GENETIC OPTIMIZATION OF TOWER VIBRATIONS WITH PENDULUM TMD

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Abstract. The wind turbine is supported by towers, which are slender and flexible due to their geometry and great height. As a consequence, they may experience excessive vibration levels caused by both the operation of the turbine and the wind loads [1]. One solution to this problem is the installation of passive control devices. One common passive device is the Tuned Mass Damper (TMD). Briefly, it is a pendulum-damper which transfers the energy of the vibration from the main structure to itself, working as a passive device. These passive devices need to be finely tuned to work as dampers; otherwise, they could amplify structural vibration levels. In this paper we propose to optimize the vibration performance of a pendulum TMD using Genetic Algorithm (GA). Harmonic analysis is used to describe the tower with pendulum TMD, and restricting the mass and length of the pendulum, we propose two fitness functions for the GA: (a) minimize high peaks of the vibration frequencies; (b) minimize high peaks of the vibration frequencies and maximize the anti-frequency peak (min. max. restriction). We compare the analysis of the GA optimization undertaken by report to Zuluaga [2] gradient approach.
1 INTRODUCTION

The current energy crises coupled to the depletion of fossil fuel stocks and the need to reduce emissions of carbonic gas, in order to preserve the environment makes wind power generation a viable and attractive mean for producing electricity [1].

The wind turbine is supported by a tower that can experience excessive vibrations caused by both the wind turbine and the wind forces because of its geometry and great height. A detailed analysis of the structural behavior of the tower is of great importance due to its cost, which can represent approximately 20% of the total cost of the system [3].

Given the great progress in analyses and structural dimensioning, with the advances in the material field and construction techniques, higher and slender structures have become more interesting to study. These structures, however, are more vulnerable to excessive vibrations due to dynamic loads such as earthquakes, winds, storms, waves, etc.

An alternative option, widely studied in the last few years, is the structural control. Fundamentally, the structure stiffness and damping properties is changed adding external device or applying external forces. It is classified as passive, active, hybrid, or semi-active control. Several researchers have been studying the use of structural control to help suppress the wind-induced vibrations experienced by wind turbine towers [4].

To minimize these vibrations, we implemented a structural control with a Tuned Mass Damper (TMD) Pendulum type. A TMD is a passive control device composed of a mass-spring-dashpot attached to the structure, aiming to reduce structural vibration response [5], since it exhibits a pendulum absorber geometry. After that, a genetic optimization methodology was loaded with multiple objectives to be compared to an existing mathematical and numerical optimization [2].

In this paper we propose a structural control optimization methodology to assist wind turbine design projects, enabling the selection of a configuration that targets an optimum relation between the min and max frequency peaks (Min.Max optimization). The variable parameters surveyed are: pendulum length and ratio mass between the pendulum and the wind turbine.

Depending on the result obtained for the evaluation of TMD mathematical formulation, fitness is ascribed to each configuration and inputted in a Genetic Algorithm (GA) to optimize the initial values. This optimization method was chosen for its advantages at discontinuous functions with multiple variables and complex formulations.

The following sections describe the modal analysis of a wind turbine tower, the TMD mathematical formulation for the pendulum settings, the optimization method chosen (definitions of the chromosome, fitness function, and convergence criteria), and the results.

2 MODAL ANALYSIS

The goal of the modal analysis is to reduce the complex system of partial differential equation which describes the dynamic behavior of a continuous structure. The motion of a cantilever Bernoulli-Euler beam (Figure 1) is described in Equation 1 using the theoretical treatment of the modal analysis [6].

\[
\frac{\partial^2}{\partial z^2} \left( EI \frac{\partial^2 w(z,t)}{\partial z^2} \right) + \rho \frac{\partial^2 w(z,t)}{\partial t^2} = F(z,t)
\]

where \( w(z,t) \): normal displacement of the beam axis in some directions; \( z \): distance along the beam axis; \( \rho \): mass per unit length of the beam; \( F \): external force per unit length applied normal to the beam axis in the direction of \( w \); \( E \): Young’s modulus; \( I \): area inertia moment for bending.
In general, the modulus of the inertia moment and the mass per unit length vary along the span of the beam. However, in this example they may be considered uniform over the span.

![Image of a cantilever beam with a tip mass.](image_url)

**Figure 1: Schematic description of a cantilever beam with a tip mass.**

The boundary conditions for a cantilever beam are given by Equation 2.

\[
\begin{align*}
    w(0, t) &= \partial_z w(0, t) = 0 \\
    EI \partial_z^2 w(0, t) &= \omega_n^2 \Phi'(L) J_M \\
    EI \partial_z^2 w(L, t) &= \omega_n^2 \Phi(L) M
\end{align*}
\]  

(2)

where \( M \) and \( J_M \): mass and corresponding rotatory inertia mass at the free end, respectively.

Separating a space-time function by \( w(z, t) = \Phi(z) Y(t) \) and applying the Fourier method into the associated equation for free vibration, we obtain Equation 3.

\[
\frac{1}{\Phi} \frac{\partial^4 \Phi}{\partial z^4} = -\frac{m}{EI} \frac{1}{Y} \frac{\partial^2 Y}{\partial t^2} = \text{const.} 
\]

(3)

The solution of Equation 3 is given by Equations 4 and 5.

\[
\Phi(z) = C_1 \sin \alpha_n z + C_2 \sin \alpha_n z + C_3 \sinh \alpha_n z + C_4 \cosh \alpha_n z \\
Y(t) = A \sin \omega_n t + B \cos \omega_n t, \text{ with } n = 1, 2, 3, \ldots
\]

(4)

(5)

where \( \omega_n = (n^2 \pi^2 / L^2) \sqrt{EI/m} \): natural frequency of vibration; \( \alpha_n = \omega_n^2 m / EI \).

Using boundary conditions (Equation 2) and \( J_M = 0 \) we determined the constants \( A, B \) and \( C_i, i = 1, 2, 3, \ldots \). Thus the spatial end-conditions of the cantilever beam with tip mass were determined using Equation 6 [7].

\[
\frac{\Phi_n(z)}{C_1} = \sin \alpha_n z - \sinh \alpha_n z + \left( \frac{\sin \alpha_n z + \sinh \alpha_n z}{\cos \alpha_n z + \cosh \alpha_n z} \right) \left( \cosh \alpha_n z - \cos \alpha_n z \right)
\]

(6)

Using the fundamental mode shape \( \Phi_1(z) \), as first vector, the Gram-Schmidt orthogonality process is going to be use to orthogonalize the other mode shapes. To determine the equivalent
one-degree-of-freedom (one-DOF) for a dynamic system with distributed mass and stiffness, it can be assumed that the fundamental mode shape is described by Equation 7 [8].

\[ w(z, t) = Y(t)\Phi(z) = Y(t)\left[1 - \cos\frac{\pi z}{2L}\right] \tag{7} \]

Substituting Equation 7 into Equation 1, we obtain Equation 8.

\[ M_s \ddot{Y} + K_s Y = F \tag{8} \]

Then the generalized stiffness and mass of the tower are computed, respectively in Equations 9 and 10.

\[ K_s = \int_0^L EI [\Phi''(z)]^2 dz = \int_0^L EI \frac{\pi^4}{16L^4} \cos \frac{\pi z}{2L} dz \Leftrightarrow K^* = \frac{\pi^4}{32L^3} EI \tag{9} \]

\[ M_s = M + \int_0^L \left(1 - \cos \frac{\pi z}{2L}\right)^2 dz = M + \frac{mL}{2\pi} (3\pi - 8) \Leftrightarrow M^* = \frac{mL}{2\pi} \left[\pi \left(3 + 2\frac{L_e}{L}\right) - 8\right] \tag{10} \]

where the tip mass \( M_s = mL_e \) is defined proportional as an equivalent length \( L_e \).

3 TUNED MASS DAMPER MATHEMATICAL FORMULATION

Figure 2 shows a schematic description of a pendulum TMD attached to a main system composed of a two-DOF. The main system is reduced to a one-DOF model, which corresponds to the mode to be controlled [5]. The motion considering small displacements to the system is given by the matrix of the Equation 11 [2].

\[
\begin{bmatrix}
(M_s + M_p) & M_p L \\
M_p L & M_p L^2
\end{bmatrix}
\begin{bmatrix}
\ddot{y} \\
\ddot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
C_s & 0 \\
0 & C_p
\end{bmatrix}
\begin{bmatrix}
\dot{y} \\
\dot{\theta}
\end{bmatrix}
+ \begin{bmatrix}
K_s & 0 \\
0 & (K_p + M_p g L)
\end{bmatrix}
\begin{bmatrix}
y \\
\theta
\end{bmatrix}
= \begin{bmatrix}
F_s(t) \\
0
\end{bmatrix}
\tag{11}
\]

where \( M_s \): main system modal mass; \( C_s \): main system modal damping; \( K_s \): main system modal stiffness; \( M_p \): pendulum mass; \( C_p \): pendulum damping; \( K_p \): pendulum stiffness; \( L \): cable length; \( g \): gravity acceleration; \( F_s(t) = F_{s0} e^{i\omega t} \): excitation modal force; \( y(t) \): main system displacement; \( \theta(t) \): pendulum angular displacement.

---

Figure 2: Structure with a linear pendulum attached: excitation due to a force \( F_s(t) \).
Considering \( F_s(t) = e^{i\omega t} \), then was obtained \( y(t) = H_y(\omega)e^{i\omega t} \). Substituting in Equation 11, we obtain the linear equation system given by Equation 12.

\[
\begin{bmatrix}
-(M_s + M_p)\omega^2 + C_s i\omega + K_s \\
-M_p L\omega^2
\end{bmatrix}
\begin{bmatrix}
M_p L\omega^2 \\
-M_p L^2\omega^2 + C_p i\omega + (K_p + M_p g L)
\end{bmatrix}
\begin{bmatrix}
H_y(\omega) \\
H_\theta(\omega)
\end{bmatrix}
= \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

where \( H_y(\omega) \): structure response function in the frequency domain; \( H_\theta(\omega) \): pendulum response function in the frequency domain.

Solving this linear equation system (Equation 12), Zuluaga [2] obtained response functions \( H_y(\omega) \) and \( H_\theta(\omega) \) in the frequency domain. These responses are present in Equations 13 and 14.

\[
H_y(\omega) = \frac{-\omega^2 B_2 + i\omega B_1 + B_0}{\omega^4 A_4 - i\omega^3 A_3 - \omega^2 A_2 + i\omega A_1 + A_0}
\]

\[
H_\theta(\omega) = \frac{-\omega^2 B_2 + i\omega B_1 + B_0}{\omega^4 A_4 - i\omega^3 A_3 - \omega^2 A_2 + i\omega A_1 + A_0}
\]

where: \( B_0 = K_p + M_p g L; B_1 = C_p; B_2 = M_p L^2; A_0 = M_p K_s g L + K_s K_p; A_1 = M_p C_s g L + C_s K_p + C_p K_s; A_2 = M_s K_p + M_p K_p + M_s M_p g L + M_p^2 g L + C_s C_p + M_p K_s L^2; A_3 = M_s C_p + M_s C_p + M_p C_s L^2; A_4 = M_s M_p L^2.\)

4 \hspace{1cm} OPTIMIZATION

In this section we discuss how to reduce wind turbine vibrations currently, analyzing the analytic formulation and we propose a Genetic Algorithm (GA) optimization to reach this goal, with length pendulum and mass ratio restrictions.

4.1 \hspace{1cm} Current optimization

The structural optimization proposed by Zuluaga [2] considers some criteria such as reducing the mean square displacement, velocity, and acceleration. A mean square value of \( x \) is \( E[x^2] \), where \( E \) is the expected value of a random variable, given by:

\[
E[x^2] = \int_{-\infty}^{\infty} x^2 p(x) dx
\]

where \( p(x) \) is the probability density function of the random variable \( x \).

Some power spectra was used in the optimization. In undamped structures subjected to random excitations with a white-noise spectrum, Zuluaga [2] determined an analytical solution for the TMD optimum parameters, such as: cable length and pendulum damping ratio (Equations 16 and 17, respectively).

\[
L_{\text{optim}} = \frac{2g(\mu + 1) + 2\sqrt{(\mu g + g^2) + 2\omega_0^2(\mu + 2)}}{2\omega_0^2(\mu + 2)} (\mu + 1)
\]

\[
\xi_{\text{p(optim)}} = \frac{\sqrt{\mu(\mu + 2)(3\mu + 4)(\mu + 1)A}}{2\omega_0^2(\mu + 2)^2}
\]
where \( A = \left( \mu g + g \right)^2 + \omega_a \omega_s^2 (\mu + 2) + g(\mu + 1) \sqrt{\left( \mu g + g \right)^2 + 2\omega_a \omega_s^2 (\mu + 2)}; \mu = \frac{M_p}{M_s}; \) mass ratio; \( \omega_s = \sqrt{\frac{K_s}{M_s}}; \) structure natural frequency; \( \omega_p = \sqrt{\frac{K_p + M_p g L}{M_p L^2}}; \) pendulum natural frequency; \( \omega_a = \frac{K_p}{M_p}; \) ratio between stiffness and pendulum mass.

A white-noise is an idealized spectrum, but it can reach useful approximations of a physically possible excitation. A white-noise has been used by many researchers in the study of random vibrations such as the seism and wind [2, 9, 10].

Taking a damped structure in consideration, it is impossible to ascertain an analytical solution of the best pendulum settings. Therefore it is necessary to perform a numerical search. This procedure consists of a setting search of the minimum value of the mean square response to be controlled (displacement, velocity, and acceleration) of the main system. In this study it was adapted by a Min.Max numerical search, which registers the peak of the frequency response function and the forced frequency ratio corresponding to different setting combinations, searching for minimum peak.

Furthermore, other criteria used by different researchers can be included, such as [5]:

- Maximum dynamic stiffness of the main structure;
- Maximum effective damping of the structure system/TMD;
- A mixed criterion involving tuning frequency using minimum displacement criterion and determining the TMD damping using the criterion of maximum effective damping;
- Minimum force of the main portico structure.

Repeated attempt variations of each parameter were made to reach Zuluaga’s goal.

In the study carried out by Zuluaga [2], it was observed that the optimum pendulum length decreases when the mass ratio increases. Therefore the pendulum period also decreases when the mass ratio increases. Also, no significant difference was found between white-noise, Davenport or Kanai-Tajimi spectra. In many cases, the white-noise provides good approximations in random vibrations.

Knowing the hard work in the search of variable combinations (pendulum length and mass ratio between the main system and the pendulum) and the uncertainty to reach analytical solutions, in this paper we propose a best way to optimize the wind turbine vibration, with a pendulum TMD. To reach this goal, a genetic algorithm optimization routine was created using a white-noise spectrum.

### 4.2 Genetic Algorithm optimization

A Genetic Algorithm (GA) is a stochastic algorithm that mimics natural phenomena as operators, proposed to find the best pendulum setting. GAs are search techniques based on the processes of natural selection for survival through population genetics [11]. For the GAs to start evolving, we can use the steps selection, recombination (crossover), mutation, and replacement, where the survival-of-the-fittest mechanism can be applied to the candidate solutions [12, 13].

In the initialization, the algorithm replies the evolutionary genetics and generates a random population by uniform distribution. The chromosomes represent the elements in evolutionary algorithm space, where their features, called genes (quantified by values called alleles), are the problem inputs to be rated by the fitness function [14].

The selection allocates more copies of those solutions with higher fitness values and the roulette-wheel select is the procedure chosen to accomplish this idea. After that, recombination
combines parts of two parental solutions to create new, possibly better, solutions. The mutation, on the other hand, randomly modifies the solutions’ allele. At the end, a percentage of the best elements is simply copied to the next generation using an elitism probability. This is a mechanism to guarantee the best solutions are transmitted to the current generation [12].

The algorithm feeds the system back and evaluates each element from the population. Using the aforementioned evolutionary strategies, the fittest elements will have more probability to pass their features to the next generations. They are depicted by real base, where each gene matches to the hereditary characteristics to be combined and evaluated. The chromosome is described by the following function:

$$C = [L; \mu]$$  \hspace{1cm} (18)

where \(L\): pendulum length; \(\mu = M_p/M_s\), where \(M_p\): pendulum mass; \(M_s\): main system mass.

The real base is useful when the parameters to be optimized are continuous variables [15]. Working with any decimal digits of precision the computer uses flow point numbers to represent the chromosome and its size is equal to the vector that represents the problem solution; thereby each gene represents one problem variable [16].

Variable restrictions were inputted to control this optimization faster. The lanes for pendulum length \(L\) (m) and mass ratio \(\mu\) are:

$$0.70 \leq L \leq 2.00$$ \hspace{1cm} (19)
$$0.05 \leq \mu \leq 0.20$$ \hspace{1cm} (20)

The performance of each element of the population is measured by the fitness function. It rates the capability of the element (given a pendulum setting) to minimize high vibration level and minimize high vibration level and the difference of frequency and anti-frequency peaks (Min.Max restriction).

5 RESULTS

Given the presented model (Figure 1), the tower with realistic dimensions was considered for the GA optimization and modeled as a uniform cantilever beam of circular hollow cross-section with a tip mass (nacelle) at its free end. The tower was made of steel with a hub height of 60m, a width of 3m, and a shell thickness of 0.015m, considering elastic modulus and density of the steel with \(2.1 \times 10^{11} N/m^2\) and \(7850 kg/m^3\), respectively. The tower carries a nacelle and a rotor system mass of 19.876kg. The pendulum stiffness was calculated as 1% of the system stiffness, \(C_s = 0\), and the pendulum damping was fixed in \(\xi_p = 0.024\) [3].

Using Equations 9 and 10 and considering wind force as a harmonic excitation, we obtained the following generalized rigidity, mass, and force parameters:

$$M_s \ddot{Y} + K_s Y = F_s(t), \text{ where } M_s \cong 34.899 kg \text{ and } K_s \cong 46.3671 N/m$$ \hspace{1cm} (21)

Inputting these values in the algorithm, many tests were made to determine the best probabilities of crossover, mutation, and elitism, as well as the population and generation sizes. The parameters were adjusted with: 7% probability of mutation; 70% crossover; 10% elitism; 150 chromosomes per generation. After 80 generations, the figures were plotted containing the curves of the best general fitness for each generation with their mean values.

Two function fitness were loaded with distinct objectives. First, a simple case aiming to reduce the two high system response peaks. Second, a complex case trying to reduce, simultaneously, the high system response peaks and the difference between the high and low response peaks. These two cases are detailed below.
5.1 Reducing the high peaks

In this first case, the low peak of the response was not considered for the optimization. The best fitness × generation curve was dramatically converged as shown in Figure 3, maximizing Equation 22.

\[
f_{fitness_i} = \frac{1}{\max_y H_y(\omega)_i}, \text{ where } i = 1, 2, \ldots, N_{ind}
\]  

where \( \max_y H_y(\omega)_i \): maximum response of each generation; \( N_{ind} \): size of the population.

![Figure 3: Optimization results by fitness per generation of the best fitness of each generation (squares) and their means (asterisks).](image)

The frequency response of the structure was plotted, as well as the structure without a pendulum TMD (Figure 4).

![Figure 4: Displacement harmonic response of the wind turbine: with TMD (using GA), with TMD (using Zuluaga optimization), and without TMD.](image)
Although the best fitness values look the same in the Figure 3, the convergence is in the
decimal places. The GA gives the best pendulum length and the best mass ratio between
the pendulum and the main system, respectively:

\[
L = 0.9737\text{m}; \mu = 0.1090
\]  

(23)

Using the mass ratio \(\mu\) determined by the GA, the optimum pendulum length was calculated
using Zuluaga’s formulation (Equation 16), which gives:

\[
L_{(\text{opt})} = 0.9713\text{m}
\]  

(24)

The difference almost cannot be viewed between the best pendulum length given by the GA
optimization and Zuluaga optimization (0, 24\%). This can validate the GA, because for the
same ratio mass \(\mu\), we can reach the same goal (\(L\)) with distinct ways.

We make a parametric analysis searching the maximum displacement of the transfer func-
tion. With the optimum Zuluaga length (Equation 23), we obtained the maximum displacement
of harmonic response as function of mass ratio \(\mu\) (Figure 7). The minimum response found is
close to \(\mu = 0.1088\).

\[
\text{Figure 5: Relation between response function in the frequency domain } H_y(\omega) \text{ and the mass ratio } \mu \text{ between the pendulum and the main system.}
\]

5.2 Reducing the high peaks and increasing the low peak

In this second case, we searched to minimize the main system pole peaks and maximize the
zeros peaks. Thus was necessary analyzed more than one situation, because probably existed
local maxima. The fitness function it was maximized is given by Equation 25.

\[
f_{\text{fitness 2}} = \min \frac{H_y^*(\omega)_i}{\max H_y(\omega)_i}, \text{ where } i = 1, 2, \ldots, N_{\text{ind}}
\]  

(25)

where \(\min H_y^*(\omega)_i\): minimum response of each generation between the natural frequencies
(\(\lambda_1 \leq \Omega_f \leq \lambda_2\)); \(N_{\text{ind}}\): size of the population.

The fitness evolution converges after 10 generations. The optimum combination between
the pendulum length \(L\) and the mass ratio \(\mu\) was summarized in Table 1. For the same GA
optimum mass ratio, we can estimate a pendulum length by Zuluaga (Equation 16). The pendulum damping ratio \( \xi_p \) was remain the same of previous case (Subsection 5.1).

Figure 6 compares harmonic responses \( H_y(\omega) \) of GA and Zuluaga optimum values. GA results present an harmonic response asymmetric with a pronounced resonant frequency. The pendulum damping ratio \( \xi_p \) has a role importance in dynamic response. To obtain better harmonic results, we need set \( \xi_p \) as a project variable.

The pendulum damping ratio has a crucial role to attain a min. max. harmonic response of the structure. We inserted this pendulum damping as a chromosome variable, \( C = [L; \mu; \xi_p] \). The interval lane assumed was \( 0.00 \leq \xi_p \leq 0.20 \), limiting the pendulum TMDs to lightly damped oscillator. The assumed range was quite extensive to provide a large amount of possibilities.

Figure 7 compares GA results with Zuluaga approaches. To obtain Zuluaga’s length \( L_{(opt)} \) and damping ratio \( \xi_{(opt)} \) optimum results, we used the mass ratio \( \mu \) obtained by GA optimization.

The displacement harmonic response of the min.max. optimization shows a flat level similar to reject-band filter. Its harmonic response is worse compared to a Zuluaga optimum values, as expected, due to a pendulum damping ratio increase.

Optimum project variables for GA and Zuluaga approaches was summarized in Table 2. For the same mass ratio \( \mu \), the pendulum length was quite similar. Perhaps the min.max. optimization point to underdamped system with highest pendulum mass ratio.

### Table 1: Length results obtained by GA and Zuluaga optimization for the same mass ratio \( \mu \) and pendulum damping ratio \( \xi_p \).

<table>
<thead>
<tr>
<th></th>
<th>( L(m) )</th>
<th>( \mu )</th>
<th>( \xi_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>0.76</td>
<td>0.0787</td>
<td>0.024</td>
</tr>
<tr>
<td>Zuluaga</td>
<td>0.97</td>
<td>0.0787</td>
<td>0.024</td>
</tr>
</tbody>
</table>

![Figure 6: Displacement harmonic response of the wind turbine: with TMD (using GA), with TMD (using Zuluaga optimization), and without TMD.](image_url)
Figure 7: Displacement harmonic response of the wind turbine: with TMD (using GA), with TMD (using Zuluaga optimization), and without TMD.

![Figure 7: Displacement harmonic response of the wind turbine](image)

Table 2: Length $L$ and pendulum damping ratio $\xi_p$ results obtained by GA and Zuluaga optimization for the same mass ratio $\mu$.

<table>
<thead>
<tr>
<th></th>
<th>$L$ (m)</th>
<th>$\mu$</th>
<th>$\xi_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>1.01</td>
<td>0.1051</td>
<td>0.18</td>
</tr>
<tr>
<td>Zuluaga</td>
<td>0.97</td>
<td>0.1051</td>
<td>0.15</td>
</tr>
</tbody>
</table>

We undertake a parametric analysis optimizing pendulum length $L$ and damping ratio $\xi_p$ for a fixed mass ratio $\mu$. The mass ratio values was assumed as $\mu \in [0.04; 0.06; 0.08; 0.10; 0.12; 0.14]$. Figure 8 shows a displacement harmonic response for each mass ratio $\mu$.

Figure 8: Evolution of displacement harmonic response of the wind turbine: with TMD (using GA) and without TMD.

![Figure 8: Evolution of displacement harmonic response of the wind turbine](image)
The increase of mass ratio $\mu$ presents an expected reduction of dynamic displacement of the main structure. But reject-band filter shape has a degradation with the $\mu$ increase.

6 CONCLUSION

This paper present a GA application to optimize the vibration performance of a pendulum TMD. We use harmonic analysis to obtain the frequency response function of a tower with a pendulum TMD. The continuous system that describe the tower was reduced to the first mode. Then the structure system was described as a 2-DoF dynamic system.

Using a genetic algorithm with floating point chromosome, we propose two restrictions in objective function: (a) minimize high peaks of the vibration frequencies; (b) minimize high peaks of the vibration frequencies and maximize the anti-frequency peak (min. max. restriction).

We compare the analysis of the GA optimization undertaken by report to Zuluaga [2] gradient approach. The first restriction obtain an optimum harmonic response identical to Zuluaga approach, as expected. This result validate present GA implementation. The second restriction provide several results. Pendulum TMD is mainly influenced by mass ratio an damping ratio.

The optimum mass ratio $\mu$ to 10% of main structure mass. But the optimum pendulum damping ratio $\xi_p$ has a high value of 18%. More studies must be undertake to analyze damping forces in other mode shapes.

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REFERENCES


