

## UNDERSTANDING SHEAR WAVE VELOCITIES CORRELATIONS WITH N-SPT AND QC-CPT VALUES

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**Keywords:** Shear Wave Velocity, SPT, CPT, Soil Liquefaction.

**Abstract.** *Prediction of the ground shaking response at soil sites requires knowledge and understanding of soil shear wave velocity  $V_s$ , a parameter that is strongly related to small strain shear modulus  $G_{max}$ . While it is preferable to measure soil parameters in situ, this is often not economically feasible at all locations. For this reason, and in order to improve correlations between geophysical measurements and key geotechnical parameters, this paper attempts to propose new general correlations between shear wave velocity  $V_s$  and SPT standard penetration resistance  $N$  as well as the CPT cone penetration resistance  $q_c$ . The studied site is located on the Mediterranean coast, in the North Eastern suburb of Beirut, the capital of Lebanon, where the level of water is present near the ground surface. The variable near surface geological conditions throughout the city results in spatially variable ground shaking characteristics. In order to better understand seismic soil response, geotechnical data comprising both in situ and laboratory tests was gathered. This extensive data allows determining good correlations that may be validated by comparing the calculated shear wave velocity with previously obtained profiles from the inversion parameterization of dispersion estimates in order to derive shear-wave velocities ( $V_s$ ) within each geological layer. In some relations, the depth factor  $D$  or effective stress  $\sigma'_v$  and the corrected SPT- $N_{60}$  were also introduced. The proposed relations are divided into three soil categories: sand, clay and all soils (mixed clay, sand and silt). The equations developed in this study compare very well with most of existing correlations in the literature. The proposed correlations are very important for both Lebanese and other soils, and may be used both in the design of a foundation system as well as in the assessment of soil behavior under earthquake loading conditions (Settlement and liquefaction, soil response...). These correlations should be used with caution in geotechnical earthquake engineering and should be checked against measured  $V_s$ .*

## 1 INTRODUCTION

Geophysical measurements are becoming important and common for geotechnical projects where earthquake loading is anticipated. Older site investigations lack geophysical measurements and mainly provide beside the geologic setting, SPT blow counts or CPT cone tip penetration resistances. Correlations between shear wave velocity, SPT blow counts  $N$  and  $q_c$  cone tip resistance, including geologic setting and site stratigraphy are therefore potentially useful and beneficial at least as a primary screening tool before identifying the necessity for further geophysical testing. Shear wave velocity  $V_s$  has become one of the preferred methods for estimating the small strain shear properties and has been incorporated into site classifications systems and ground motion prediction equations. Shear wave velocity ( $V_s$ ) is an important pointer of the dynamic properties of soil and rock because of its relationship with  $G_{max}$ , given by  $G_{max} = \rho V_s^2$ , where ( $\rho$ ) is the soil density. Characterization of the stress-strain behavior of soils is a fundamental component of many seismic analyses, including hazard analysis, site response analysis, and soil–structure interaction. The shear modulus ( $G$ ) of soils is highly reliant upon strain level. The small-strain shear modulus ( $G_{max}$ ) is typically linked with strains on the order of  $10^{-3}\%$  or less [1]. With knowledge of  $G_{max}$ , the shear response at various levels of strain can be estimated using modulus reduction ( $G/G_{max}$ ) curves [2, 3]. The importance of  $V_s$  has been widely recognized in ground motion prediction equations by implementation of site factors that modify ground motion based on the difference between a site  $V_s$  and a reference  $V_s$  or by direct incorporation of a  $V_s$  term in the ground motion regression equations. New relations include  $V_{s30}$  as a constant required for ground motion prediction, where  $V_{s30}$  is the average shear wave velocity in the upper 30m. Geophysical methods are being increasingly used for analyzing the subsurface, whether for landslides or site effects, because of their mainly non destructive nature and relatively simple execution; nevertheless, the measured geophysical parameters cannot be yet directly used by geotechnical engineers to perform stability or resistance calculations.

In the present study, an overview of some of the existing correlations between shear wave velocity  $V_s$  with the SPT blow count number  $N$ , or with the cone tip resistance  $q_c$  of CPT is given [4, 5, 6, 7]. Afterwards the general setting of the study site is presented with specific emphasis on available geological as well as geotechnical data [8]. And since there are actually no correlations available for Lebanese soil deposits between geophysical measurements such as shear wave velocity and geotechnical measurements such as SPT and CPT in situ tests, the existing correlations in the literature are used to establish specific correlations for the Lebanese context. Finally discussion and recommendations are proposed to help the geotechnical earthquake engineering community make a better assessment of the soil properties as far as liquefaction potential or seismic response are concerned.

## 2 BACKGROUND AND EXISTING CORRELATIONS

Many researchers have proposed correlations mostly between  $N$ -SPT or  $q_c$ -CPT and  $V_s$  for different soils (sand, silt and clay). There have been attempts to incorporate some additional dependent variables in describing the relationships, among these some researchers tried to include a depth factor or the vertical effective stress in their correlations. The following tables present an overview of the relations that will be the point of departure of the research carried on by the authors. Table 1 presents the correlations with  $N$ -SPT and  $V_s$ , Table 2 shows correlations involving the depth factor  $D$ , and finally Table 3 presents correlations  $q_c$ -CPT and  $V_s$ . Further discussions on the influence of the parameters as well as propositions of new correlations will be given later in the paper.

Reference	All soils	Sand	Clay
[9]	$V_s = 76N^{(0.33)}$		
[10]	-	$V_s = 100.5N^{(0.29)}$	-
[11]	-	$V_s = 57.4N^{(0.49)}$ $V_s = 57N^{(0.49)}$	$V_s = 114.43N^{(0.31)}$ $V_s = 114N^{(0.31)}$
[12]	$V_s = 51.5N^{(0.516)}$	-	-
[13]	$V_s = 22N^{(0.85)}$	-	-
[14]	-	-	$V_s = 27N^{(0.73)}$
[15]	-	$V_s = 125N^{(0.3)}$	-
[16]	$V_s = 107.6N^{(0.36)}$	-	$V_s = 76.55N^{(0.445)}$
[17]	$V_s = 90N^{(0.309)}$	$V_s = 90.82N^{(0.319)}$	$V_s = 97.89N^{(0.269)}$

Table 1: Correlations between shear wave velocity  $V_s$  and N-SPT.

Reference	All soils
[18]	$V_s = 62.4 \times N_{60}^{0.22} \times D^{0.23}$ $V_s = 72 \times N_{60}^{0.17} \times D^{0.20}$ $V_s = 93.8 \times N_{60}^{0.17} \times D^{0.20}$
[19]	$V_s = 85 \times N_{60}^{0.17} \times D^{0.20}$
[20]	$V_s = 72.3 \times N_{60}^{0.23} \times D^{0.15}$

Table 2: Correlations between shear wave velocity  $V_s$  and  $N_{60}$ -SPT including depth factor  $D$ .

Reference	Type of soil	Correlation
[21]	sand	$V_s = 17.48 q_c^{(0.13)} \sigma'_v^{(0.27)}$
[22]	sand	$V_s = 13.18 q_c^{(0.192)} \sigma'_v^{(0.179)}$
[22]	sand	$V_s = 12.02 q_c^{(0.319)} f_s^{(-0.0466)}$
[20]	sand	$V_s = 25.3 q_c^{(0.163)} f_s^{(0.029)} D^{(0.155)}$
[22]	clay	$V_s = 3.18 q_c^{(0.549)} f_s^{(0.025)}$
[23]	clay	$V_s = 1.75 q_c^{(0.627)}$
[20]	clay	$V_s = 11.9 q_c^{(0.269)} f_s^{(0.108)} D^{(0.127)}$

Table 3: Correlations between shear wave velocity  $V_s$  and  $q_c$ -CPT including depth factor  $D$ .

### 3 GENERAL SETTING OF THE STUDY SITE

Soil investigation was carried out in Antelias, the northern suburb of Beirut, to get an estimate of the physical and mechanical characteristics of the subsurface soil/rock existing beneath the site surface, in order to provide the necessary information for geotechnical aspects relevant to the proposed projects in the area. The site is a relatively flat land. It is bound by Beirut-Tripoli coastal road to the west. A water course passes along the site limits. The site has an irregular surface area of about 10000m<sup>2</sup>. The site has an average level of about 3.5mASL. The sea is present at a distance of about 150.0m to the west of the site.

#### 3.1 Field Works

Ten boreholes were drilled down to depths reaching 66.0m from existing ground levels. Some Boreholes were drilled destructively with SPT in the soil formations and using double core barrel in marl and rock formations. Other Boreholes were destructively drilled with pressuremeter Tests at an average interval of 3.0m. A total of seventy nine pressuremeter tests were carried out in these boreholes. Cone penetration test was performed at a few locations. Laboratory tests were performed on selected samples obtained from the borings to help in the classification of the soil and to determine their engineering properties. Undisturbed clay samples were extracted using Shelby Tubes for the purpose of performing Triaxial and Consolidation tests. This amount of data will help in the assessment of soil behavior during the analysis.

#### 3.2 Geology and Subsurface Conditions

According to the geological map of Beirut, the site area is underlain by alluvial sand of the quaternary age followed by in order: 1) a grey fine-bedded limestone and marly limestone of Sanine formation (C4); 2) a brown green marl of Hammana formation (C3); 3) a grey cliff forming limestone of Mdairej formation (C2b); and finally brown green variable units of limestone, marl and sandstone of Abieh formation (C2a). A fault passes at about a distance of 1000m to eastern south direction of the site.

Interpretation of the geotechnical investigation paired with the available field data revealed that the ground formation beneath the site surface consists mainly of a top layer of fill material having a variable thickness ranging between 4.5m and 6.0m, followed by alluvial deposits consisting of interbedding sand, clay, silt and gravels. This alluvial soil is underlain by interbedding marl and marly limestone.

More specifically, the fill material was encountered in all the boreholes in layers ranging in thickness between 4.5m and 6.0m. It generally consisted of beige sand with gravels and cobbles of limestone. The presence of the site close to a water course confirms the presence of alluvial deposits. These deposits encountered beneath the site surface consisted of sand, clay, silt and gravel with thicknesses varying between 21.5m and 54.5m. However, the color of these formations varied between beige, light to grayish brown, red, yellow, and grey.

Finally, the interbedding marl and marly limestone: This formation was encountered beneath the alluviums. The thickness of this layer varied between 12.0m and 33.0m. The marl was generally found to be beige to white sandy gravelly marl with cobbles of limestone. The limestone was encountered within the marl in layers having a maximum thickness of 5.0m. These were found to be beige to yellowish cream and interbedded with layers of marl at some locations. Figure 1 shows typical N-SPT results.

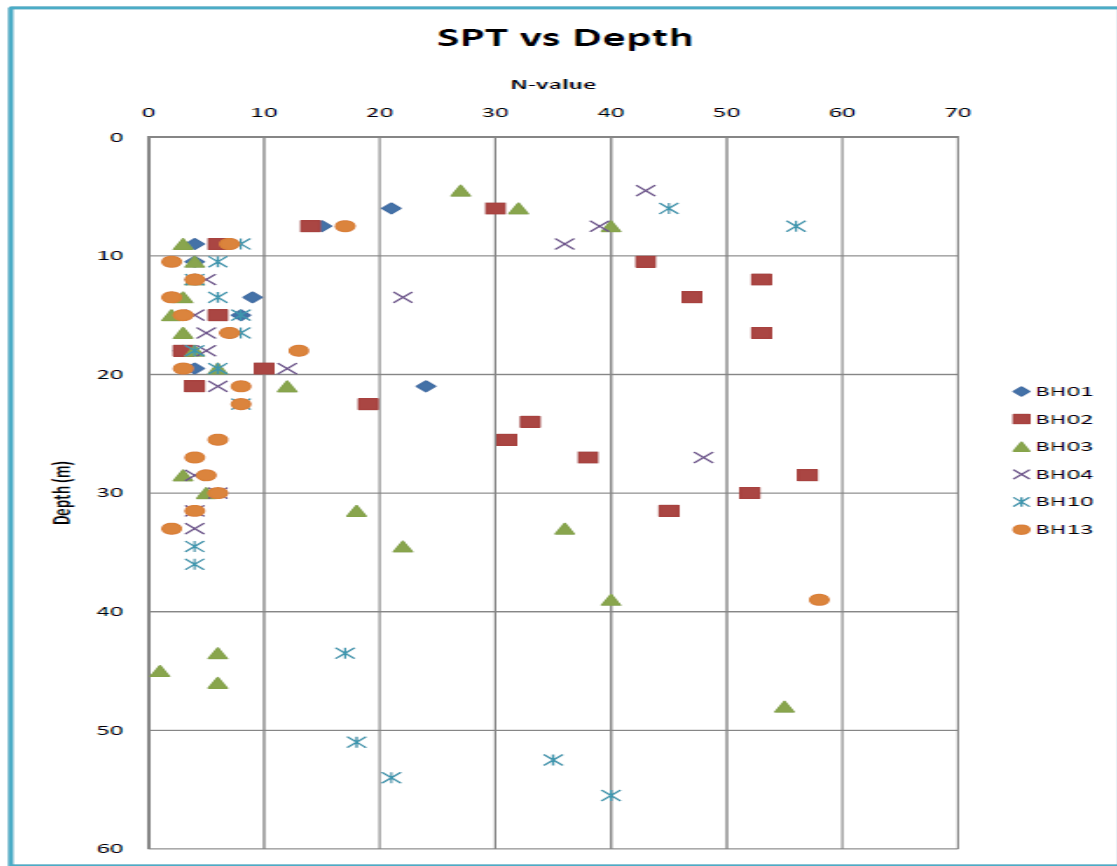


Figure 1 Typical N-SPT results from a series of boreholes.

#### 4 ANALYSIS AND DISCUSSION

In this study, analysis has been carried out for three groups of soils: Sand, Clay and All Soils. In the first phase, values of N as function of depth has been drawn based on all available data. Three envelopes were obtained for each group of soil: an upper limit envelope, a lower limit envelope and an average envelope. Regression analysis was based both on a polynomial third degree approach and a power type approach. For illustration, the polynomial third degree approach is shown in the figures. Figures 2 and 3 show the results of Vs as a function of N for sand. Figures 4 and 5 show the results of Vs as a function of N for clay. Figures 6 and 7 give the results of Vs as a function of N in the case of the group all soils. All the equations used to establish the models for our soils are presented earlier and listed in the references. Of course, all the measured values of Vs are in m/s. Based on the undertaken analysis, the equation for Vs as a function of N for sand based on a polynomial approach is:

$$V_s = 82.878 N^{0.3763} \quad (1)$$

And if the depth term D is to be included in the case of sands, the following equation to find Vs may be considered using a polynomial approach as a function of  $N_{60}$ :

$$V_s = 77.17 N_{60}^{0.1899} D^{0.1960} \quad (2)$$

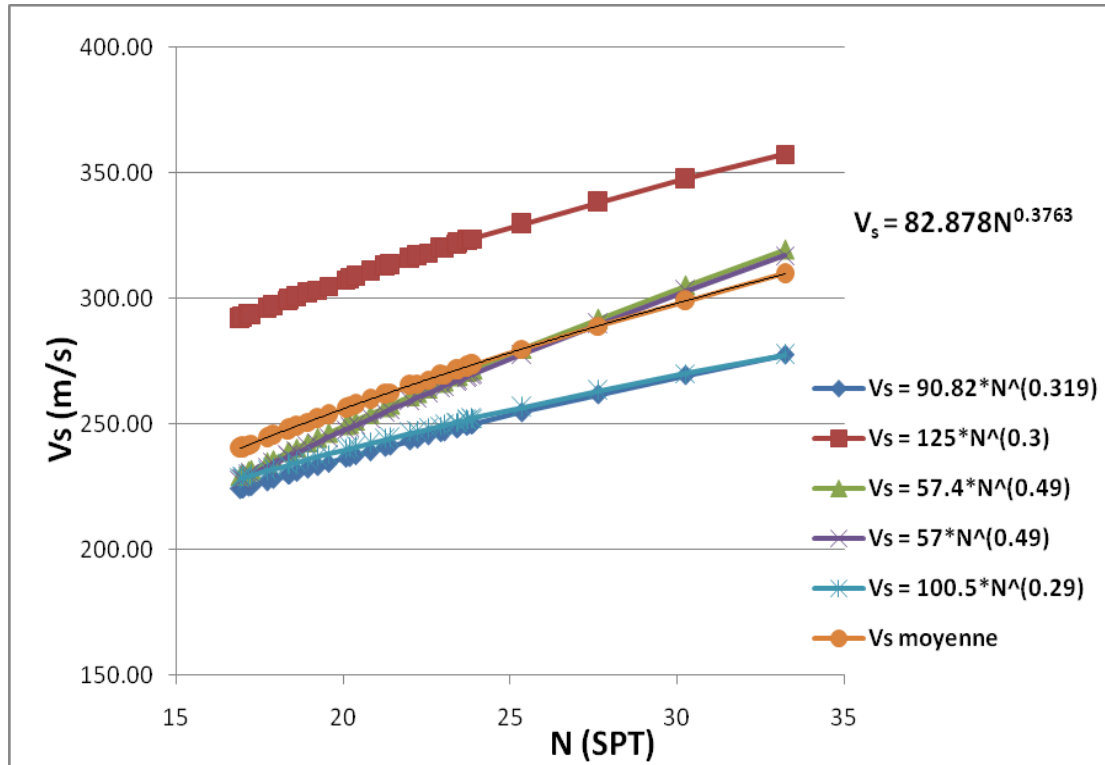


Figure 2 Shear wave velocity  $V_s$  as a function of  $N$  for Sand

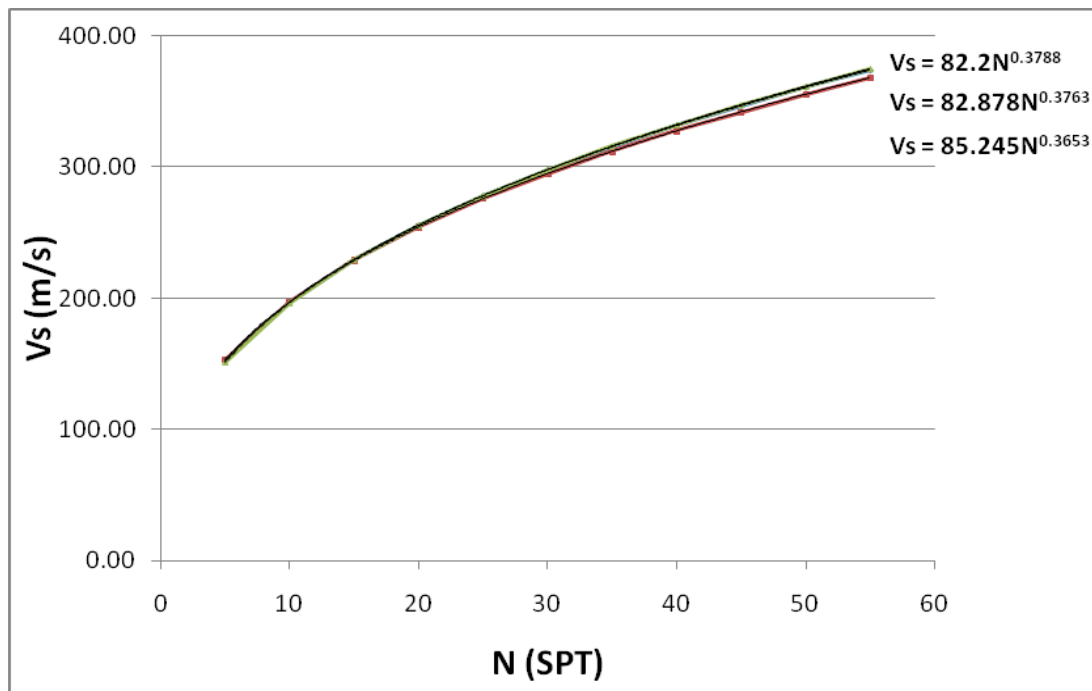


Figure 3  $V_s$  as a function of  $N$  for Sand, with upper limit, lower limit and average curves

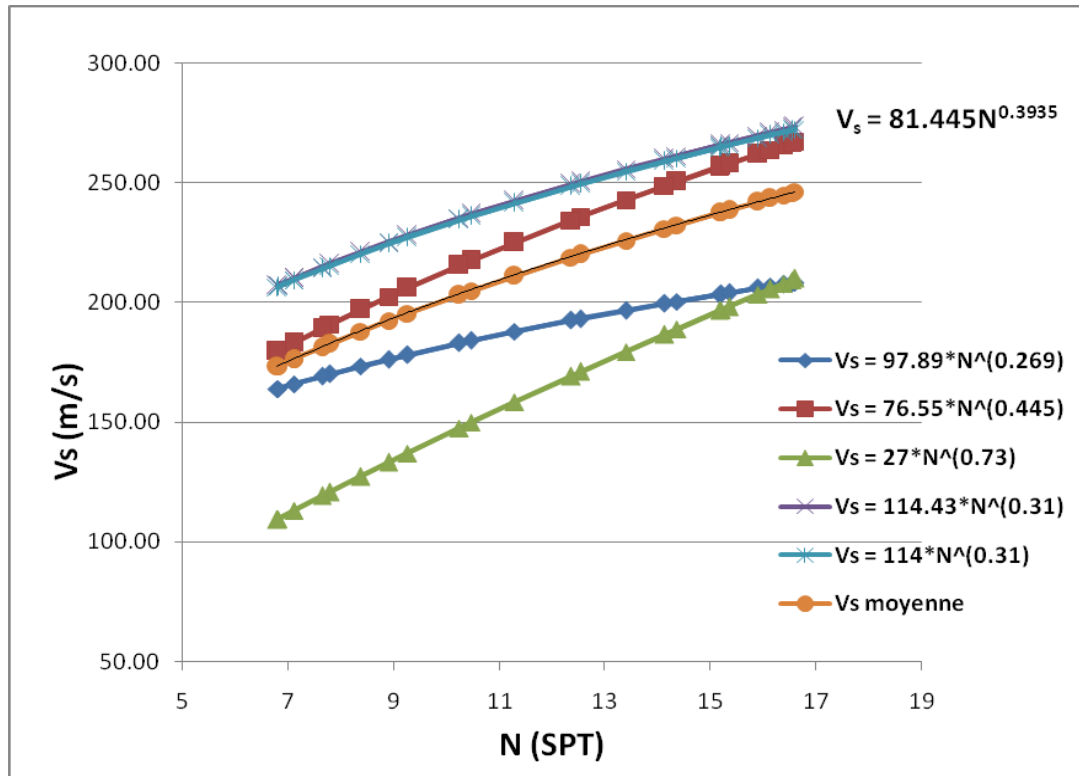


Figure 4 Shear wave velocity  $V_s$  as a function of  $N$  for Clay

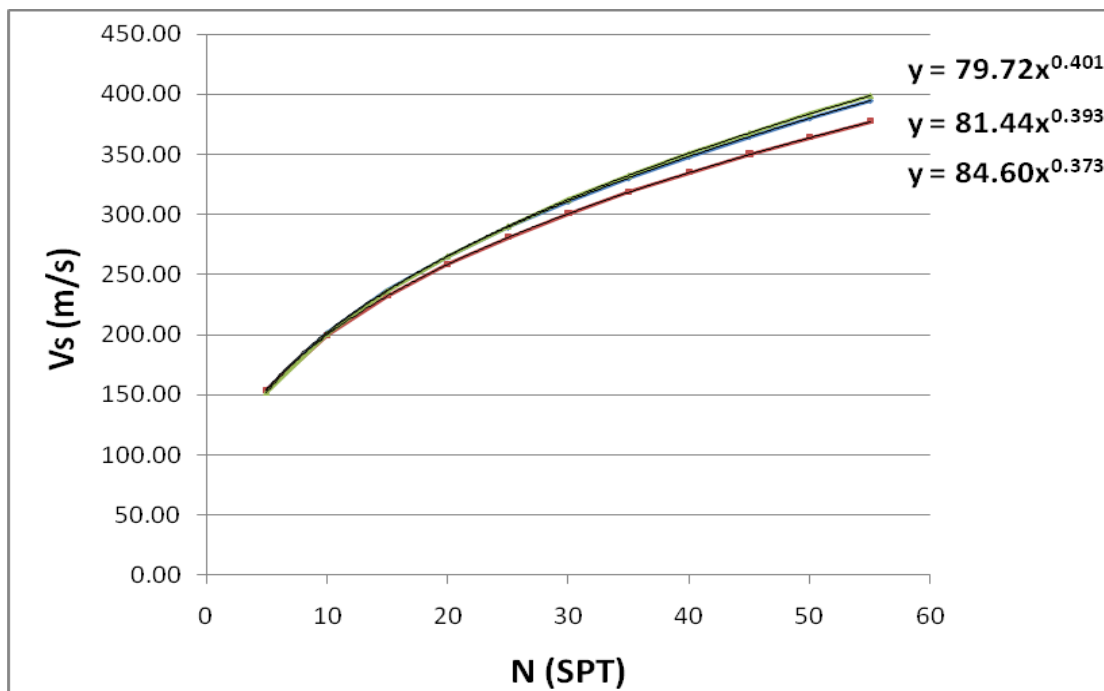


Figure 5 Shear  $V_s$  as a function of  $N$  for Clay, with upper limit, lower limit and average curves

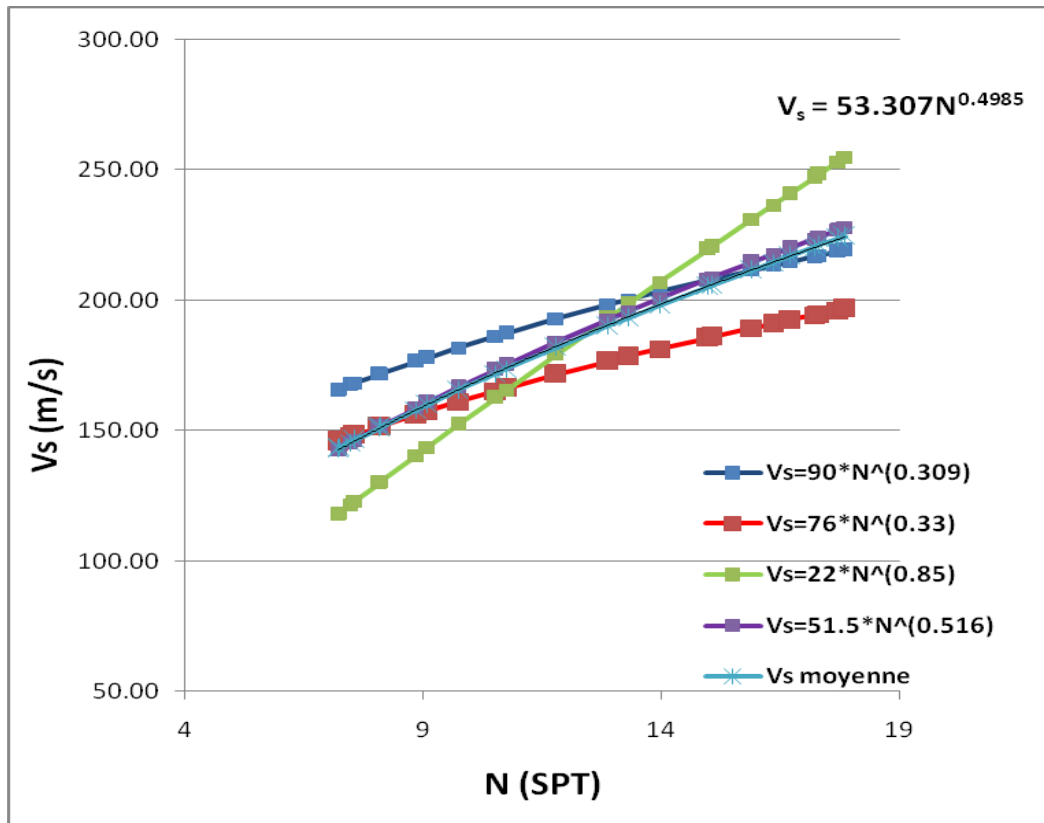


Figure 6 Shear wave velocity  $V_s$  as a function of  $N$  for All Soils Group

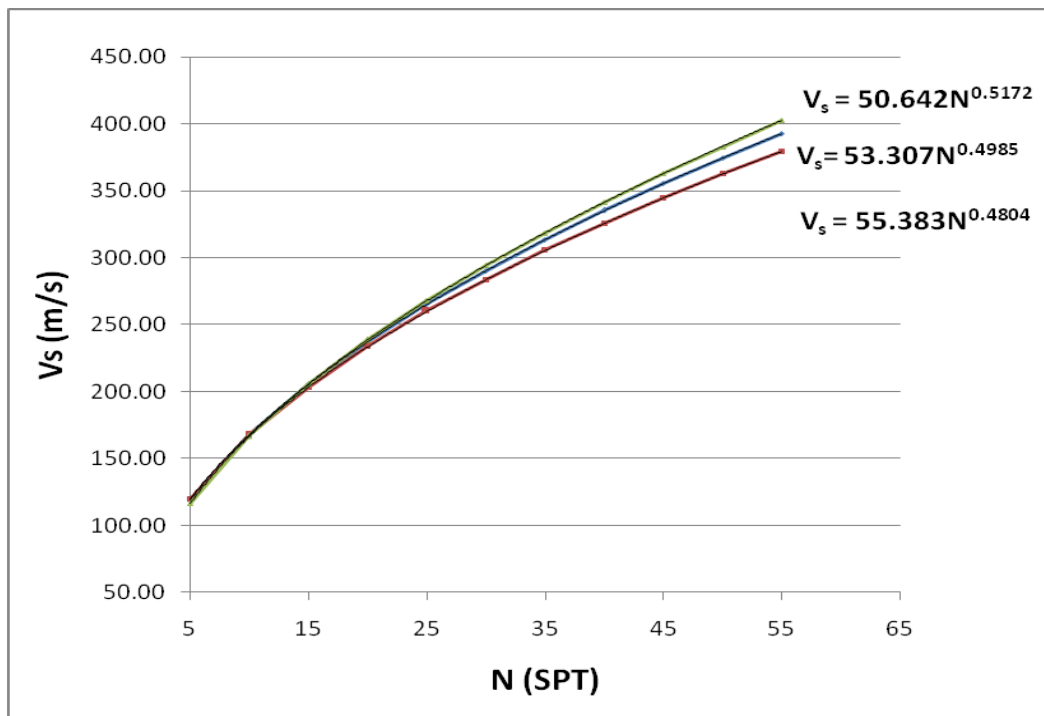


Figure 7 Shear  $V_s$  as a function of  $N$  for All Soils, with upper limit, lower limit and average curves



For the same data analyzed, the equation to be considered for  $V_s$  as a function of  $N$  for sand based on a power approach is:

$$V_s = 83.455 N^{0.3739} \quad (3)$$

And if the depth term  $D$  is to be included in the case of sands, the following equation to find  $V_s$  may be considered using a power approach, as a function of  $N_{60}$ :

$$V_s = 82.03 N_{60}^{0.1742} D^{0.1879} \quad (4)$$

As far as clay is concerned, the equation for  $V_s$  as a function of  $N$  based on a polynomial approach is:

$$V_s = 81.445 N^{0.3935} \quad (5)$$

And if the depth term  $D$  is to be included in the case of clays, the following equation to find  $V_s$  may be considered using a polynomial approach as a function of  $N_{60}$ :

$$V_s = 76.39 N_{60}^{0.1573} D^{0.2007} \quad (6)$$

For the same data analyzed, the equation to be considered for  $V_s$  as a function of  $N$  for clay based on a power approach is:

$$V_s = 82.679 N^{0.3862} \quad (7)$$

And if the depth term  $D$  is to be included in the case of clays, the following equation to find  $V_s$  may be considered using a power approach, as a function of  $N_{60}$ :

$$V_s = 136.76 N_{60}^{-0.046} D^{0.1204} \quad (8)$$

The third group of soils to be considered is called all soils, which means that correlations found should be applied to all types of soils regardless of their classification. The equation for  $V_s$  as a function of  $N$  based on a polynomial approach is:

$$V_s = 53.307 N^{0.4985} \quad (9)$$

And if the depth term  $D$  is to be included in the case of all soils, the following equation to find  $V_s$  may be considered using a polynomial approach as a function of  $N_{60}$ :

$$V_s = 71.22 N_{60}^{0.2227} D^{0.1994} \quad (10)$$

The equation to be considered for  $V_s$  as a function of  $N$  for all soils based on a power approach is:

$$V_s = 54.308 N^{0.49} \quad (11)$$

And if the depth term  $D$  is to be included in the case of clays, the following equation to find  $V_s$  may be considered using a power approach, as a function of  $N_{60}$ :

$$V_s = 71.59 N_{60}^{0.2206} D^{0.1990} \quad (12)$$

All the above equations show that results are very close whether considering a polynomial third degree or a power approach. The thing to be noted is that in the case of the group all soils, results are more conservative because all points are considered. Values of  $V_s$  found using the equations range from 150 m/s to 400 m/s and these are reasonable values for these types of soils.

In the last part of the discussion of results, CPT based results are considered. Based on the references [20, 21, 22, and 23], the following equations are derived. The correlation to be considered for  $V_s$  as a function of  $q_c$  and the effective stress  $\sigma'_v$  for sand based on a polynomial approach is:

$$V_s = 225 q_c^{-0.002} \sigma'^{0.0423}_v \quad (13)$$

The correlation to be considered for  $V_s$  as a function of  $q_c$  and the effective stress  $\sigma'_v$  for sand based on a power approach is:

$$V_s = 234 q_c^{-0.001} \sigma'^{0.0405}_v \quad (14)$$

As far as clay is concerned, the equation for  $V_s$  as a function of  $q_c$  based on a polynomial approach is:

$$V_s = 156 q_c^{0.024} \quad (15)$$

For the same data analyzed, the equation to be considered for  $V_s$  as a function of  $q_c$  for clay based on a power approach is:

$$V_s = 141 q_c^{0.029} \quad (16)$$

The correlations obtained shall be very helpful for the earthquake geotechnical engineering community. It is very important nowadays to work both with geotechnical as well as geophysical data when available. Not all geotechnical projects include geophysical prospection; this is why the use of correlations might be of interest for a first assessment only. Any soil seismic response analysis requires however a complete geophysical investigation couples of course with the traditional geotechnical investigation. Results obtained in the present study compare very well with an earlier published [24]  $V_s$  profile based on inversion parameterization of dispersion estimates obtained in the same area. More geophysical measurements are to be made in the near future.

## 5 CONCLUSIONS

Geotechnical and geophysical tests are of primary importance and complementary as far as characterizing the soil subsurface. Since geophysical testing is not systematically carried out, the use of existing correlations between in situ geotechnical testing (SPT and CPT) and shear wave velocity for soils is beneficial. In this paper, existing N-SPT and  $q_c$ -CPT data has been thoroughly analyzed. Correlations between  $V_s$ , N-SPT and  $q_c$ -CPT were established. Some of the proposed correlations include the depth parameter  $D$  as well as the effective earth pressure  $\sigma'_v$ . The developed correlations are based on other correlations already available in

the geotechnical literature. The obtained results compare well with previous geophysical measurements conducted at the same site. The proposed correlations should be used with precaution by the earthquake geotechnical engineering community. Further analysis is being conducted in the hope of yielding still more representative correlations. Vs in situ measurements are to be recommended in parallel with the use of the correlations.

## REFERENCES

- [1] G. Lefebvre, D. Leboeuf, M.E. Rahhal, A. Lacroix, J. Warde, K.H. Stokoe, Laboratory and field determinations of small strain shear modulus for a structured Champlain clay, *Canadian Geotechnical Journal*, vol. **31**, 61-70, 1994.
- [2] M.E. Rahhal, G. Lefebvre, Understanding the Cyclic Threshold Strain in Soil Dynamics, *Proceedings 11<sup>th</sup> International Conference on Soil Dynamics and Earthquake Engineering*, and *3<sup>rd</sup> International Conference on Geotechnical Earthquake Engineering*, University of California, Berkeley, USA, Volume **1**, 453-459, 2004.
- [3] M.E. Rahhal, G. Lefebvre, Characterizing Shear Moduli Reduction in Soils Cyclic Behavior, *Proceedings ASCE Geo Congress*, Atlanta, USA. Permalink: [http://dx.doi.org/10.1061/40803\(187\)164](http://dx.doi.org/10.1061/40803(187)164), 2006.
- [4] R.D. Andrus, P. Piratheepan, B. S. Ellis, J. Zhang, C. H. Juang, Comparing Liquefaction Evaluation Methods using Penetration-Vs relationships, *Soil Dynamics and Earthquake Engineering*, **24**, 713-721, 2004.
- [5] R.D. Andrus, N.P. Mohanan, P. Piratheepan, B.S. Ellis, T.L. Holzer, Predicting shear-wave velocity from cone penetration resistance, *Proceedings, 4th International Conference on Earthquake Geotechnical Engineering*, Thessaloniki, Greece, 2007.
- [6] U. Dikmen, Statistical correlations of shear wave velocity and penetration resistance for soils, *Journal of Geophysics and Engineering*. **6** (1), 61-72, 2009.
- [7] L. Paoletti, Y. Hegazy, S. Monaco, R. Piva, Prediction of shear wave velocity for offshore sands using CPT data – Adriatic sea; *2nd International Symposium on Cone Penetration Testing*, Huntington Beach, CA, USA. Volume **2&3**: Technical Papers, Session 2: Interpretation, Paper No. 29, 2010.
- [8] L. Dubertret, Geological Maps of Lebanon, 1/50,000. Ministry of National Defense, Geography Department, Lebanon.
- [9] T. Imai, T. and Yoshimura, Y., The relation of mechanical properties of soils to P and S-wave velocities for ground in Japan, *Technical Note*, OYO Corporation (1975).
- [10] D.E. Sykora, K.H. Stokoe, Correlations of in-situ measurements in sands of shear wave velocity, *Soil Dynamics and Earthquake Engineering*, **20**:125–36, 1983.
- [11] S. H. Lee, Regression models of shear wave velocities, *J. Chin. Inst. Eng.*, **13**, 519–32, 1990.
- [12] R. Iyisan, Correlations between shear wave velocity and in situ penetration test results (in Turkish), *Chamber of Civil Engineers of Turkey, Teknik Dergi* **7**(2):1187–1199, 1996.

- [13] M.K. Jafari, A. Asghari, I. Rahmani, Empirical correlation between shear wave velocity (VS) and SPT N-value for south of Tehran soils, *Proceedings, 4th International Conference on Civil Engineering*, Tehran, Iran, 1997.
- [14] M.K. Jafari, A. Shafiee, A Ramzkhah, Dynamic properties of the fine grained soils in south of Tehran, *Journal Seismology and Earthquake Engineering*, **4** (1):25–35, 2002.
- [15] T. Okamoto, T. Kokusho, Y. Yoshida, K. Kusuonoki, Comparison of surface versus subsurface wave source for P–S logging in sand layer, *Proceedings 44th Annual Conference. JSCE*, vol. 3, 996–7 (in Japanese), 1989.
- [16] G.A. Athanasopoulos, Empirical correlations Vs-NSPT for soils of Greece: a comparative study of reliability, *Proceedings. 7th International Conference on Soil Dynamics and Earthquake Engineering (Chania, Crete)* ed. A. S. Cakmak (Southampton: Computational Mechanics), 19–36, 1995.
- [17] N. Hasancebi, R. Ulusay, Empirical correlations between shear wave velocity and penetration resistance for ground shaking assessments, *Bulletin Engineering Geology and the Environment*, **66**: 203–213, 2007.
- [18] Y. Ohta, N Goto, Empirical shear wave velocity equations in terms of characteristic soil indexes, *Earthquake Engineering Structural. Dynamics*, **6**:167–187, 1978.
- [19] H.B. Seed, R.T. Wong, I.M. Idriss, K. Tokimatsu, Moduli and damping factors for dynamic analyses of cohesionless soils, *Journal of Geotechnical Engineering.*, **112**(11):1016–1032, 1986.
- [20] P. Piratheepan, *Estimating Shear-Wave Velocity from SPT and CPT Data*. Master of Science Thesis, Clemson University, 2002.
- [21] G. R. Baldi, V.N. Bellotti, M. Ghionna, M. Jamiolkowski, D.C.F LoPresti, Modulus of sands from CPTs and DMTs, *Proceedings, 12th International Conference. Soil Mechanics and Foundation Engineering*, Vol. **1**, Rio de Janeiro, 165–170, 1989.
- [22] Y.A. Hegazy, P.W. Mayne, Statistical correlations between  $V_s$  and cone penetration data for different soil types, *Proceedings, International Symposium on Cone Penetration Testing, CPT '95*, Linkoping, Sweden, Vol. **2**, 173–178, 1995.
- [23] P.W. Mayne, G.J. Rix, Correlations between shear wave velocity and cone tip resistance in natural clays, *Soils and Foundations*, Vol. **35**, No. 2, 193-194, 1995.
- [24] C. Cornou, M. Brax, N. Salloum, M. Rahhal, F. Harakeh, J. Harb, D. Youssef Abdel-Massih, A. Mariscal, C. Voisin, D. Jongmans, P.Y. Bard, Shear-wave velocity structure and correlation with N-SPT values in different geological formations in Beirut, Lebanon, *Proceedings, Second European Conference on Earthquake Engineering and Seismology*, Istanbul, TURKEY, August 25-29, 2014.