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A GROUPED MAXIMAL CONDITIONED SPECTRAL ORDINATE METHOD (MCSO) FOR GROUND MOTION SELECTION

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Abstract. *This article intends to present a computationally simple ground motion selection method (MCSO) whose spectral accelerations are matched at specified period in the desired uniform hazard spectrum (UHS). The deficiencies of the Conditional Mean Spectrum and Conditional Spectrum (CS) approaches are monitored. MCSO benefits from the advantages of consistency with the predefined target Magnitude-Distance. It comes up with positive epsilons, scaling factors close to one, larger magnitudes than those of the two mentioned methods, and associated with the actual earthquake variability. The seismic demands of the three reinforced concrete structures are calculated and compared with those of MCSO, POC, CMS, and CS.*

1 INTRODUCTION

Relating regional hazard curve to mean structural collapse capacity by providing a suite of strong motion (e.g., [1] due to its ability in reducing the strong motion scaling factors is a promising approach in earthquake engineering. Recently, two techniques in the context of GMSM, termed the Conditional Mean Spectrum (CMS) and the Conditional Spectrum (CS) approach, have been proposed [2, 3]. These two methods are developed on the basis of epsilon (ε) at the spectral ordinate at the first mode vibration and relating it to those of the multiple spectral ordinates at the other periods to form the mean spectral values termed "Conditional Mean Spectrum".

The key feature of these methods are the scaling spectral acceleration at the fundamental period Sa(T1) to the desired hazard level (e.g., 2% in 50 years) and selecting sets of ground motions compatible with the predefined conditioned mean spectrum [2]. As mentioned by Jayaram et al [3], matching the selected ground motions only to a predefined mean-response-spectrum and ignoring the inherent variability existing in the ground motions is not sufficient for appropriately selecting sets of ground motions.

We believe that the two approaches especially CMS method suffer from being associated with: (a) small earthquakes likely to be associated with negative epsilons; (b) considerably large scaling factors;, and (c) ground motions associated with mean magnitude and distance different from those of the target magnitude M and distance R. The two above-mentioned methods have been cited by many researchers (e.g., [4, 5, 6, 7]), despite their deficiencies that will be discussed later. These points have been the main motivation of this article.

2 OBJECTIVE AND SCOPE

The major purposes of this article are to present a simple methodology for selecting suites of appropriate ground motions, including the following characteristics:

- Associated with scaling factor close to one i.e., not being suffered from large scaling factor.
- Correspond to magnitude (M) and distance (R) and consistent with the desired hazard level.

3 PROPOSED APPROACH

A simplified GMSM for selecting suites of ground motions being matched at a specific period (T*) in the desired hazard level termed the Maximal Conditioned Spectral Ordinate (MCSO) is proposed.

In this approach, the process of computing the mean target response spectrum $\mu_{lnSa}(\bar{M},\bar{R},T^*)$ established by $\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}(T^*)$, given Sa(T*), through a disaggregation process is similar to the two above mentioned methods. It will be demonstrated that this approach results in the selection of suites of large ground motions associated with positive epsilons, scaling factors close to one, and correspond to the target magnitude and distance.

4 SUCCESSIVE STEPS OF THE METHOD

The mathematic formulation and the successive steps of the proposed algorithm are:

(I) For each ground-motion candidate, the Scaling Factor Effect (SFE) is calculated by Eq. 4 expressed as:

$$scaling\ factor\ effects = \left| \frac{\ln sa_{hazard}(T^*)}{\ln sa(T^*) - \ln sa_{hazard}(T^*)} \right| \quad (1)$$

Where $\ln sa_{hazard}(T^*)$ and $\ln sa(T^*)$ are the logarithmic response spectrum in the hazard curve and that of the candidate at the fundamental period T*, respectively

(II) The Selection Criterion Function (SCF) for each individual candidate is calculated by Eq. 5 expressed as:

$$SCF = scaling\ factor\ effects * \sum_{i=1}^{n} (element\_mean(i) * element\_drop(i)) \tag{5}$$

Where Element-Mean (i) for each candidate is calculated by Eq. 6 (see Fig. 1) expressed as:

$$element\_mean(i) = \left(\frac{\ln Sa_{scaled}(T_i) + \ln Sa_{scaled}(T_{i+1})}{2} - \frac{\mu_{\ln Sa}(T_i) + \mu_{\ln Sa}(T_{i+1})}{2} \right) \tag{6}$$

Where $\ln Sa_{scaled}$ is the scaled logarithmic spectral acceleration corresponding to each ground motion candidate (i.e., each candidate is scaled in the hazard curve at T*), μ_{lnSa} is the logarithm of the mean hazard spectral acceleration. The Element-Drop(i) is calculated by Eq. 7 expressed as:

$$element\_drop(i) = \left(\ln Sa_{scaled}(T_i) - \ln Sa_{scaled}(T_{i+1}) \right) + \left(\mu_{\ln Sa}(T_i) - \mu_{\ln Sa}(T_{i+1}) \right) \tag{7}$$

Where Element-Drop(i) determines the difference between the logarithmic spectral accelerations for each element.

(III) The selected ground motion candidates are listed in a descending order on the basis of their SCF values, i.e., larger SCF value is placed at the upper numbers of the list as the more appropriate candidate place.

Lastly, the orderly listed desired numbers of strong motions are selected as the most appropriate suite of ground motion candidates.

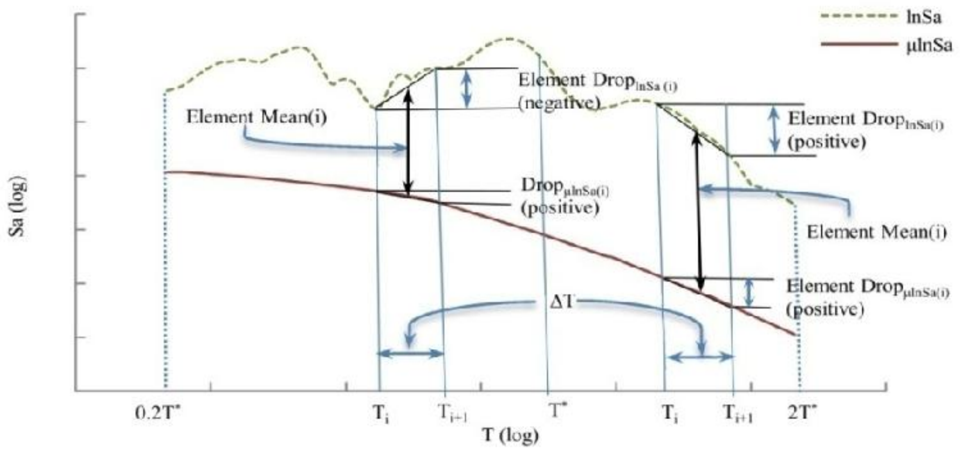

Figure 1: Graphical illustration of the Element-Mean and the Element-Drop associated with a typical response spectrum. Note that Element-Drop at the left side of the element has a negative sign, whereas that of the right side is positive. The vertical line (termed T* line) drawn from the fundamental period of the structure under study (T*) is also shown.

Table 1: The selected structures used, along with the corresponding code compliances, and their three natural periods.

Building	No. of Story	Type	Code compliance	Period
A	4	Modern RC special moment frame	2003 IBC, ASCE 7-02, ACI 318-02	0.97, 0.35, 0.18
B	12	Modern RC special moment frame	2003 IBC, ASCE 7-02, ACI 318-02	2.01, 0.68, 0.39
C	20	Modern RC special moment frame	2003 IBC, ASCE 7-02, ACI 318-02	2.63, 0.85, 0.46

5 EXAMPLES: MODELED STRUCTURES

As mentioned earlier, the selected three reinforced concrete special-moment frames used are those previously used by Haselton and Deierlein [8]. The 4-story, 12-story, and 20-story R/C structures used (see Table 1) have been previously designed according to modern building codes and checked by practicing engineers as part of the Applied Technology Council Project [9]. Table 1 lists the three buildings (A, B, and C), number of stories, structural resisting systems, code compliances, and the three successive natural periods of the structures. The structures subjected to the selected suite of ground motions are dynamically analyzed using the proposed method. The resulting response histories are evaluated by comparing to those of the POC [10]. The process implicitly compares and evaluates the results of MCSO algorithm to those of the CMS and CS. The Open-SEES software [11] is used for modeling the structures and performing dynamic analysis procedure. The structural deterioration is taken into account, and the structural collapse is assumed to occur once dynamic instability is reached [12], i.e., a large increase in drift as the result of a small increase in the applied loading. Tables 2,3, and 4 compare the maximum inter-story drift ratios (MIDR) obtained from POC and those of the MCSO, CMS, and CS methods in the form of mean values. The number of the selected ground motions, the MIDR ratios to POC, the dispersions, and lastly the collapse probabilities corresponding to each method are also shown.

Table 2: The maximum inter-story drift ratio (MIDR) corresponding to the 4-story building

method	POC	No. Of Rec.	Median MIDR	Ratio to POC	Dispertion MIDR	Collapse probability
Conditional Mean Spectrum	0.027	28	0.0274	1.014	-	-
		40	0.0279	1.033	0.28	0.1
Conditional Spectrum	0.027	40	0.0237	0.87	0.46	0.2
Proposed algorithm	0.027	28	0.0293	1.08	0.44	0.035
		40	0.0287	1.06	0.43	0.025

6 GROUND-MOTION SELECTION

For demonstrating the reliability of the proposed algorithm, the following circumstances are made aimed at comparing the structural response histories as the result of implementing the proposed algorithm with those of the POC [10] under the same conditions:

--A total of 7102 single components from the Next Generation Attenuation (NGA) [13] were used as a database. Each horizontal component (recorded at the same station) in the NGA database is treated as an individual ground motion.

--The Campbell and Bozorgnia ground-motion prediction model [14] was used to estimate the target scenario using the disaggregation method, i.e., a mean with magnitude = 7; distance to rupture of 10 km, assuming Vs30 = 400 m/s; and a strike-slip mechanism, which is the same as those of Haselton et al. [10].

--Each ground-motion candidate is preliminarily scaled at the fundamental period of the three selected structures in the probabilistic hazard curve [14] with $\varepsilon(T^*) = 2$. This step is the same as that of Haselton et al. [10]).

The proposed algorithm is used for selecting three sets of appropriate ground motions (e.g., 28 and 40 in this study) for the purpose of dynamically analyzing the three structures. The sets of ground motions in orderly descending SCFs are selected following the above-mentioned steps. Lastly, the first three sets of 28 and two sets of 40 single strong motions were selected.

Table 3: The maximum inter-story drift ratio (MIDR) corresponding to the 12-story building

method	POC	No. Of Rec.	Median MIDR	Ratio to POC	Dispertion MIDR	Collapse probability
Conditional Mean Spectrum	0.022	28	0.0199	0.90	-	-
		40	-	-	-	-
Conditional Spectrum	0.022	40	-	-	-	-
Proposed algorithm	0.022	28	0.0227	1.03	0.41	0.10
		40	0.0247	1.12	0.43	0.075

Table 4: The maximum interstory drift ratio (MIDR) corresponding to the 20-story building

method	POC	No. Of Rec.	Median MIDR	Ratio to POC	Dispertion MIDR	Collapse probability
Conditional Mean Spectrum	0.019	28	0.0186	0.98	-	-
		40	0.0186	0.98	0.25	0
Conditional Spectrum	0.019	40	0.0196	1.03	0.43	0.05
Proposed algorithm	0.019	28	0.0192	1.01	0.37	0.10
		40	0.0192	1.01	0.40	0.125

7 DYNAMIC ANALYSIS PROCEDURE

In POC procedure, the accuracy of each GMSM method has been evaluated by comparing the results of the nonlinear response histories of each structure with the true response, or, alternatively, the high-end prediction [10]. The CMS approach [1] has been recognized as the most appropriate approach among the 16 GMSMs [10]. The reader is referred to Haselton et al. [10] for more information. The proposed method is also evaluated by comparing the nonlinear responses of the same structures subjected to the same number of the selected ground motions as those of POC [10]. To this end, the 2-dimensional forms of the selected frames having 4, 12, and 20 stories are dynamically analyzed calculating the response histories. The dynamic nonlinear analysis of the structures subjected to the following strong motions are performed using CMS, CS, and MCSO methods.

(a) Sets of 28 and 40 selected earthquakes.

(b) sets of 7, 14, and 21 selected earthquakes

Furthermore, the median, the dispersion of response histories of each structure, and the probability of collapses are calculated and explicitly compared with those of the POC [10], while implicitly are compared to those of the CMS and the CS methods.

8 RESULTS AND COMPARISON

The Maximum Inter-story Drift Ratio (MIDR), defined as the relative displacement between two consecutive story levels normalized by the story height, is used as the primary measurement of the structural response history. The mean MIDR of the three selected structures subjected to the set of 28 selected records, among 7102 single components, are calculated. The obtained seismic demands of the structures having 4, 12, and 20 stories are explicitly compared with those of POC [10] and implicitly with those of CMS and CS [3].

The statistical characteristics of the obtained MIDRs, in the median and dispersion forms of the non-collapse response histories and the probability of collapse (i.e., counted as the fraction of response histories indicating collapse), are also calculated and compared. Similar comparisons are performed between the selected 28 strong motions and those of the Haselton's are made [10]. The selected ground motions, as listed in Table A-1, are in the form of four sets of seven ground motions obtained from the CMS method and this study along with PEER-NGA Filenames and PEER-NGA Numbers for the structures B and C. The scaling factors associated with the individual ground motions as well as the mean value of sets of the seven earthquakes are also shown.

The agreement of the target scenario (M-R) with those of the average magnitude M and distance R, the positive epsilons, and the scaling factors, having values close to one as the objectives of the article, confirm the reliability of the proposed technique in selecting the appropriate suite of ground motions. Notably, the resulted scaling factors, as our objective, imply that the method does not suffer from scaling-factor problems. The comparison of the selected suite of strong motions obtained from this study along with those of the CMS (left-hand side plots), and CS (right-hand side plots) are shown in Fig. 3. The response spectra (from 0.2T*-2T*), corresponding to the selected sets of 28 and 40 ground motions which are scaled at the fundamental periods of the three structures (T*) in the hazard curve using the proposed method, along with those of the CMS, and CS methods are also shown.

Moreover, median plus two standard deviations (from Campbell and Bozorgnia GMPE 2008), at: (a) 4 stories; (b) 12 stories; and (c) 20 stories are displayed in the Figure 2. The right panel three figures, b, d, and f show the comparison of the standard deviation variations versus the period obtained from this study and the CS as the results of applying the sets of 28 and 40 selected strong motions upon the three structures. The differences of standard deviation variations at periods in the two methods are quite visible. As seen, the mean nonlinear responses have slight differences with those of the CMS, especially CS, whereas the dispersions are closer to those of the CS than to the CMS. The latter result implies that the inherent variances existing in the spectral ordinates of the selected ground motions selectd by the proposed MSCO are sufficiently accounted for.

Furthermore, a series of dynamic analyses on the structural types B and C subjected to the sets of 7, 14, and 21 numbers of the selected strong motions are performed to better understand the effects of increasing the strong motion numbers on the mean MIDRs (see Table 5).

Table 5: Comparison of the MIDR's median results for 2 structural types, B and C, obtained from the CMS and this study.

No. Of records	Structure B (8 story)			Structure C (20 story)		
	CMS	This study (MSCO)	MSCO/CMS ratio	CMS	This study (MSCO)	MSCO/CMS ratio
7	0.0224	0.0159	0.7	0.0173	0.0207	1.19
14	0.0199	0.022	1.1	0.0178	0.0197	1.1
21	0.0185	0.182	0.98	0.0182	0.0192	1.05
28	0.0199	0.0227	1.14	0.0185	0.0192	1.03

9 DISCUSSION

As mentioned earlier, the two CMS and CS methods are developed to select those candidates whose spectral ordinates are restricted to the predefined mean spectrum ignoring their intensity, frequency content, and duration [15]. Additionally, their restriction at periods smaller than T* causes the selection of candidates associated with small spectral ordinates at the periods smaller than T* where the demands of irregular structures are influenced by the higher modes [16]. Figure 2 shows the selected sets of 28 earthquakes in the forms of epsilons versus periods obtained from the CMS (right-hand side) and this study (left-hand side). The trends of the mean responses because of the above-mentioned restriction are quite visible. The problem is also confirmed by showing the mean responses of the structures A, B, and C subjected to the sets of 28 and 40 earthquakes obtained from CMS, CS, and this study (see Fig.3). The three dashed lines in the left-hand side figures at periods larger than T* (this study) are associated with spectral ordinates equal to or smaller than those of the CMS's, whereas those at smaller periods are larger. On the other hand, the spectral ordinates associated with smaller periods are closer to those in the hazard curve, compared to those of the CMS, meaning that use of the MCSO method, to some extent, provides earthquakes more sensitive to the structural higher mode. Because regular structures are used in this study (i.e., the responses are less influenced by the higher-mode effects). This problem notably has no significant impact on the response histories.

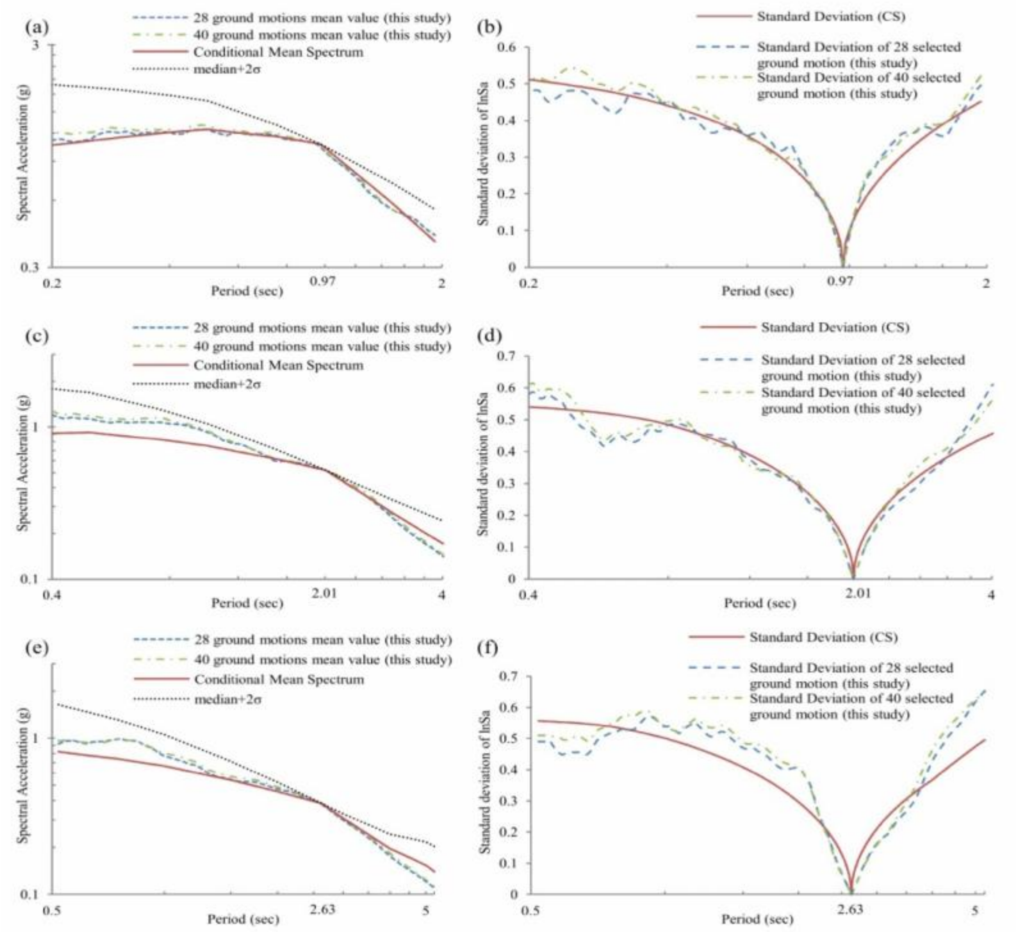

Figure 2: Comparison illustration of the selected suit of earthquakes (in logarithmic form) obtained from this study as compared with those of CMS (left-hand side plots) and CS (right-hand side plots). The left three figures, a, c, and e, display the comparison of the median response spectra (from 0.2T* -2T*) corresponding to the selected 28 and 40 ground motions scaled at the fundamental periods of the three structures (T*) using the proposed method, CMS, and CS methods. The median plus two standard deviations (from Campbell and Bozorgnia GMPE, 2008) are also shown in the three left-hand sides. The examined structures are (a) 4 stories, (b) 12 stories, and (c) 20 stories. The right-hand side three figures, b, d, and f, show a comparison of the standard deviation variations versus the periods obtained from this study and those of CS as the results of applying 28 and 40 selected ground motions to the three structures. The standard deviation variations at different periods in the two methods are quite visible.

The right-hand side plots in Fig. 3 show the comparison of the response history standard deviations obtained from this study and those of the CS. As seen, the standard deviation obtained from this study, which reflects the dispersions of the three structures' response histories, poses similar and, to some extent, more standard deviation than those of the CS reflecting more-realistic data.

the proposed method, in addition to its simplicity, is free from the above mentioned shortcomings, The reasons are: its very small scaling factors (close to one), selecting more real earthquakes, achieving the target magnitude, distance, and positive epsilon (M-R-ε) considerably higher than those of the CMS and CS (see Table 6), and finally accounting for the ground motion variability.

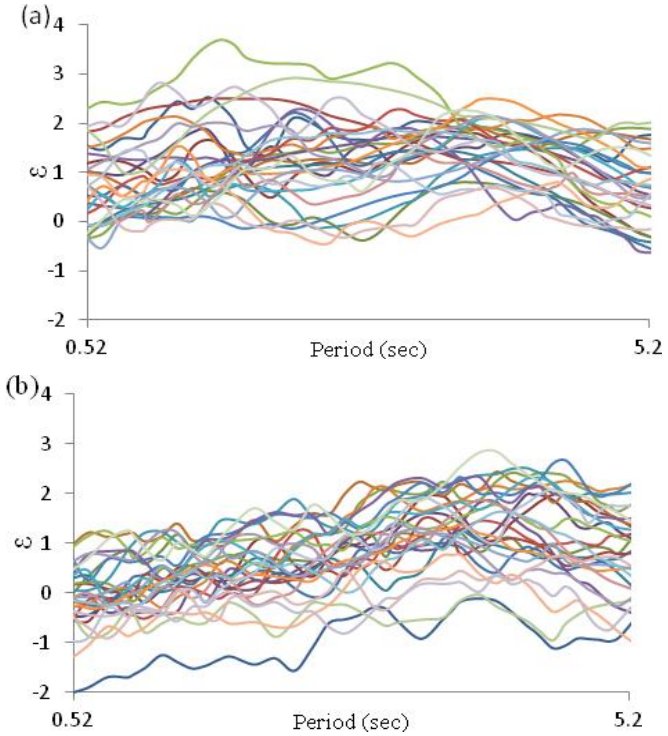

Figure 3: Illustration of the epsilon's trends associated with the selected set of 28 earthquakes for the 20-story frame: (a) MCSO method, and (b) CMS method. The downward trends of CMS (right-hand side) earthquakes are quite visible.

Tables 2, 3, and 4 compare the median MIDRs and their ratios to POC obtained from the dynamic analysis of the three selected structures subjected to the selected sets of 28 and 40 ground motions as the result of using the proposed algorithm, CMS, and CS. The average magnitudes M, distances R, and epsilon of the sets of seven ground motions and their total averages are listed in Table 6 demonstrating the M-R-ε (Richter-Km-dimensionless) values of 6.85-94.25-0.13, 6.75-71-0.43 (by CMS) against 7.1-8.8-1.46, 7-5.75-1.25 (by this study), respectively. As seen, this study has produced results more consistent with those of the target moment, distance, and epsilon with no suffering from scaling problems.

Table 6: Comparison illustration of the mean magnitude M, distance R, and epsilon ε corresponding to the ground motions selected by the MSCO and CMS methods.

	Mean M-R-ε (Richter-Km)	
Method \ Structural Type	B	C
CMS	6.85-94.25-0.13	6.75-71-0.43
This Study	7.1-8.8-1.46	7-5.75-1.25

10 CONCLUSION

A computationally GMSM, termed MCSO, is proposed to select suites of appropriate ground motions that, in addition of its simplicity, emerges with earthquakes associated with positive epsilons, have scaling factors close to one, consistent with the target magnitude-distance M-R, and provide real ground-motion variability. The selected ground motions are matched in a predefined Uniform Hazard Spectra, (UHS) (e.g., 10% or 2% in 50 years) at a specified period (T*) (fundamental period of the structure under study). The basic concept of the method and its mathematic formulations are comprehensively declared, tested, and evaluated. The effectiveness of the Selection Criteria Function "SCF," whose value reflects the appropriateness of the ground-motion candidate, is evaluated through a selection of four couples of the candidate ground motions provided by MCSO and CMS and their influences in the selection procedure are compared. The MCSO performance is evaluated through a comparison process by intentionally selecting the same assumptions made by Haselton and his co-workers. These assumptions are: the three special reinforced concrete structures having 4, 12, and 20 stories; the 7102 single ground motions obtained from the Next Generation Attenuation (NGA) database; the Campbell and Bozorgnia ground-motion prediction equation [14]; a mean target scenario with magnitude = 7; distance of 10 km to rupture (as the result of using the disaggregation method; the Vs30 = 400 m/s, and the strike-slip mechanism. The traditional CMS and the CS methods and their deficiencies are briefly explained, aimed at posing the simplicity and merits of the proposed method. The 2-dimensional form of the selected three reinforced concrete structures are dynamically analyzed, and the MIDR response histories are calculated taking into account the structural deterioration. Two series of dynamic analyses of the three structures having 4, 12, and 20 stories subjected to 28 and 40 sets of the selected ground motions are performed using the successive steps of the proposed algorithm (see Fig 2). The Point Of Comparison (POC) procedure, by which the accuracies of the 16 GMSMs have been evaluated, as the high-end prediction [16], are followed as the comparison purpose for evaluating the results of the proposed method. Comparison of the results are explicitly made between those of the MCSO and the POC methods and implicitly with those of the CMS and CS. These are as follows:

-The median of Maximum Inter-story Drift Ratios (MIDRs) and their ratios to those of the POCs.

-The corresponding dispersions and the scaling factors associated with each selected set of ground motions.

-The magnitudes and distances (M-R), the epsilons, the dispersions, and the probability of collapse.

Stricktly speaking, the suite of ground motions selected by the proposed approach and its simplicity (i.e., free from calculating the correlation at multiple vibration periods as a necessary step in CMS and CS), benefits from the following major advantage:

(a) Coming up with a very small scaling factor, thus keeping up the natural links between the signal amplitudes and associated frequencies; (b) good consistency with the target earthquake scenario (M-R of 6.85-94.25 and 6.75-71 from the CMS against 7.1-8.8 and 7.5-5.75 from this study); (c) ending up with higher positive epsilons (see Table 6); (d) and lastly, providing the natural variability of the suites of earthquake.

The aforementioned points are quantitatively demonstrated through calculating and comparing the response histories (MIDR) of the three R/C frames subjected to 28 and 40 sets of the selected ground motions as the result of implementing the CMS, CS, and MCSO methods.

The material in this article is meant to be neither comprehensive nor complete, and more structures including different types of irregularities are necessary for analysis to more-convincing results.

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