the internal forces, which are chosen to represent the exact solution. While members modelled with the FB formulation only require a single element per member, DB members require several elements to obtain sufficient accuracy. Distributed inelasticity elements commonly have their sections discretized into several fibres with different material properties. This is called fibre modelling, and enables representing inhomogeneous element sections with reasonable accuracy. The first concentrated inelasticity elements where designed with nonlinear springs at the member ends [1]. However, it is now more common to use fibre modelling in the plastic hinge lengths to achieve better numerical results.

Both DB and FB element formulations experience localization issues for strain-softening, called non-objective response. This is a numerical failure issue due to the analysis not converging into one single solution as the member is further discretized. In the case of DB elements, this implies the increased number of elements in the member, while for FB elements it is the increased number of integration points. Several regularization procedures have been proposed to achieve objective response for strain-softening. DB elements can be regularized by adjusting the extremity element lengths, or by adding a damage variable to the constitutive relation [2, 3]. The regularization of FB elements is often done by adjusting stress-strain relationships by a constant fracture energy criterion, and by post-processing moment-curvature response based on plastic hinge lengths [4]. One of the most recent methods to achieve objective response is using the Improved Gauss-Radau integration scheme for concentrated inelasticity elements; thus reducing computational power as well as gaining increased accuracy [5].

The objective of the research presented in this paper is to compare the efficiency of the aforementioned modelling approaches for the analysis of reinforced concrete (RC) frame structures subjected to earthquakes. A single-storey industrial building was analysed using displacement- or force-based elements with distributed inelasticity and concentrated inelasticity ones. The analytical results were compared to experimental data obtained from a series of pseudo-dynamic tests of a full-scale model.

## 2 SEISMIC TEST

The Joint Research Centre of the European Commission has performed several experiments on the seismic behaviour of buildings and other structures. A full-scale one-story RC industrial building is examined in this paper [6]. The prototype was made of cast-in-situ beams and columns, consisting of two two-bay frames connected by a slab (Fig. 1). The column heights were 5050 mm, while the beams were 8300 mm long. The prototype was designed for a dead load equal to 27 kN/m$^2$, including the self-weight of the slab. All six columns were identical with section dimensions 300x300 mm, having 8Ø14 longitudinal reinforcement throughout their whole length (Fig. 2). The transverse reinforcement consisted of 6 mm bars. In the critical regions, which are 1 m from the bottom and top cross sections, stirrups were placed with 50 mm spacing. Outside these regions, the spacing was 150 mm. The beams had 600x300 mm sections, with such reinforcement that the plastic hinges form in the columns. Concrete cube specimen test gave cylindrical compressive strengths of $f_{cm}$ = 42.7 MPa for the columns and $f_{cm}$ = 47.2 MPa for the beams. The longitudinal steel reinforcement had a yielding strength of $f_y$ = 550 MPa, and a tensile strength of $f_y$ = 657 MPa.

Seismic tests of the cast-in-situ prototype were performed using the pseudo-dynamic method. Horizontal displacements were applied with the use of hydraulic jacks. In addition, vertical jacks were used to apply additional load on the slab, in total approximately 600 kN including the self-weight of the slab. The vertical jacks were always oriented towards a fixed base point, thus not producing any second-order P-delta effects. These effects were taken into account in the dynamic equilibrium model employed for the control of the pseudo-dynamic tests. The seismic ground motion was simulated by an artificial accelerogram (Fig. 3), which was generated to yield a response spectrum similar to the Type 1 response spectrum of Eurocode 8 [7] for ground type B. The prototype was subjected to four different acceleration time-histories based on the scaling of the ground motion for peak ground acceleration (PGA) equal to 0.05g, 0.32g, 0.64g, and 0.80g. The acceleregram with PGA = 0.05g was applied to calibrate the testing devices. The time-histories of top displacement measured during the tests are plotted in Fig. 4.

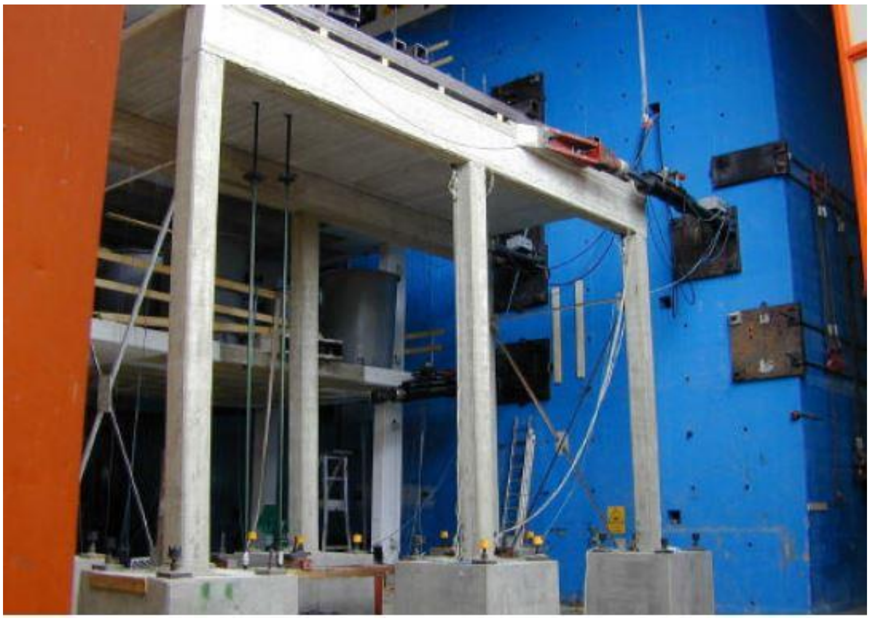

Figure 1: One-story RC frame [6].

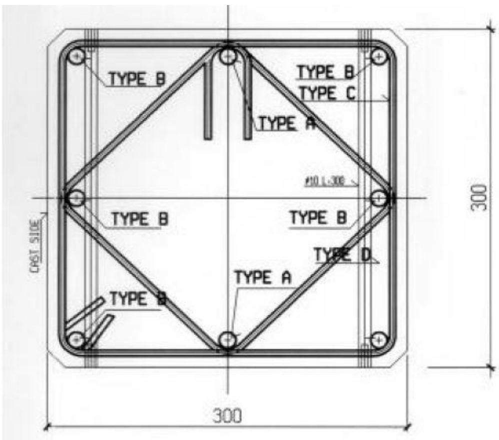


Figure 2: Column geometry and reinforcement [6].

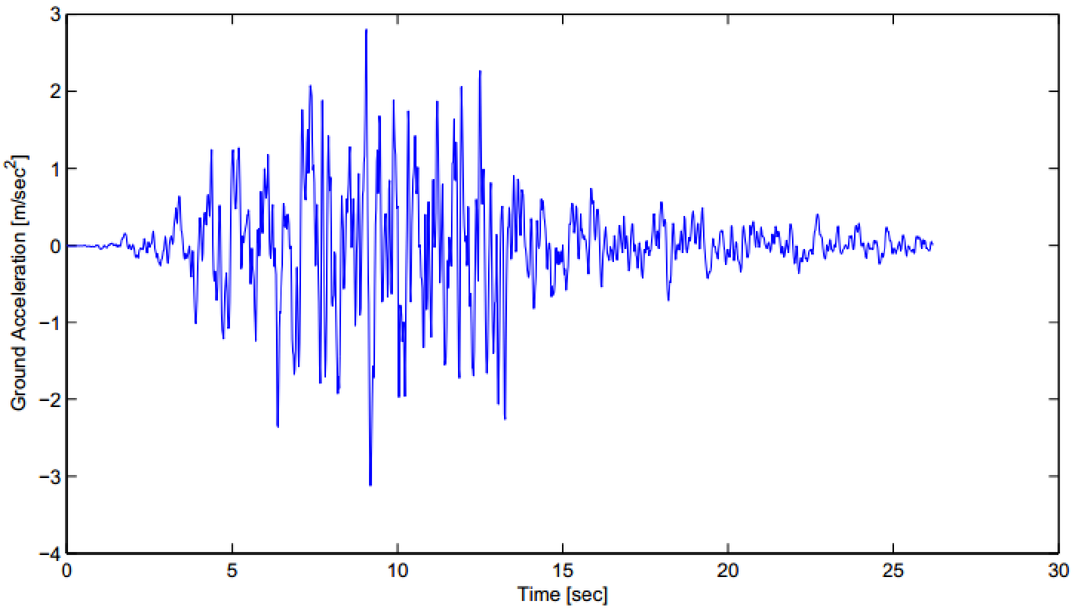


Figure 3: Artificial ground motion time-history with PGA 0.32g.

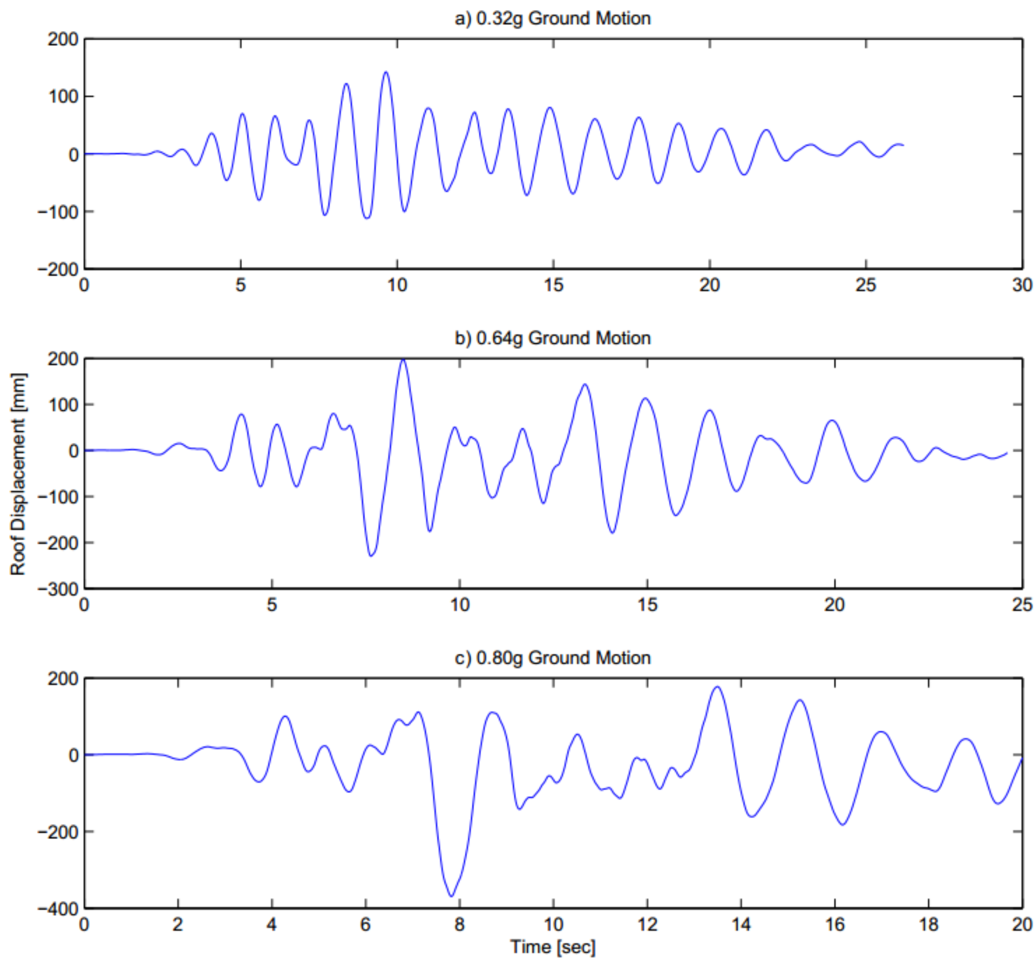


Figure 4: Displacement time-histories from pseudo-dynamic tests.

# 3 NUMERICAL MODELING

## 3.1 General

The numerical analysis has been performed in the finite element software SeismoStruct. It provides all the element types discussed in this paper, as well as options to modify their discretization properties. Because the structure is symmetric, only one of the frames has been modelled, carrying half of the total distributed vertical load, which is 300 kN including the self-weight of the slab. The column bases were fixed, with their heights set to 5350 mm, such that the structural nodes were situated in the centre of the beam section. The models developed by Chang and Mander [8] and Menegotto-Pinto [9] were used for the concrete and steel reinforcement elements respectively. The material properties were set equal to the ones measured from the prototype, with confinement factors calculated in SeismoStruct based on the amount of transverse reinforcement. The beams were modelled with large amounts of reinforcement, such that they had a significant overstrength compared to the columns. A static analysis was then performed by imposing the top displacement time-history provided from the experiment.

## 3.2 DB distributed inelasticity

The discretization of the columns for the different element types is illustrated in Figure 5. Based on earlier research, objective hardening response is expected when using a minimum of six DB elements per structural member with a two-point Gauss-Legendre integration scheme [10, 11]. Fewer elements tend to produce overly stiff solutions. In an effort to also achieve objective response in the case of strain-softening, the extremity elements of the column member have their lengths adjusted such that localization concentrates in one single integration point. This was done by choosing the element length as double the predetermined plastic hinge length, and was calculated as 1.146 m [12].

## 3.3 FB distributed inelasticity

Due to the fact that FB element members have exact solutions of the internal forces, it is not necessary to divide structural members into several elements [13]. A five-point Gauss-Lobatto integration scheme was used to achieve objective hardening response. Since no information on the concrete fracture energy was available for the prototype, no regularization procedure was applied for the FB element member.

## 3.4 FB concentrated inelasticity

The concentrated inelasticity members employed FB fibre modelling of the prescribed plastic hinges, with a linear elastic interior span. To ensure objectivity for both strain-hardening and strain-softening, the Modified Gauss-Radau integration scheme was used for the plastic hinges. It consists of two integration points: one is situated at the member end, and the second in the element interior.

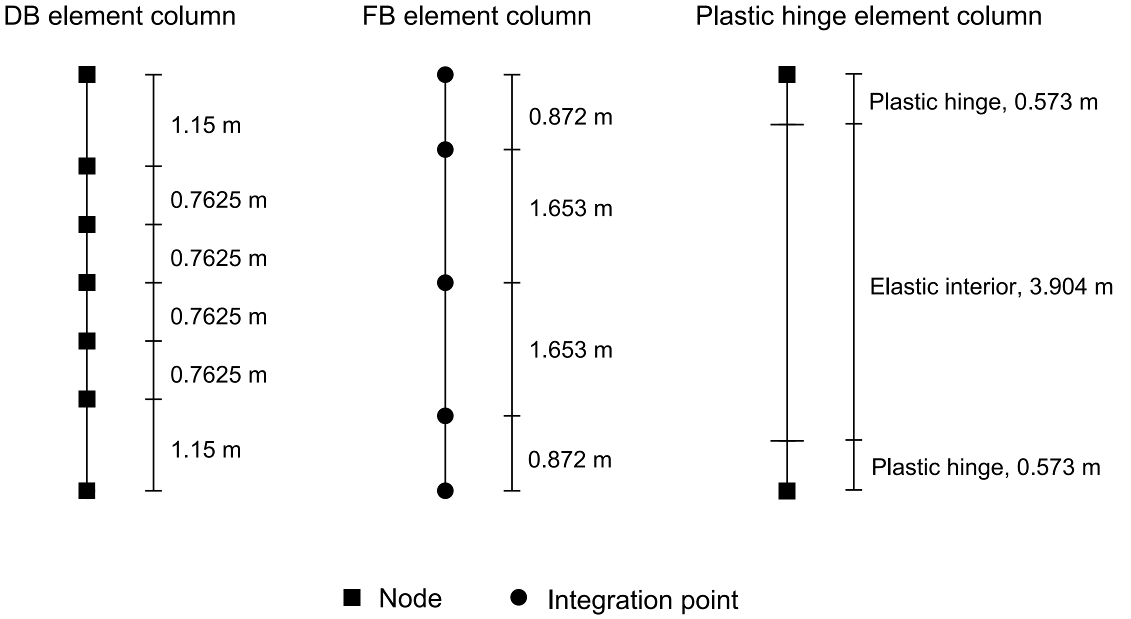


Figure 5: Discretization of RC columns.

## 4 COMPARISON OF EXPERIMENTAL AND NUMERICAL RESULTS

The frame prototype was subjected to the acceleration input motions without any repair between the tests. Thus, to obtain the most realistic numerical results, the 0.32g, 0.64g, and 0.80g motions have been applied consecutively in one single run. The computation times for each of these analyses are listed in Table 1. As expected, the frame modelled with the DB formulation requires more computational effort than the FB formulation elements. This is because of the higher number of nodes required for the DB beam-column members.

| Element formulation | Elements | Integration points per element | Total time |
|---|---|---|---|
| DB distributed inelasticity | 6 | 2 | 00:02:58 |
| FB distributed inelasticity | 1 | 5 | 00:02:33 |
| FB plastic hinge | 1 | 4 | 00:02:11 |

Table 1: Element discretization and computation times of analysis.

Due to the small number of structural nodes in the examined frame models, the differences between the computation times are not that significant. Indeed, the studied frame is very small compared to common residential and industrial buildings. The finite element models for these structures often consist of several thousands of nodes, which increase immensely if DB element members are used instead of FB elements. The fastest known matrix multiplication and inversion algorithms have running times of $n^{2.373}$ for a square a n x n matrix [14]. This means that a large number of nodes causes a significant increase in computational effort.

The base shear time-histories from the analysis are compared to the experimental data in Figs. 6, 7 and 8. All element types result in identical response paths as the experimental results for all three ground motion intensities. There is however, overestimation of the force values at some cycles. The difference is larger for the DB elements and smaller for the FB distributed and plastic hinge elements, which is reasonable because the DB formulation tends to provide stiffer solutions. The time histories from the analysis with the 0.64g and 0.80g ground motions do not start at zero force. This is due to the fact that, as the three time-histories were applied consecutively, the base shear at the first time step was equal to the force at the last analysis step of the previous time history. The peak base shear values from the time-histories are listed in Table 2. The overestimation of force by the DB formulation is confirmed. It also shown that the FB distributed inelasticity and the plastic hinge elements result in almost identical values of base shear.

The modelling issues and the comparison of analytical and experimental results are discussed in detail elsewhere [11].

| **Peak base shear [kN]** | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | Error (%) | | |
| PGA | Experimental | DB | FB | Plastic hinge | DB | FB | Plastic hinge |
| 0.32g | 109.4 | 138.4 | 116.0 | 115.2 | 26.5 | 6.0 | 5.3 |
| 0.64g | 122.0 | 145.7 | 117.0 | 117.4 | 18.9 | -3.9 | -3.8 |
| 0.80g | 118.3 | 143.5 | 115.7 | 114.8 | 23.4 | -0.9 | -0.8 |

Table 2: Peak base shear values from analysis.

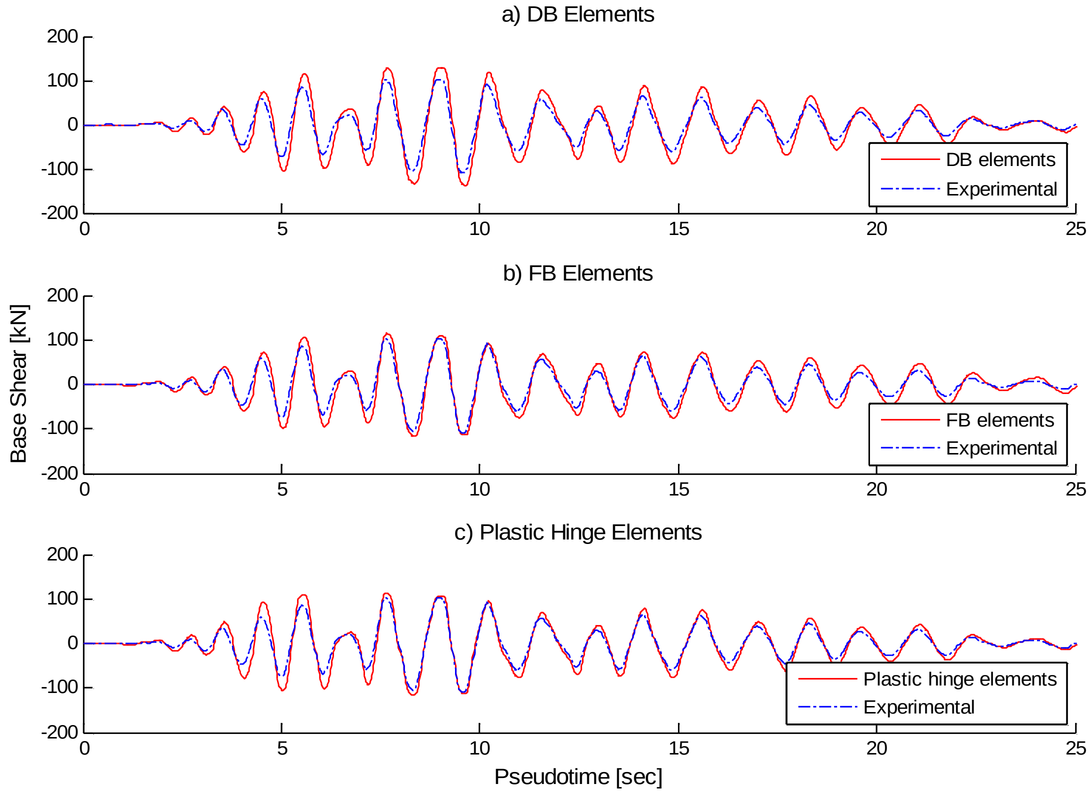


Figure 6: Base shear time-histories for 0.32g ground motion.

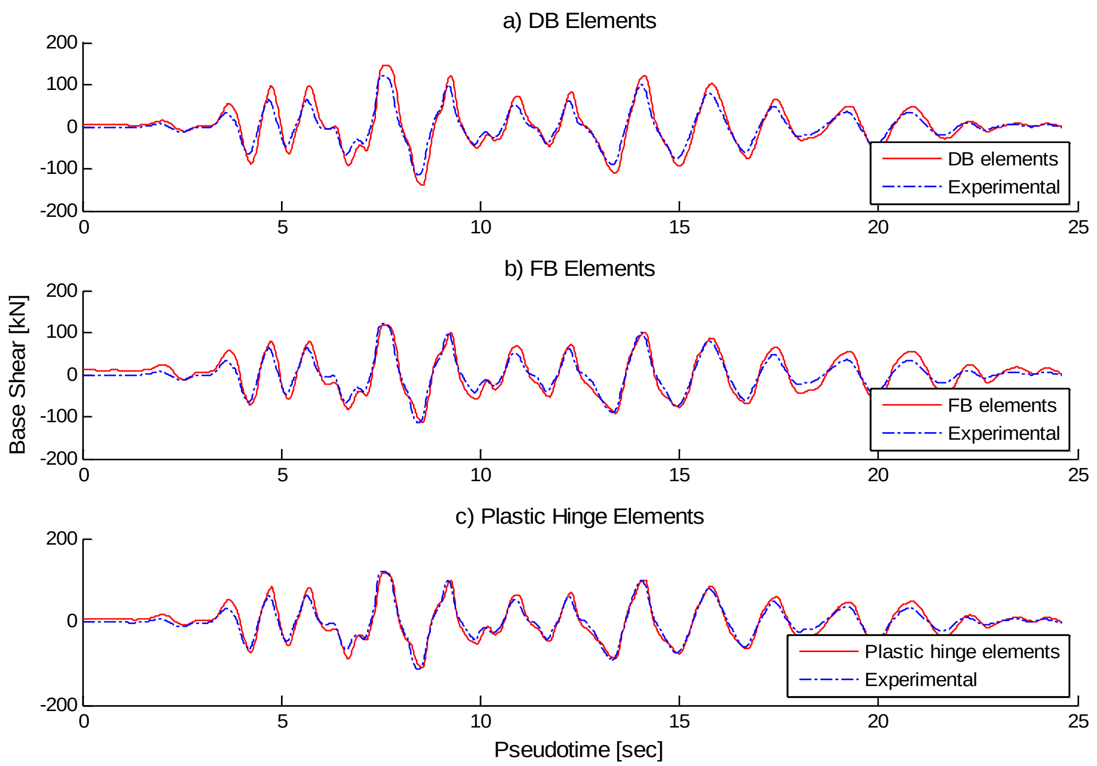


Figure 7: Base shear time-histories for 0.64g ground motion.

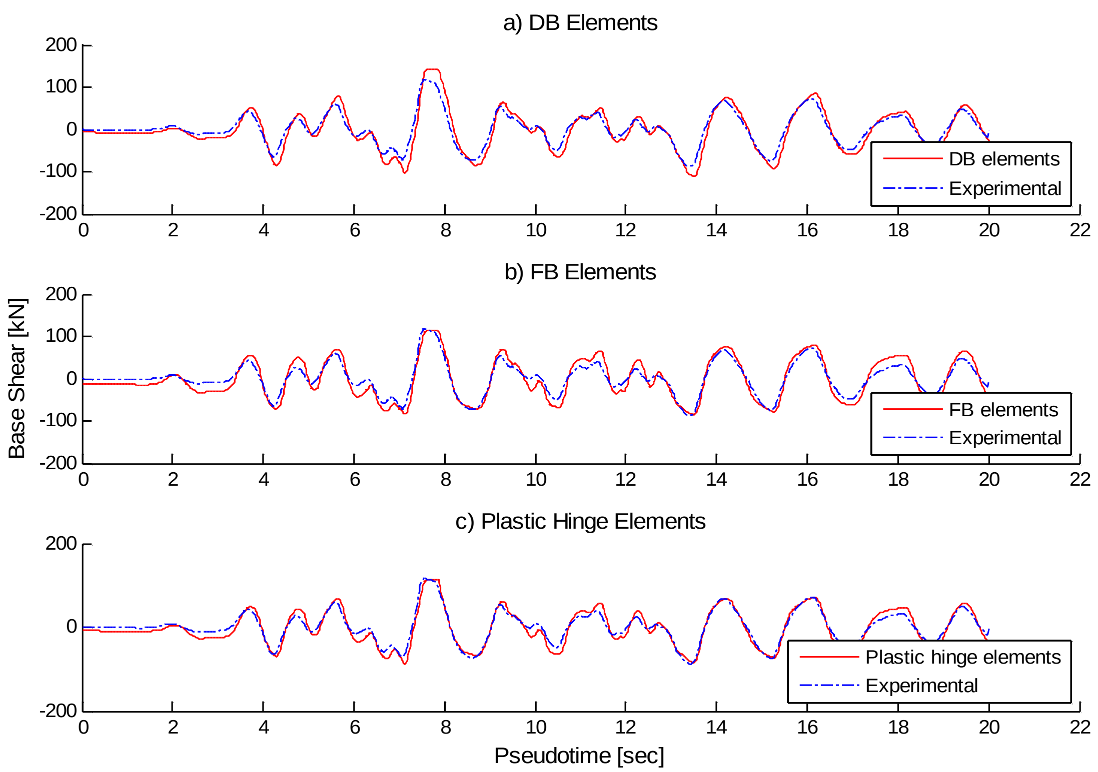


Figure 8: Base shear time-histories for 0.80g ground motion.

## 5 CONCLUSIONS

The running times of the different analyses clearly show that the computational effort of models with DB elements is higher than the one of FB elements. In addition, concentrated-inelasticity elements require fewer calculations than distributed-inelasticity elements, resulting in decreased computation time. While all three element types display satisfactory accuracy compared to experimental values for a simple RC frame, the FB elements offer a slightly better approximation of the response. Thus, the analyses confirm that both FB distributed- and concentrated-inelasticity elements achieve greater accuracy and require less computational effort than DB elements.

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