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DESTRUCTION OF THIN FILMS WITH A DAMAGED SUBSTRATE AS A RESULT OF WAVES LOCALIZATION CAUSED BY PERIODIC IMPACT

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Abstract. *The present paper is devoted to an issue of possible localization of waves propagating within a structure that consists of a film connected to a backing material through a substrate. The substrate is initially damaged.In the first approximation the film model in the present paper is assumed to be a string on an elastic foundation with a coefficient depending on the substrate damage degree. The elastic foundation imitates the substrate and backing material effects on the film.Initiation of a string delamination resulted from the structure damaged at localized oscillations caused by a periodic impact load has been considered.At loading the initial damage of the substrate is changing in time and space according to the proposed law of the damage growth.It has been shown that at impact the cause of the string substrate material damage increase can be localized oscillation modes. The localized mode existence depends on relation between the initial substrate rigidity and the main material rigidity.There is also possible a passage through a sequence of resonances under the action of a periodic impact force.*

1 INTRODUCTION

Multilayered structures coated with thin films are widely used in various fields of engineering. An illustrative example is auto windshields that either are coated with special films or are laminated structures consisting of thin films bound together by some other material [1]. Shock resistance capability of such multilayered structures depends on a backing material strength and parameters of the film itself. Thin films coating a backing material are employed in microelectronics of flexible screens that can be rolled up, electronic skin production and flexible solar batteries convenient for transportation and application. One of the main problem of developing and application of the aforementioned multilayered structures is their shock resistance capability. An important aspect of the problem is wave processes in the multilayers structures initiated by an impact action. Issues of the multilayered structure shock resistance, in particular the composite material structure, have been investigated during the last 50 years and are still of great interest. The in-depth review of the problem can be found in [2, 3]. The present paper is devoted to an issue of possible localization of waves propagating within a structure that consists of a film connected to a backing material through a substrate. The substrate is initially damaged. Modeling and analysis of waves propagation in elastic solids undergoing damage and growth process were reported in [4]. It was found that a certain low-frequency band exists where the group velocity is antiparallel to the phase advancement,thus indicating an acoustic metamaterial phenomenon. In the first approximation the film model in the presenting paper is assumed to be a string on an elastic foundation with a coefficient depending on the substrate damage degree. The elastic foundation imitates the substrate and backing material effects on the film.A similar spring model has been used by the authors of [5, 6] for comparing the calculated results with the obtained experimental data on ultrasonic investigation of defects in multilayered structures. Papers [7, 8, 9] have revealed that in structures with delamination zones the oscillations localization is possible in those zones. It is explained by the fact that rigidity and density parameters of the delamination zones differ from those in other parts of the structure. The present paper studies initiation of a string delamination resulted from the structure damaged at localized oscillations caused by impact loads. The thin film is simulated as a string in the present paper, and, therefore the substrate and backing material reactions to outside impact effects are taken into account only as the Winkler foundation. The elastic string foundation coefficient is equal to the total rigidity of substrate strings linked together in series plus the rigidity of the backing materials. At loading the initial damage of the substrate is changing in time and space according to the proposed law of the damage growth. The spring model, as the name implies, substitutes the problem of a wave propagation in a medium with the analog problem of exciting an equivalent set of springs. The method is stable and convergent as long as the equivalence between the two problems is justified.The 1-D case is, of course, very elementary. Nevertheless, it is discussed because it allows us to define in a very natural way the ingredients - internal springs,and condition of a string defoliation, which can later be extended to 2D.

2 STATEMENT OF PROBLEM

The main equation of the string dynamics has the form:

$$\gamma u_{xx} - K(n)u - \rho_0 u_{tt} = Q(t,x), \quad x \in (-\infty, +\infty), \quad t \geq 0 \tag{1}$$

Where $K(n)$ is the reduced coefficient of the elastic foundation, n is the damage function (see for the classical definition of n [10]), u is the vertical displacement of the string, ρ_0 is the string material density, $Q(x,t)$ is the external force. In order to underline the purposes of our approach let us introduce the following definitions.The total volume of a substrate V of M particles is divided into equal cells $w = V/M$,every cell being either filled by "damaged" particles (N is the number of "damaged" particles per cell) or occupied by an undamaged particle(N_0 is number of undamaged particles per cell). The nature of these cells can,in principle,be choose arbitrarily;in practice,it would probably be most satisfactory to chose cells sufficiently small so that the probability of two particles occupying a single cell is negligible.Thus, we can introduce the damaged function n as follows: $n = 1 - \frac{N_0}{N+N_0}$. There is the following expression for the damage function n (see [11]): $\frac{\partial n}{\partial t} + n\nabla V_* = J$, where V_* is the velocity of the particles of the substrate, J is the damage growth source.The term $n\nabla V_*$ in that equation is neglected in the first approximation , i. e. the particle diffusion of the damaged material is not taken into consideration. Assume, that $G = G_0(1-n)$ is the tension/compression rigidity of the substrates, where G_0 is the rigidity of the interstitial layer of the acoustic medium (substrate) with compression modulus $G = \rho_1 c^2$. Here c is a wave speed in the material of substrate, and ρ_1 is the density of the substrate material.Then, a pressure value in the substrate layer in quasi-static approximation is $p_* = \frac{\rho_1 c^2 (u-v)}{h}$, where v is the vertical displacement of the foundation,h is the substrate layer thickness, and $\frac{u-v}{h}$ defines the relative deformation . Assume , that $J = \beta H(p_* - p_{cr})$. Here H is the Heaviside function. Thus, we assume that the damage growth source starts increasing the damage when the pressure in the layer reaches its critical value $p_{cr} = G_0\Delta$. Note, that $\frac{\Delta}{h}$ is the maximum value of the interstitial layer relative deformation. Finally, as a result we have :$J = \beta H[(u-v) - \Delta]$. Since the substrate rigidities and the rigidity of the main material of the structure (designated as k_0) are joint in series, one has : $v = \frac{Gu}{k_0+G}$ and $u - v = \frac{k_0 u}{k_0+G}$. Thus, finally the following system of equations describing dynamics of a film connected to an elastic foundation and containing damaged material has been obtained:

$$\gamma u_{xx} - K(n)u - \rho_0 u_{tt} = Q(x,t), \quad x \in (-\infty, +\infty), \quad t \geq 0, \tag{2}$$
$$K(n) = \mu(n)G(n),$$
$$\mu(n) = \frac{k_0}{k_0+G(n)};$$
$$\frac{\partial n}{\partial t} = \beta H\left(u\frac{k_0}{k_0+G(n)} - \Delta\right)(1-n),$$

where $n = n(x,t), n_0 = n(x,0)$; $k_0 > 0$ is a constant,

$$G(n) = G_0(1-n), \quad 0 \leq n \leq 1,$$

$$G(n) = 0, \quad n > 1,$$

In the case when $n = 1$ the film comes off the substrate. It may be assumed that the film separation starts at some critical value of the damage coefficient $n = n_*$. Thus, the present paper investigates the process of the damage growth leading to the initial film detachment. Let us consider the system of Eqs.(2) describing the elastic system model with the damaged substrate. Here $K(n)$ is the complicated functional of u that includes a damage function n.

The time evolution of the damage function $n(x,t)$ is defined by the following differential equation

$$\frac{\partial n}{\partial t} = \beta H(\mu(n)u - \Delta)(1-n), \tag{3}$$

where $\beta, \Delta > 0$ are positive constants, and H is the Heaviside step function. It is assumed that Q is a smooth function fast decreasing in $|x|$, and the boundary and initial conditions are as follows:

$$u(x,t) \to 0 \quad (|x| \to \infty) \tag{4}$$

$$u(x,0) = 0, \quad u_t(x,0) = 0, \quad x \in (-\infty, +\infty) \tag{5}$$

$$n(x,0) = n_0(x), \quad x \in (-\infty, +\infty) \tag{6}$$

where n_0 also is a smooth function fast decreasing in $|x|$.

For bounded domains, if $x \in [-h, h]$, we set the Dirichlet boundary condition

$$u(h,t) = u(-h,t) = 0. \tag{7}$$

Below we will show that weak solutions for the case $x \in (-\infty, +\infty)$ can be obtained as a limit of solutions satisfying (7) as $h \to +\infty$.

Note that Eq. (3) implies $0 \le n(x,t) \le 1$ for all n if $0 \le n(x,0) \le 1$. This can be shown by the maximum principle. In fact, if $n > 1$ at some $x_0, t_0 > 0$, then there is a point (x_*, t_*) where $n_t > 0$ and $n = 1$, but this is in contradiction with Eq.(3). Therefore, Eq. (3) correctly defines the damage function. There are possible alternative versions of Eq.(3) for the damage function but they are not considered in the paper. Eq. (3) means that the destruction (the damage growth) starts when the displacement reaches the particular threshold.

To describe the time of the elastic system destruction the criterion of critical damage level is used.

Criterion of critical damage level

Assume that the elastic material is destroyed at point x and at the moment t when the damage function $n(x,t)$ reaches some critical level n_* at that moment:

$$n(x,t) = n_* \tag{8}$$

The minimal time T_F such that Eq.(8) holds for some x, is called the full destruction time.

The front of full destruction $L_F(t)$ is defined by the following relation

$$n_* = n(L_F(t), t), \tag{9}$$

where n_* is a critical damage level. In the general case when $u(x,0) \neq 0$ the function $L_F(t)$ is increases in time.

3 ASYMPTOTIC FOR SMALL β

If β is small, then $dn/dt = O(\beta) << 1$ and the coefficient $K(n)$ is a function of the slow time $\tau = \beta t$. Therefore, we can use a quasi-stationary approximation by the two time scales perturbation method. Let us use the ansatz

$$u = U_0(x,t,\tau) + \beta U_1(x,t,\tau) + ...$$

Then, by substituting this series into Eq.(2) one obtains for terms of the order $O(1)$ the following equation

$$\gamma U_{0xx} - K(n)U_0 - \rho_0 U_{0tt} = Q(t,x), \quad x \in (-\infty, +\infty), \quad t > 0 \tag{10}$$

In this equation $n = n(x,\tau)$ can be considered as a slow parameter. Different asymptotic of U_0 will be found in coming sections. The parameter n is defined, in the first approximation, as follows:

$$\frac{\partial n}{\partial \tau} = H\left(\frac{k_0 U_0(x,t,\tau)}{k_0 + G(n)} - \Delta\right), \quad \tau = \beta t. \tag{11}$$

The destruction process starts in points where the following conditions hold:

$$\frac{k_0 U_0(x,t,\tau)}{k_0 + G_0(1 - n(x,\tau))} = \Delta. \tag{12}$$

The roots $T_{beg}(x,\tau)$ of eq. (12), can be named the *time when the destruction process start*.

The correction U_1 can be found from the following equation :

$$\gamma U_{1xx} - K(n)U_1 - \rho_0 U_{1tt} = Q_1(t,x,U_0), \quad x \in (-\infty, +\infty), \quad t > 0$$

where $Q_1 = 2\rho_0 U_{0t\tau}$. In general, it is not easy to find explicit relations for U_0 and U_1, and because of that, different cases where asymptotic can be obtained in the coming sections are considered.

4 PERIODIC FORCES AND RESONANCE EFFECT

4.1 Resonance condition

In some experiments one studies external loads consisting of some periodical strikes. This can be described by equation (1), where

$$Q(x,t,\epsilon) = \delta_\epsilon(x - x_0) \sum_{j=0}^{M} \delta(t - j\Delta t), \quad x \in (-\infty, +\infty), \quad t > 0. \tag{13}$$

where Δt is a time step for the strikes. The solution depends on the localization parameter ϵ and M. Let us assume that the initial data are as follows:

$$u(x,0,\epsilon) = \phi_0(x), \quad u_t(x,t,\epsilon)|_{t=0} = \phi_1(x).$$

Let us construct some particular solutions of (1) describing resonances between the time periodic external load and localized modes. We consider the limit $\epsilon \to 0$ and suppose that $M >> 1$. To investigate this initial value problem, we can use the Fourier transformation :

$$u(x,t,\epsilon) = 2\pi^{-1/2} \int_{-\infty}^{\infty} \exp(i\omega t)\hat{u}(x,\omega,\epsilon)d\omega,$$

$$Q(x,t) = 2\pi^{-1/2} \int_{-\infty}^{\infty} \exp(i\omega t)\hat{Q}(x,\omega)d\omega, \quad i = \sqrt{-1}$$

The Fourier coefficients $\hat{Q}(x,\omega)$ can be computed and have the form:

$$\hat{Q}(x,\omega) = \delta_\epsilon(x - x_0)2\pi^{-1/2}\hat{S}(\omega), \quad \hat{S}(\omega) = 2\pi^{-1/2} \sum_{j=0}^{M} \exp(-ij\omega\Delta t).$$

Let us make a remark. For large M the quantity $\hat{Q}(x,\omega)$ has the order M only if $\Delta t\omega \approx 2m_0\pi$, where m_0 is a positive integer. Indeed, if $|\Delta t\omega - 2m_0\pi| >> M^{-1}$, then $\hat{S}(\omega)$ is bounded for large M

$$\sum_{j=0}^{M} \exp(-ij\Delta t\omega) = \frac{1 - \exp(-(M+1)i\omega\Delta t)}{1 - \exp(i\omega\Delta t} = O(1),$$

and it is large if $|\omega\Delta t - 2m_0\pi| = 0$ for some integer m_0. Then

$$\sum_{j=0}^{M} \exp(-ji\omega\Delta t) = M$$

Therefore, we observe a resonance for ω close to $\omega(m_0) = 2m_0\pi(\Delta t)^{-1}$.

For $\hat{u}(x, \omega, \epsilon)$ one finds the following equation

$$\gamma\hat{u}(x, \omega, \epsilon)_{xx} - K(n)\hat{u}(x, \omega, \epsilon) + \rho_0\omega^2\hat{u}(x, \omega, \epsilon) = \delta_\epsilon(x - x_0)\sqrt{2\pi}^{-1/2}\hat{S}(\omega). \qquad (14)$$

The left hand side of (14) is the Schrödinger operator, which has the localized eigenfunctions $\Psi_j(x)$ with eigenvalues E_j, and non-localized eigenfunctions $\Psi(x, k)$ with eigenvalues $E(k)$, where k is a wave number. Note that E_j is a complicated functional of $n(x, \tau)$, and these quantities depend on slow time τ as a parameter.

4.2 Asymptotics describing resonance effect

There is also possible a passage through a sequence of resonances. We consider the following cases: (**a**) initial stage of destruction when n is small, (**b**) final stage where $n \approx 1$.

Remind some facts about the Schrödinger operator with shallow potentials wells. Consider the operator

$$H\Psi = (-\Delta + \rho\Phi(x))\Psi$$

in $\mathbf{R}^1$, where $\Phi(x)$ is a smooth function fast decreasing as $|x| \to \infty$. Then, under condition

$$I_\Phi = \int_{-\infty}^{+\infty} \Phi(x)dx < 0 \qquad (15)$$

for sufficiently small $\rho > 0$ we have a single eigenfunction $\Psi_1(x)$ with the eigenvalue $E_1(\rho)$ defined by

$$E_1 = -\eta^2, \quad \eta = \frac{\rho I_\Phi}{2}. \qquad (16)$$

If $I_\Phi > 0$, we have no eigenfunctions. For Ψ_1 we have asymptotics

$$\Psi_1 = \eta^{3/2}\int_{-\infty}^{+\infty} \exp(ipx)a(p)(p^2 + \eta^2)dp, \quad a(p) = \frac{\tilde{\Phi}(p)}{\tilde{\Phi}(0)}, \qquad (17)$$

where

$$\tilde{\Phi}(p) = (2\pi)^{-1/2}\int_{-\infty}^{+\infty} \Phi(x)\exp(-ipx)dx$$

is the Fourier image of the potential Φ.

Let us apply these results to the cases (**a**) and (**b**).

In the first case (**a**) the expression $K(n)$ has the form $a_0^2 - b_0 n + O(n^2)$, where $a_0^2 = k_0G_0/(k_0 + G_0)$ and $b_0 = k_0/(k_0 + G_0) > 0$. Then we obtain a Schrödinger operator with a small potential well. The condition (15) holds. Thus we have a single localized mode and the corresponding energy E_1 admits the estimate $E_1 = -a_0^2 + O(n)$. For $n << 1$ The resonance condition takes the form

$$k_0G_0/(k_0 + G_0) + O(||n||) = \rho\omega(m_0)^2 \qquad (18)$$

for some integer m_0. To obtain this condition in a more precise and general form, below we consider a particular case when Φ is a rectangle potential, where resonance condition can be derived by a straight forward way, and after we derive this condition in general form.

Let us introduce the detuning parameter κ by

$$\kappa^2 = \gamma^{-1}(a^2 - \rho\omega(m_0)^2), \quad a^2 = k_0G_0/(k_0 + G_0). \qquad (19)$$

Let us investigate the resonance effect in more detail. Suppose $n << 1$, then we can use asymptotics

$$K(n) = a^2 - \frac{k_0 G_0}{(k_0 + G_0)^2} n(x) + O(n^2) = a^2 + \gamma W(x)$$

for the case when $\Phi(x)$ is a rectangle potential

$$\Phi(x) = -h, \quad |x| < l, \quad \Phi = 0, \quad |x| > l. \tag{20}$$

Equation (14) takes the form

$$\hat{u}(x,\omega)_{xx} - \bar{k}_0 \hat{u}(x,\omega) - \Phi(x)\hat{u}(x,\omega) = \gamma^{-1}\delta_\epsilon(x - x_0)\sqrt{2\pi}^{-1/2}\hat{S}(\omega). \tag{21}$$

Due to the basic properties of the Schrödinger equation mentioned above the localized eigenfunction exists if

$$h - \kappa^2 = \beta^2 > 0, \quad \kappa > 0.$$

We assume that these conditions hold. In the case (20) and $\epsilon \to 0$ the solution of eq. (21) can be found. The solution $U = \lim_{\epsilon\to 0} \hat{u}(x,\omega,\epsilon)$ has the form (we assume that $|x_0| < l$, i.e., the external force is localized within the destruction zone). For $x < x_0$ we have

$$U = C_-(\frac{\kappa}{\beta}\sin(\beta(x+l)) + \kappa\cos(\beta(x+l)), \quad x \in (-l, x_0), \tag{22}$$

$$U = C_- \exp(\kappa(x+l)), \quad x < -l \tag{23}$$

and for $x < x_0$ we obtain

$$U = C_+(-\frac{\kappa}{\beta}\sin(\beta(x-l)) + \cos(\beta(x-l)), \quad x \in (x_0, l), \tag{24}$$

$$U = C_+ \exp(-\kappa(x-l)), \quad x > l. \tag{25}$$

Let $x_0 = 0$. Then the constants $C_\pm$ are defined by

$$C_+ = C-- = \frac{1}{2(\kappa\sin(\beta l) - \beta\cos(\beta l))}. \tag{26}$$

Finally, the resonance condition has the form

$$\kappa\sin(\beta l) - \beta\cos(\beta l) = 0. \tag{27}$$

The left hand side of this equation is the eigenvalue E_1 of the corresponding Schrödinger operator. This condition does not depend on x_0 however the solution U depends on x_0.

It is clear that the general form of the resonance condition is

$$k_0 G_0/(k_0 + G_0) + E_j = \rho\omega(m_0)^2, \quad j = 1, ... \tag{28}$$

where E_j are eigenfunctions of the corresponding Schrödinger operator. An explicit expression for E_j can be obtained only in some cases. For small damage densities $n_0(x)$ we obtain, by asysmptotics (16)

$$E_1 = \frac{k_0 G_0}{(k_0 + G_0)^2} \int n_0(x) dx. \tag{29}$$

4.3 Wave regime

Consider the case when the external load is defined by (13) and $n \approx 1$. In the case (**b**) the expression $K(n)$ has the form $G_0(1-n) + ((1-n)^2)$. Then again we obtain a Schrödinger operator with a small potential well. However, now condition (15) does not hold. We have no single localized modes and no resonances. Therefore, we conclude that close to a final destruction stage, localized mode and resonances vanish. In this case we observe purely wave regime. Namely, taking into account that $K(n) \approx 0$, we note that eq. (1) takes the form

$$\gamma u_{xx} - \rho_0 u_{tt} = Q(x,t,\epsilon), \quad x \in (-\infty, +\infty), \quad t > 0. \tag{30}$$

Using (13) in limit $\epsilon \to 0$ we can find the following solution of this equation. One has $u(x,t,\epsilon) \to u(x,t)$ as $\epsilon \to 0$ and

$$u = (2c)^{-1} \sum_{j=1}^{M} (H(x - c(t - j\Delta) - (H(x - c(t + j\Delta))H(t - j\Delta t), \quad t > 0. \tag{31}$$

where H is the Heaviside step function. This solution describes M propagating waves.

5 DESTRUCTION FRONT DYNAMICS.

The formal solution of Eq.(3) can be written down as follows:

$$n(x,t) = 1 + (n_0(x) - 1)\exp(-\beta S(x,t)), \quad S = \int_0^t H(\mu u - \Delta)ds. \tag{32}$$

The equation for L_F is as follows

$$n_* = 1 + (n_0(L_F) - 1)\exp(-\beta S(L_F, t)). \tag{33}$$

It is natural to assume that the critical parameter $n*$ is close enough to 1. The integral S can be computed using expression for u. This gives the following asymptotic for large t:

$$S = C_1 t, \quad |x - x_0|/c < t,$$

$$S = 0, \quad |x - x_0|/c > t,$$

where C_1 is a constant depending on the system parameter in a complicated way. Therefore, if $|x - x_0| > ct$, i.e., the wave does attain x within time t, Eq.(33) has no solutions for $n* \approx 1$ since $n_* > \max n_0(x)$. If the right hand side of Eq.(33) is smaller than n_*, then there are no solutions for all x and the destruction process does not start. If the right hand side of Eq.(33) is bigger than n_* then the solution L_F of (33) exists. So, finally, the following result is obtained: for large t we have two destruction fronts such that

$$L_F(t) \approx \pm c\, t + O(1). \tag{34}$$

Consider an influence of the wave contribution W and the localized mode contribution V to a destruction process. It is seen that this effect depends on x and t. For some x the localized mode contribution reinforces the destruction process, while for others that contribution diminishes it. So,we have two qualitatively different dynamics of a destruction front. If the localized mode amplitude is smaller than the amplitude of the wave term, then the dynamics of the front is linear. If the localized mode amplitude has the same order as the wave term amplitude, we have alternation of time intervals,where L_F increases, and time intervals where $L_F(t)$ is a constant. The transition from oscillating dynamics to wave dynamics with a linear growth, when the front leaves the zone where the localized mode is not small, can be observed. Finally, for large times the front moves in a linear manner, and for very large times the front stops since the amplitude of the wave contribution slowly decreases as $t^{-1/2}$. The time when the front stops can be estimated roughly with the help of the asymptotic for the wave contribution, which finally gives the following expression:

$$T_{stop} \approx A\epsilon^2 \bar{k}_0 \gamma^{-1} \sqrt{\rho_0}.$$

Moreover, there is a possibility for a transition region in which oscillating front dynamics with stop intervals change to the monotone linear dynamics if the zones of the impact force application, and the localized mode concentration, are different.

6 PASSAGE OF FAST LOCALIZED IMPULSE THROUGH DESTRUCTION ZONE

An inhomogeneous distribution of the damage parameter influences the localized solution propagation. The goal of this section is to estimate the impulse amplitude. Experimental data show that the impulse amplitude decreases during this propagation process. In this section, we will find asymptotic solutions, which are valid for any $\beta > 0$ and also for not small inhomogeneities. We can describe a localized solution by the asymptotic solution in the form:

$$u(x,t,\epsilon) = a(x,t)\exp(-\epsilon^{-1}S(x,t)), \tag{35}$$

where $S(x,t) \geq 0$ is a smooth function such that $S(x,t) = 0$ at a point $x = X(t)$, $\epsilon > 0$ is a small parameter. Then this solution describes a displacement impulse localized at $x(t)$. One can get the following equation

$$\epsilon^{-2}a(\gamma S_x^2 - \rho_0 S_t^2) + \epsilon^{-1}(\gamma(-2a_x S_x - aS_{xx}) + \rho_0(2a_t S_t + aS_{tt}) + K(n)a) + \gamma a_{xx} - \rho_0 a_{tt} = 0. \tag{36}$$

We consider first terms of the orders ϵ^{-2}. Then one has the eikonal equation in the form:

$$\gamma S_x^2 - \rho_0 S_t^2 = 0. \tag{37}$$

For $a(x,t)$ one has

$$\gamma(-2a_x S_x - aS_{xx}) + \rho_0(2a_t S_t + aS_{tt}) + \epsilon(K(n)a + \gamma a_{xx} - \rho_0 a_{tt}) = 0. \tag{38}$$

One of simple solutions of equation (37) is $S = \frac{(x-ct)^2}{2}$. Then $X(t) = ct$. The form of solution for $a(x,t)$ (which is not so simple, in general) can be simplified. In fact, in this case we can set $a(x,t) \approx a(ct,t) = A(t) + O(\epsilon)$ and $n(x,t) \approx n(ct,t) + O(\epsilon) = \bar{n}(t) + O(\epsilon)$ with accuracy ϵ, since for $|x - ct| >> \epsilon$ the solution u is exponentially small. It is difficult, however, to use (38) in order to find $a(t)$. We apply the energy balance equation. Let us compute the main asymptotic contribution of the localized solution into the energy, up to small corrections. This gives

$$E[u,u_t] = \frac{1}{2}\int_{-h}^{h}(\gamma u_x^2 + \rho_0 u_t^2 + K(n)u^2)dx \approx R(\epsilon,t)\frac{\bar{a}^2(t)}{2}, \tag{39}$$

where

$$R(\epsilon,t) \approx \int_{-\infty}^{\infty}\exp(-2S/\epsilon)(\epsilon^{-2}(\gamma S_x^2 + \rho_0 S_t^2) + K(\bar{n}(t)))dx. \tag{40}$$

Let us denote

$$M_0(\epsilon) = \int_{-\infty}^{\infty}\exp(-2S/\epsilon)dx.$$

For $S = (x-ct)^2/2$ the third term $K(\bar{n}(t)$ in the right hand side of the equation can be removed, since the contributions of the first and second ones have the order $O(\epsilon^{-1}M_0)$, where the third term give $O(M_0)$. Then $R(\epsilon, t) = R_0(\epsilon)$ is independent on t, and up to small corrections vanish as $\epsilon \to 0$. For the dissipative functional D one has:

$$D(\epsilon, a) \approx \frac{1}{2}\bar{n}_t \bar{a}^2(t)\frac{dK}{dn}(\bar{n}(t))H, \quad M_0 = \int_{-\infty}^{\infty} \exp(-2S/\epsilon)dx. \tag{41}$$

If we suppose that the impulse is powerful enough to start the destruction process at $x = ct$,then:

$$\frac{da}{dt} = M_0 R_0^{-1}(\epsilon)\beta K_n(\bar{n}(t))a. \tag{42}$$

So, the impulse amplitude decrease according to the following expression:

$$a(t) = \Theta(t)a(0), \quad \Theta(t) = \exp(M_0 R_0^{-1}(\epsilon)\int_0^t \frac{K(\bar{n}(s)}{d\bar{n}}ds). \tag{43}$$

For a small β we can replace $\bar{n}$ in this formula to $\bar{n}_0(t) = n_0(ct)$, where n_0 is initial damage distribution. Finally,we obtain for Gaussian impulses with $S = (x - ct)^2/2$ that

$$\Theta(t) = \exp(-const\epsilon\gamma^{-1}\beta k_0^2 G_0 \int_0^t (k_0 + G_0(1 - n_0(cs)))^{-2}ds, \tag{44}$$

and for small G_0 or $n_0 \approx 1$ one has:

$$\Theta(t) = \exp(-const\epsilon\gamma^{-1}\beta k_0 G_0 t). \tag{45}$$

7 CONCLUSIONS

In the paper an influence of the wave contribution and the localized mode contribution to a destruction process has been considered. This effect depends on x and t. For some x the localized mode contribution reinforces the destruction process, while for others that contribution diminishes it. So,we have two qualitatively different dynamics of a destruction front. If the localized mode amplitude is smaller than the amplitude of the wave term, then the dynamics of the front is linear. If the localized mode amplitude has the same order as the wave term amplitude, we have alternation of time intervals,where L_F increases, and time intervals where $L_F(t)$ is a constant. The transition from oscillating dynamics to wave dynamics with a linear growth, when the front leaves the zone where the localized mode is not small, can be observed. Finally, for large times the front moves in a linear manner, and for very large times the front stops since the amplitude of the wave contribution slowly decreases as $t^{-1/2}$. Some particular solutions describing resonances between the time periodic external load and localized modes has been obtained. Also possible passage through a sequence of resonances has been observed. The conclusion that close to a final destruction stage, localized mode and resonances vanish can be made. In this case one can observe purely wave regime. An inhomogeneous distribution of the damage parameter influences the localized solution propagation. It has been shown that at impact the cause of the string substrate material damage increase can be localized oscillation modes. The localized mode existence depends on a relation between the initial substrate rigidity and the main material rigidity. The impact energy is redistributed between propagating waves and localized waves in such a way that the latter may make the main contribution to the growth of the material damage.

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