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SEISMIC RESILIENCE OF A BRIDGE BASED ON FUZZY-PROBABILISTIC APPROACH

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Abstract. *In this paper, a new model based on a fuzzy-probabilistic approach for predicting seismic resilience of a bridge is developed. The main purpose is to use seismic resilience in decision making process for disaster management during the pre-disaster period. Another aim is to include contingency in resilience assessment. In previous research, uncertainties have not been fully considered in the proposed models for seismic resilience assessment. Since the residual functionality of bridges depends on its vague damage states, so it is presented by fuzzy triangular numbers. However, the both idle time interval and recovery duration are random in nature, so they are described as random variables. The Monte Carlo simulation is used for generating 10000 samples of these variables. Further, resilience is represented as fuzzy-random variable with corresponding fuzzy mean value and fuzzy standard deviation obtained from the generated and estimated data. The functionality of the system is therefore described as a fuzzy-random function whose shape depends on the disaster preparedness of the system. The resilient curves are illustrated using fuzzy functions. A Java application is developed for purpose of resilience assessment. For a case study, a bridge in Santa Barbara is chosen. The result of resilience assessment process is used for decision making in disaster management and emergency responses.*

1 INTRODUCTION

Transportation systems represent critical infrastructures that play a major role in any country and their failure would have a great impact on the health, safety, economics and social well-being of society [1]. Societies are impacted by natural disasters such as earthquake, hurricane, floods; and man-made disasters. When a disaster strikes, the performance of highway networks is vital for emergency response and recovery activities. Also, the reduced functionality of transportation network caused economic lossess. In recent years, a lot of attention is paid to build resilient infrastructure systems that show reduced failure probabilities, reduced consequences from failure and reduced recovery time [2]. Although in this paper, the focus is on the bridges, since these highway infrastructure components are the most vulnerable and fragile elements of the transportation network in case of seismic event [3]. Earthquakes produce direct and indirect economic losses to bridges [4]. Furthermore, the direct losses include bridge repair, reconstruction and rehabilitation, and the indirect losses are caused by disruption of a traffic flow.

Many studies have been conducted on the topic of seismic resilience assessment. Bocchini and Frangopol (2010, 2011a, 2011b, 2011c, 2012a, 2012b) have studies the reliabilty, fragility and resilience of bridge network [5-10]. They proposed a method using seismic resilience as a criterion in decision making process for optimal restoration process of bridge network. Deco et al (2013) have extended the research by proposing a probabilistic model for seismic resilience assessment of bridge [11]. Karamlou and Bocchini (2014) in their latest research have suggested a new methodology for restoration process of bridges based on Genetic Algorithms, where resilience of the transportation network and the time to connect the critical locations to the infrastructure network are used as a criterion for optimization [12]. Cimellaro et. al. (2005, 2006, 2008, 2010a, 2010b, 2011, 2013) have presented several studies focusing on the seismic resilience of hospitals, health care facilities, and hospital systems with ambulance emergency response [13-19]. In case of infrastructure resilience, Reed et al (2011) have developed a framework and a model for optimizing civil infrastructure resiliency which aim is to increase resiliency [20].

The above research only considered randomness. We noticed that there are many fuzzy uncertain factors in assessment of seismic resilience. For example, the residual functionality of bridges depends on its damage states, while the definition and classification are generally vague and modeled by linguistic variables of experts. The purpose of this research is to include fuzzy uncertainties in resilience assessment considering randomness and fuzziness. An approach for seismic resilience of bridges is developed based on the idea of fuzzy-random variables, where resilience is modeled by random triangular fuzzy numbers, which values correspond to three case scenarios: optimistic, pessimistic and the most probable. The result of resilience assessment process is to represent seismic resilience as a fuzzy random variable, with corresponding fuzzy mean value and fuzzy standard deviation.

The paper is organized into five parts. In the second part, the description and definition of resilience are given. The fuzzy-probability model is illustrated in the third part. The case study is shown in the fourth part, and the conclusion with the suggested further research is presented in the last part.

2 DEFINITION AND MEASURE OF RESILIENCE

Resilience characterizes a system to '*bounce back*' and return its previous state after the disastrous event. In literature, there are different definitions of the term *resilience*. The concept of resilience appears in two different aspects: as outcome and as a process leading to desire outcome [21]. Cimellaro et. al. (2010a) gave a framework for analytical quantification of disaster resilience and defined resilience as a function indicating the capability to sustain a level of functionality or performance for a given building, bridge, lifeline network, or community, over a period defined as the control time that is usually decided by owners, or society [16]. Terje (2011) explained the concept of resilience as the ability of a system to withstand a major disruption within acceptable degradation parameters and to recover within an acceptable time, and composite costs, and risks [22]. Bocchini and Frangopol (2014) represented resilience as a metric that measures the ability of a system to withstand an unusual perturbation and to recover efficiently from the damage [23]. In civil engineering, resilience represents the ability to deliver a certain service level even after the occurrence of an extreme event, and to recover the desired functionality as fast as possible.

The most widely used definition of resilience is the definition which is given by Bruneau et al (2003). They provided the definition of community seismic resilience as the ability of social units to mitigate hazards, contain the effects of disasters when they occur, and carry out the recovery activities in ways that minimize social disruption and mitigate the effects of future disasters [24]. The characteristics of a resilient system are to robustness, redudancy, resourcefulness, and rapidity.

The resilience is based on the functionality of a system *Q(t)* over the time *t* as shown in Figure 1. It is defined for the quality of service of the infrastructure system. Mathematically, resilience is expressed by:

$$R = \int_0^{t_h} \frac{Q(t)}{t_h} dt \tag{1}$$

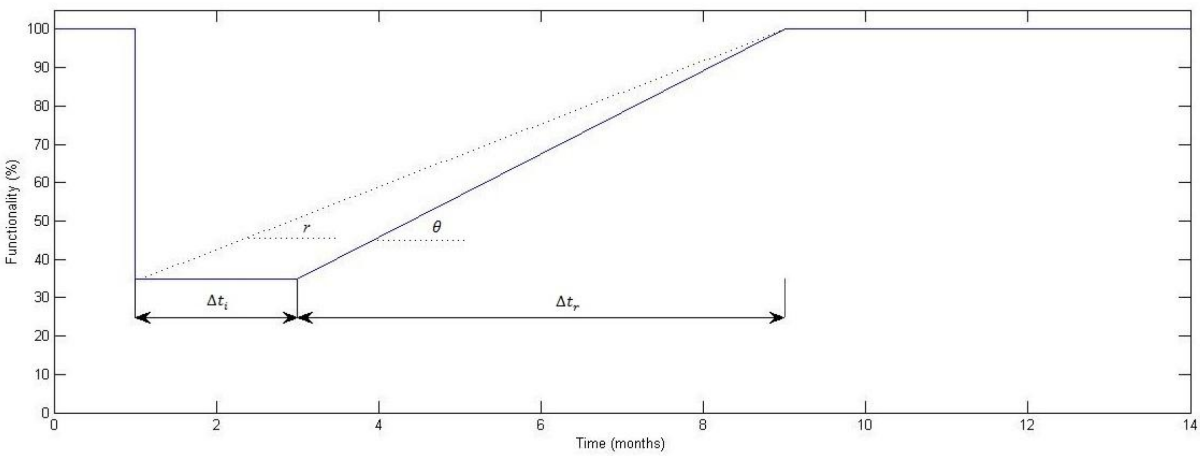

Figure 1: Schematic representation of resilience

After the recovery process, the system can return its previous functionality, or the functionality of the system can be higher or lower than the previous one [25].

3 FUZZY-PROBABILITY MODEL FOR SEISMIC RESILIENCE ASSESSMENT

The resilience is a complex issue which depends on a lot of factors such as residual functionality, target functionality, idle time interval, recovery time and recovery function [11]. Due to this, the prediction of seismic resilience of bridges is a hard task which is determined by many highly uncertain factors. For example, the residual functionality is highly uncertain factor because it is directly associated with the level of bridge damage [11].

The seismic event will cause damages or collapse to bridges. The possible damage states of a bridge are defined in *HAZUS-MR 4* as [26]: no damage, slight/minor damage, moderate damage, extensive damage and complete damage (collapse). The fragility functions are generally used as a tool for evaluating damage states of the bridge. Essentially, the level of the damage is quantified by linguistic variables, thus it represents lexical uncertainty. The residual functionality is directly associated with the levels of bridge damage as the aftermath of the disaster event. According to this, the residual functionality also includes lexical uncertainty. In addition, the characteristics of lexical uncertainty are fuzziness [27]. Hence, the residual functionality can be represented by a fuzzy variable in the model.

In this research, the residual functionality is defined as a convex, normalized fuzzy set $\overline{Q}_r = <Q_{r_1}, Q_{r_2}, Q_{r_3}>$, which is determined by interval bounds of supporting the smallest value Q_{r_1} and the largest value Q_{r_3}, whose membership functions are segmentally continuous and has the functional value $\mu(Q_r)=1$, at exactly one of the value for $Q_r = Q_{r_2}$. For different damage states, they have different values. The membership function for the residual functionality is given as:

$$\mu(Q_{r_i}) = \begin{pmatrix} \dfrac{Q_{r_i} - Q_{r_1}}{Q_{r_2} - Q_{r_1}}, Q_{r_1} < Q_{r_i} < Q_{r_2} \\ \dfrac{Q_{r_3} - Q_{r_i}}{Q_{r_3} - Q_{r_2}}, Q_{r_2} < Q_{r_i} < Q_{r_3} \\ 0, otherwise \end{pmatrix} \tag{2}$$

The recovery path depends on the preparedness for disasters and on the rehabilitation process as well as its techniques and scheduling. The first phase of the recovery is the idle time interval, which is a period between the occurrence of a disaster event and the starting of restoration process on the bridge. During this period, the activities for rehabilitation process are planned. Also, it is used for contacting construction companies and contracting the future reconstruction works. In some cases, during the idle time the restoration activities which do not increase the functionality are finished. In the proposed approach, the idle time interval is a random variable, which follows uniform distribution with values between the minimum and the maximum. Furthermore, the functionality of a bridge in this phase is constant and it is equal to residual functionality.

In the second phase, reconstruction and rehabilitation works are carried out. The recovery time is a period from the beginning of the restoration process until the system gain a desired level of functionality which can be higher, equal or lower than before the disastrous event. The construction industry is more uncertain compare to other industries due to its unique features of construction, reconstruction or repair activities. Usually, these activities are performed outside, so in great extent these processes depend on the weather conditions, soil conditions, and underground water, among others. Bad weather conditions can make delay bridge reconstruction works. Due to this, the recovery period is also modeled by a random variable that follows uniform distribution.

At the end of a restoration process, the system will achieve the target functionality. Generally, the target functionality can be equal to the functionality which the system had before the disaster occurred, lower than before, higher than before, or to totally loose its functionality. The target functionality is defined as a convex, normalized fuzzy set $\overline{Q}_t = <Q_{t_1}, Q_{t_2}, Q_{t_3}>$, that is determined by interval bounds of supporting the smallest value Q_{t_1} and the largest value

Q_{t_3}, whose membership functions are segmentally continuous and has the functional value $\mu(Q_t)=1$, at exactly one of the value for $Q_t = Q_{t_2}$. The membership function for the residual functionality is given as:

$$\mu(Q_{t_i}) = \begin{pmatrix} \dfrac{Q_{t_i} - Q_{t_1}}{Q_{t_2} - Q_{t_1}}, Q_{t_1} < Q_{t_i} < Q_{t_2} \\ \dfrac{Q_{t_3} - Q_{t_i}}{Q_{t_3} - Q_{t_2}}, Q_{t_2} < Q_{t_i} < Q_{t_3} \\ 0, otherwise \end{pmatrix} \tag{3}$$

The shape and the position of the recovery curve depend on the already mentioned factors in previous paragraphs: the residual functionality, the idle time interval, the recovery duration, the target functionality, and the selected recovery function. The residual functionality is modeled as a fuzzy triangular number. However, the idle time and recovery time are treated as independent random variables. Further, the recovery function that covers the recovery pattern depends on the bridge preparedness to disaster. In addition, according to the system preparedness, the system can be classified into three classes: well-prepared systems, average prepared systems and poor-prepared systems. For modeling recovery process, the fuzzy functions are used as fuzziness of recovery process, attributable to a variable of residual functionality which constitutes lexical uncertainties.

If the system is well-prepared to disasters, the recovery process is modeled by a negative-exponential recovery function. The well-prepared systems at the beginning of the restoration process have the recovery speed faster than at the end of the process. The function for functionality of a system for a well-prepared system is proposed as a fuzzy negative-exponential function:

$$\overline{Q}(t) = \begin{pmatrix} \overline{Q_r}, t < t_i \\ -(\overline{Q_t} - \overline{Q_r}) * e^{-\ln(200)*\frac{t-t_i}{t_r}} + \overline{Q_t}, t_i < t < t_i + t_r \\ \overline{Q_t}, t > t_i + t_r \end{pmatrix} \tag{4}$$

where: $\overline{Q_t}$, $\overline{Q_r}$, t_i and t_r are target functionality, residual functionality, idle tme interval and recovery time, respectively.

For the systems which are poor prepared for disasters, the situation is different than from previous case. At the beginning of the restoration process, the recovery speed is less than at the end of the process. To describe the recovery path for this type of systems, the positive-exponential recovery function is proposed. The fuzzy function for a poor-prepared/unprepared system is modeled as a fuzzy positive-exponential function:

$$\overline{Q}(t) = \begin{pmatrix} \overline{Q_r}, t < t_i \\ (\overline{Q_t} - \overline{Q_r}) * e^{\ln(200)*\frac{t-t_i}{t_r}} * \dfrac{1}{200} + \overline{Q_r}, t_i < t < t_i + t_r \\ \overline{Q_t}, t > t_i + t_r \end{pmatrix} \tag{5}$$

The average-prepared systems are modeled by a linear recovery function, because the average-prepared system has uniformly recovery speed during this phase. The fuzzy function for functionality in the case of an average-prepared system is proposed as a fuzzy linear function:

$$\overline{Q}(t)=\begin{pmatrix}\overline{Q_r},t<t_i\\(\overline{Q_t}-\overline{Q_r})*\dfrac{t-t_i}{t_r}+\overline{Q_r},t_i<t<t_i+t_r\\\overline{Q_t},t>t_i+t_r\end{pmatrix} \tag{6}$$

In this research, resilience is defined as a fuzzy random variable with fuzzy mean value and fuzzy standard deviation. To model fuzzy mean value and fuzzy standard deviation fuzzy triangular numbers are preferred since they are suitable to describe the three case scenarios (optimistic, the most-probable and pessimistic) of bridge functionality. Mathematically, the resilience can be measured by the fuzzy integral:

$$\overline{R}=\int_0^{t_h}\frac{\overline{Q}(t)}{t_h}dt \tag{7}$$

4 A CASE STUDY

The case study represents a bridge in Santa Barbara in the USA, which is located on a crossroads of Route 217 and Route 101 [9]. The resilience of the bridge is considered for seismic hazards. The research on the risk assessment has been conducted and the expected level of bridge damage is extensive damage [9]. For the expected damage state of the bridge, the resilience assessment is obtained and the possible recovery paths are obtained.

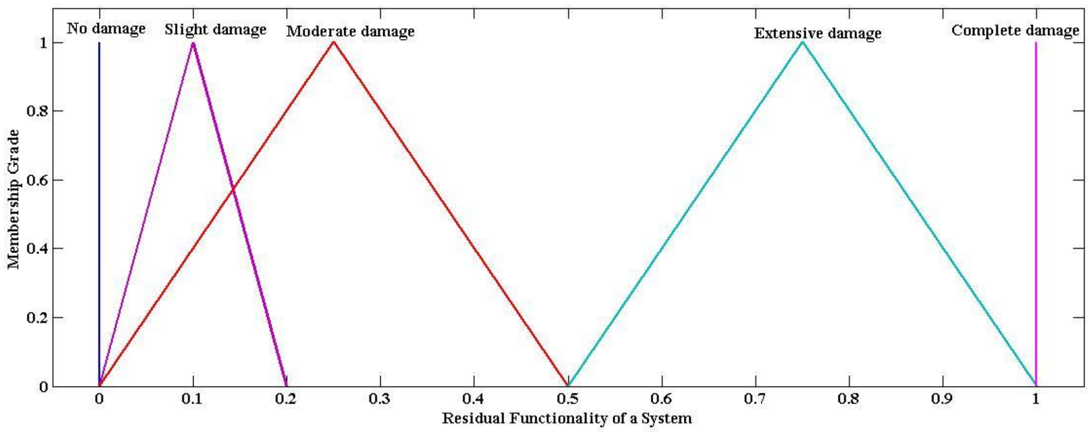

Figure 2:Membership functions for Residual functionality of the bridge

The first step is forming fragility curves for this bridge. In this research, the fragility curves are obtained from FEMA HAZUS-MR4 [26]. According to the damage degree, the residual functionality is estimated and these data is taken from *HAZUS-MR 4.* Furthermore, the residual functionality is represented as fuzzy triangular numbers with the values from *HAZUS MR 4* (Figure 2). Also, this step includes defining the horizon time. The value for time horizon is assumed to be equal to 14 months. Normally, the time horizon value lies between 1 to 2 years.

The estimated values from *HAZUS-MR4* of the residual functionality for each damage state of the bridge are:

1. Residual functionality in case of "no damage": $\overline{Q}_{r_1} = <1,1,1>$
2. Residual functionality in case of "slight damage": $\overline{Q}_{r_2} = <1,0.75,0.50,>$
3. Residual functionality in case of "moderate damage": $\overline{Q}_{r_3} = <0.5,0.25,0>$
4. Residual functionality in case of "extensive damage": $\overline{Q}_{r_4} = <0.2,0.1,0>$
5. Residual functionality in case of "complete damage": $\overline{Q}_{r_5} = <0,0,0>$

The target functionality is 100%. In this case study, we assume that after the rehabilitation process, the bridge returns to its previous functionality, which is the target functionality. According to that, the target functionality is presented as single fuzzy number. The value for the target functionality for each damage state and recovery path is:

$$\overline{Q}_t = <1,1,1>$$

Damage level	Minimum (days/months)	Maximum (days/months)
Slight damage	10/0.333	150/5
Moderate damage	20/0.667	200/6.667
Extensive damage	60/2.0	250/8.333
Complete damage	75/2.5	300/10.0

Table 1: Uniform distribution parameters for recovery time of bridge [1, 28]

Idle time	Min (months)	Max (months)
	1	2

Table 2: Uniform distribution parameters for idle time

The recovery time is given by a uniform probability distribution for each damage state of the bridge, as shown in Table 1. Also, the idle time interval follows uniform distribution between 1 and 2 months (Table 2). The values for the recovery time and the idle time interval are generated by Monte Carlo simulation [29].

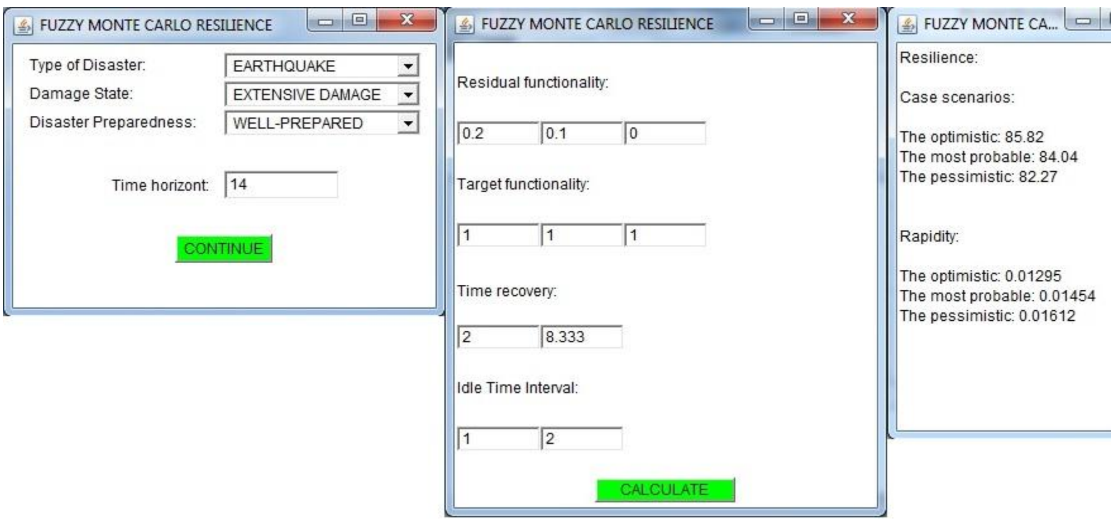

Figure 3: The Fuzzy Resilience - Java application

An In-house application tool has been developed in Java programming language [30] named *Fuzzy Monte Carlo Resilience*, as shown in Figure 3, and it is used for the process of resilience assessment. It contains options to choose the type of disasters: earthquake, tsunami, hurricane, floods, scour, drift, or multiple hazards; the damage states of the bridge: no damage, slight damage, moderate damage, extensive damage and complete damage; the disaster preparedness: well-prepared, average-prepared and poor prepared; and the time horizon given in the first window of application. The input data for the characteristics of bridges is set up in the next step.

In this case study, the residual functionality, and the recovery time are taken considering the state of "extensive" bridge damage. Each bridge damage state has three possible case scenarios for resilience assessment: optimistic, most-probable and pessimistic. The resilience when considering the extensive damage state in the optimistic case scenario takes on a value of the residual functionality after disaster equal to 20%. The most-probable case scenario which contains the values of residual functionality is only 10% of its full functionality. For the pessimistic case scenario, the minimum value of resilience which is obtained by considering the minimum residual functionality after disaster event is taken as 0%.

Then, the random numbers that follow the prescribed uniform distribution for the recovery period and the idle time interval are generated using random number generator from *Java application*. Totally, ten thousand samples in this process are used for resilience assessment in all three case scenarios. The output of the program is an Excel file with the data which contains the residual functionality, the generated idle time interval, the generated recovery time and the calculated resilience. Furthermore, the fuzzy mean value and the fuzzy standard deviation of the resilience are estimated using these 10,000 samples of fuzzy realization. The shape of the fuzzy mean value and the fuzzy standard deviation are fuzzy triangular numbers with corresponding membership functions, as shown in Figure 4. Also, the fuzzy distribution function for the resilience is obtained from these values, as shown in Figure 5. The results for resilience for each seismic damage state are presented by triangular fuzzy numbers, whose values are given in Table 3.

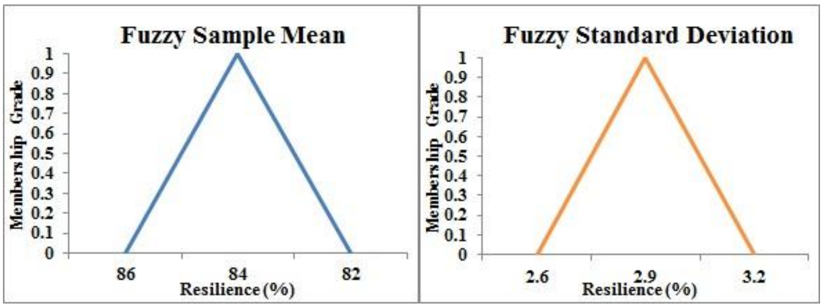

Figure 4: Fuzzy mean and fuzzy standard deviation of Bridge in Santa Barbara

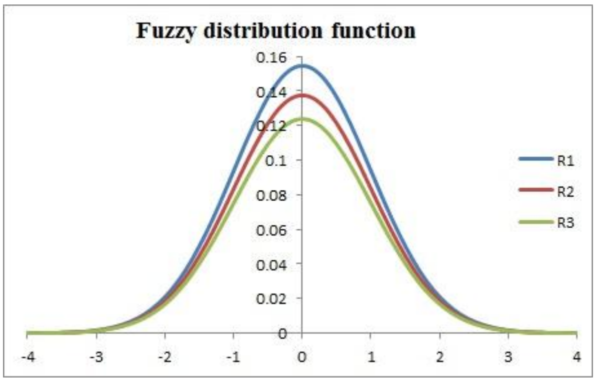

Figure 5 Fuzzy distribution functions for resilience

Damage	Disaster Preparedness	Case scenario	Resilience	
			Mean	Standard deviation
Extensive	Well-prepared	Optimistic	0.86	0.026
		The most probable	0.84	0.029
		Pessimistic	0.82	0.032

Table 3: Resilience of a bridge

The resilience of the bridge in Santa Barbara in the extensive damage state and well-prepared scenario for a future seismic disaster is (0.86, 0.84, 0.82), where the first, second and third value represent optimistic, most-probable and pessimistic case scenarios, respectively. Therefore, the possible values of the resilience lie between the optimistic and pessimistic values.

Matlab [31] is used for obtaining the resilient curves in Figure 6. The curve (1) depicts the optimistic case scenario for the bridge in the extensive damage state in a well-prepared disaster region. The residual functionality in the optimistic case scenario for the extensive damage and well-prepared system is equal to 20%, so the curve (1) begins at this point, and it is horizontal during a 1.5-months idle time. Moreover, the recovery phase follows negative-exponential function, since the bridge is well-prepared for seismic disaster and during the 5.19-months recovery period, the bridge succeeds to return to full functionality of 100%. After this, the curve is straight. The Curve (2) is obtained for the most-probable case scenario with the membership grade equals to 1. The residual functionality for the most-probable case scenario is equal to 10%, and the curve (2) starts from 10%, and it is horizontal during a 1.5-months idle time. Further, the recovery phase follows the negative-exponential function during the recovery period of 5.19-months when the bridge returns its previous functionality of 100%. The curve (3) represents the pessimistic case scenario for the extensive damage and well-prepared system with a membership grade of 0. The residual functionality in the pessimistic case scenario is 0% for a 1.5-months idle time. Also, the recovery phase follows the negative-exponential function, and during the recovery period of 5.19-months, the bridge functionality from 0% gradually proceeds to 100%. All three curves have the same recovery duration, with different starting points, because the residual functionalities are different (20%, 10%, 0%), but the same ending points, because the target functionality in all three cases is the same (100%, 100%, 100%).

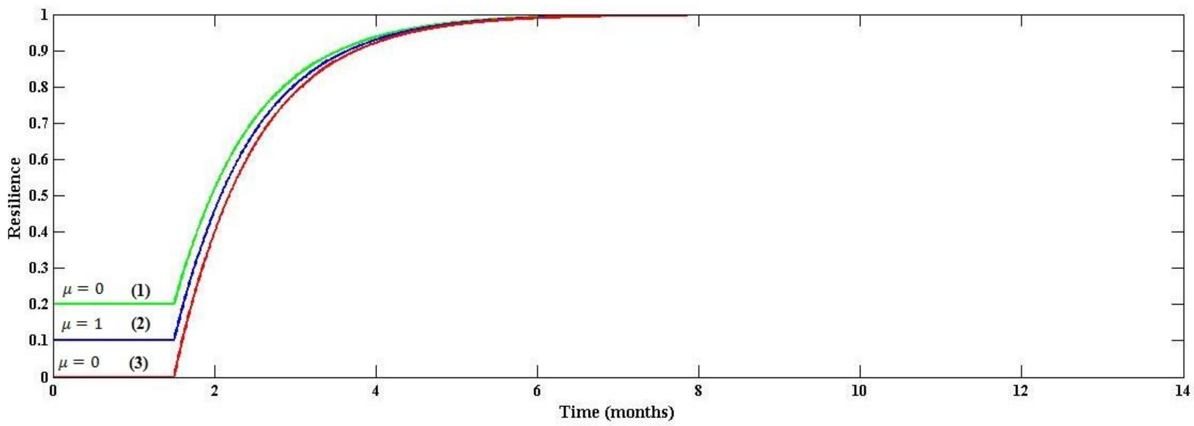

Figure 6: Resilience curves for the bridge in Santa Barbara

5 CONCLUSIONS

This paper presents a model for predicting the seismic resilience of a bridge under uncertainties based on a fuzzy-probabilistic approach. In the proposed model, the residual functionality is represented by a triangular fuzzy number; the idle time interval and the recovery period are represented by random variables that follow the uniform distributions. Compared with other methods and models for resilience assessment, the triangular fuzzy numbers are favorable for the representation of the residual functionality, because they correspond to three case scenarios: optimistic, the most-probable and pessimistic scenario of bridge damage after the disaster. The type of recovery function depends on the disaster preparedness of the bridge and its circumstances. It has been found that the negative exponential recovery functions should be applied for high-resilient, while the linear for average-resilient, and positive exponential recovery function for low-resilient bridges. The resilience is modeled by a fuzzy random variable. Also, the model can be applied to other highway infrastructure components such as tunnels and roadways, only the values for residual functionality, idle time interval and time recovery will be different. The important goal of this research is to use resilience assessment as a criterion for decision making process in disaster risk management. The prediction of resilience is used for development of the optimal strategy for disaster mitigation.

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