IMPROVED FRAGILITY FUNCTIONS FOR RC BRIDGE POPULATIONS

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Abstract. When carrying out loss assessment of a road network within a certain region, the characterization of the vulnerability of the existing population of bridges is one of the most relevant aspects. To such extent, in the last decades, the engineering community has recognised fragility curves as one of the fundamental and most effective tools in seismic risk assessment, correlating the probability of exceeding of specific limit states, which can instead be correlated with damage and loss, for different levels of intensity measures (IMs). The available literature mainly provides such curves as function of peak ground acceleration (PGA) or spectral acceleration (\(S_a\)), often preferred for the availability of national hazard information in terms of such parameters. However, when dealing with a bridge population of reinforced concrete bridges, Fajfar Index, peak ground velocity and root mean square velocity have been recently identified as the most promising intensity measures, in terms of efficiency, proficiency and sufficiency, notwithstanding the acceptable performance of PGA and \(S_a\). As such, this paper intends to further extend the analysis of RC populations of bridges by providing a statistically sound comparison of analytical fragility curves based on traditional and innovative intensity measures of an extensive bridge population. Nonlinear static analyses of 3D RC bridge models are carried out. The bridge population is randomly generated using Latin Hypercube sampling in order to include geometrical variability, in addition to aleatory and epistemic uncertainties. For what concerns the seismic ground motion, a proper selection of records was performed, according to a recent selection and scaling procedure (Conditional Spectrum Method).
1 INTRODUCTION

During the last decades, the engineering community has recognized fragility functions as fundamental tools for characterization of structural resilience due to their capability of establishing the link between seismic hazard of a certain site and the level of structural damage related to a certain seismic excitation. Even if the outcome of the most recent research in the structural fragility assessment field led to more complex methodologies for the definition of more accurate fragility curves, the lack of a unique approach for their evaluation leads to discrepancies in the results from varying initial assumptions [1]. In addition to the variability of the procedure itself, significant dispersion in seismic demand, epistemic and aleatory uncertainties are present throughout the entire process of development of fragility curves [2]. A great deal of effort has been witnessed in literature to deal with such sources of uncertainties: the logic trees for both seismic source models definition and the consideration of multiple ground motion prediction equations is only one example of the most popular strategies to face hazard variability problems. Indeed, new seismic record selection and scaling strategies arose from recent research [3] with the goal of reducing the main source of uncertainty associated to the seismic input, the record-to-record variability, which it is well known to have repercussions on the accuracy of seismic structural capacity and demand [1]. Moreover, within the process of fragility curve development, the selection of the intensity measure (IM) to be considered is a crucial aspect. Due to the nature of the earthquake record itself, the conventionally employed IMs, such as peak ground acceleration (PGA) or peak ground velocity (PGV), might not be able to take into account, as single scalars, all the information regarding the seismic signal, resulting in increased variability of the seismic structural demand. Accordingly, with a view to optimize the link between hazard and structural response, recent research [4, 5] considered different scalar IMs or couples of IMs, not only for studying the accuracy of the IM itself, but also for expressing fragility curves as function of such alternative variables.

Furthermore, most of the available literature considers building frames as case study for similar investigations, hence, to fill such gap, the present study focuses on reinforced concrete bridge structures, which are part of a road network system. In particular, a population of reinforced concrete bridges, randomly generated through Latin Hypercube sampling for including geometrical and material properties variability, already successfully employed in previous studies addressing RC bridges [6, 7], is considered to scrutinize the accuracy of fragility estimates as function of certain number of IMs with good performance in terms of efficiency, proficiency and sufficiency, such as Fajfar Index (Iv), peak ground velocity (PGV), root mean square velocity (vRMS) and spectral acceleration (Sa) [8]. Notwithstanding the optimal performance of the aforementioned IMs, the traditionally employed peak ground acceleration (PGA) is also considered as the vast majority of the available hazard information is expressed in terms of such ground motion characteristic.

Nonlinear static analysis was selected as an accurate tool for the identification of the performance point at different demand levels, validated in past studies concerning the seismic assessment of bridges and buildings through nonlinear static procedures [9-12]. As such, a reexamination of the N2 method [13-15] was featured in order to use response spectra derived from a set of selected real records. Fragility curves of the considered bridge portfolio were thus derived with different IMs with multiple stripes analysis, a methodology capable of including the variation of the ground motion records at different IMs levels, which was herein adjusted for application to structural portfolios. Finally, the dispersion level among the bridge portfolio for each IM-based fragility curve was assessed by means of statistical parameters that rapidly estimate the goodness-fit of the obtained fragility curve, enclosing in a single scalar the amount of dispersion between observed and fitted data.
BRIDGE POPULATION AND BRIDGE MODELLING

With a view to demonstrate the level of accuracy of different IM-based fragility curves in the expedite risk assessment of road networks, the case study considered in this paper does not focus on individual structural configurations but rather on a population of bridges that could represent a given bridge class, with different, yet statistically correlated, configurations. The endorsed methodology makes use of a parametric statistical characterization of Italian reinforced concrete bridges conducted in recent studies [6, 8]. One hundred bridges were randomly generated through Latin Hypercube sampling, which guarantees the reduction of computational onus when compared to other approaches, such as Monte Carlo, on the basis of the statistical distribution associated to the main geometrical and material properties shown in Table 1. As far as longitudinal and transverse reinforcement are concerned, the associated ratios were assumed to vary in the ranges of 0.5% - 1% and 0.08% - 0.1% respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Distribution parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pier height [m]</td>
<td>Lognormal</td>
<td>Mean (log): 1.95; std: 0.828</td>
</tr>
<tr>
<td>Total length of bridges</td>
<td>Lognormal</td>
<td>Mean (log): 5.10; std: 1.145</td>
</tr>
<tr>
<td>Span length [m]</td>
<td>Normal</td>
<td>Mean: 31.18; std: 11.527</td>
</tr>
<tr>
<td>Section Diameter</td>
<td>Normal</td>
<td>Mean: 2.19; std: 0.969</td>
</tr>
<tr>
<td>Reinforcement yield strength [N/mm2]</td>
<td>Normal</td>
<td>Mean: 504.40; std: 157.84</td>
</tr>
<tr>
<td>Reinforcement Young Modulus [GPa]</td>
<td>Normal</td>
<td>Mean: 203.82; std: 19.426</td>
</tr>
<tr>
<td>Concrete compressive strength [MPa]</td>
<td>Normal</td>
<td>Mean: 40.0; std: 7.44</td>
</tr>
<tr>
<td>Superstructure width</td>
<td>Lognormal</td>
<td>Mean: 2.49; std: 0.256</td>
</tr>
<tr>
<td>Number of spans</td>
<td></td>
<td>1.5+0.03*(Total length)</td>
</tr>
</tbody>
</table>

Table 1: Statistical distribution properties [1, 7].

The cross section of the deck takes the form of rectangular reinforced concrete girders, which is consistent with real Italian bridges, in which the deck is simply supported in most cases. The abutments were modeled with bilinear response springs with stiffness sets as 26’329kN/m [16] along horizontal direction. Further details on the remaining modelling assumptions can be found in a previous study [8].

An automatized procedure was coded in Matlab [17] in order to automatically carry out nonlinear static analyses, i.e. pushover analyses, of the considered bridges in OpenSees software package [18]. The analysis considered the transverse direction only, with a load path proportional to the fundamental mode of each structure of the considered bridge population, whose transverse fundamental periods are represented in the histogram in Figure 1.

![Figure 1: First transversal periods of the randomly generated bridges.](image)
3 GROUND MOTION RECORDS

In addition to the geometrical and material variability, modeled in §2, the record-to-record variability [19] was also included in the computation of the fragility curves. Within an attempt to minimize the effects of such variability in the structural demand, a recent selection and scaling procedure known as conditional spectrum method (CSM) [2] was chosen, especially for its effectiveness in selecting a suite of ground motion records that match the ground motion characteristics at each intensity measure level. Given that the CSM foresees that the record selection is conditioned at the specific structural period, its employment required a certain level of simplification in order to accommodate the analysis of groups of structures with different periods within reasonable computational onus. Accordingly, the randomly generated bridges were grouped in three classes, defined by a central period of 0.2s, 0.3s and 0.75s.

Considering that the majority of the bridges are located in central regions of Italy, the target city of Campobasso (Molise region) was fully characterized in terms of seismic hazard. Hazard curves and disaggregation results conditioned at the spectral acceleration for the selected periods of vibration (0.2s, 0.3s and 0.75s) were undertaken with a ground motion prediction equation suitable for European regions [20] using Openquake [21]. The area source model information provided by the Italian zonation (ZS9) was adopted [22]. Such outcome was used to select and scale a set of 30 records for 15 different assumed intensity measure levels and for each class of fundamental period. The response spectra obtained for T=0.2s and for one of the intensity levels are shown in Figure 2.

![Figure 2: Response spectra of the selected input records with 10% probability of exceedance in 50 years for period bridge class 0.2s.](image)

4 INTENSITY MEASURES

The selection of a proper intensity measure is very important for the reduction of the effects of the uncertainties that surround probabilistic seismic hazard analysis and nonlinear structural analysis. Reducing such uncertainties becomes even more important when dealing with the seismic loss assessment of a road network, in which there is the need to characterize the seismic behavior of different nodes, represented by bridges with different geometrical and material properties.

In line with the main findings of recent research [8], the Fajfar Index (I_v), peak ground velocity (PGV), root mean square velocity (vRMS), peak ground acceleration (PGA) and spectral acceleration (S_a) were selected as IMs to be considered in the derivation of fragility functions, due to their good performance in terms of efficiency, practicality, proficiency and sufficiency for the case of bridge populations. Moreover, the aforementioned IMs have the advantage of being easily obtained from earthquake characteristics. Coupling of IMs in vector...
fashion was not taken into account due to the not relevant improvement in terms of efficiency, practicality and proficiency, to justify their employment in the assessment of bridge populations.

5 DAMAGE LIMIT STATES

The definition of the damage limit states is a crucial phase of the process of the fragility curve definition. In literature a wide variety of different damage limit states definitions can be found for seismic assessment of bridges; however, especially in case of the assessment of a bridge population, a global damage indicator seems more appropriate. Following the approach adopted in [23] the ductility in displacement was chosen as engineering demand parameter for the quantification of the damage of each pier and the maximum ductility over all the bridge piers was considered representative of the bridge behavior. Four different values of curvature ductility, shown in Table 2, were used for the definition of the considered levels of damage: slight, moderate, extensive and collapse [24, 25].

<table>
<thead>
<tr>
<th>Slight (DS1)</th>
<th>Moderate (DS2)</th>
<th>Extensive (DS3)</th>
<th>Collapse (DS4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.29</td>
<td>2.10</td>
<td>3.52</td>
<td>5.24</td>
</tr>
</tbody>
</table>

Table 2: Damage limit states in terms of curvature ductility [23].

According to Equation (1) and Equation (2), proposed by Priestley et al. [26], the expected displacement ductility of each bridge pier was calculated as function of curvature ductility, $\mu_0$, and then compared with the values associated to each damage limit state.

$$\mu_\Delta = 1 + 3(\mu_0 - 1)(l_p/l)[1 - 0.5(l_p/l)]$$  \hspace{0.5cm} (1)

$$l_p = 0.08l + 9d_b$$  \hspace{0.5cm} (2)

Where $l_p$ is the plastic hinge length, $l$ is the pier height and $d_b$ the longitudinal reinforcement bar diameter.

6 NONLINEAR STATIC PROCEDURE

The development of fragility curves for a bridge population requires the allocation of each structure to a certain damage limit state, according to the given definition. As such, when using a nonlinear static procedure (NSP) to estimate the demand, the target displacement corresponding to each couple of the considered pushover curve and response spectrum needs to be identified. Seismic guidelines all over the world are already employing NSPs as simplified approaches for assessing seismic response of structures. A great deal of effort has been done in the last decades to approach with more accuracy the results of dynamic analyses, which, when properly used, is considered the most reliable way for predicting structural response along the entire duration of the seismic excitation. However, required computational time demand could represent an obstacle in the view of the development of rapid methodologies for deriving fragility curves, also when limited information available from national structures inventory.

In agreement with such context and despite the drawbacks of considering simplified approaches rather than dynamic response analyses, such as the impossibility of reproducing phenomena of viscous damping, strength deterioration or of considering deformations, the seismic behavior of the considered bridge population was evaluated through a conventional pushover based nonlinear static procedure – N2. The choice for N2 was based on the findings of a recent study regarding the accuracy in the fragility estimates using different approaches
to derive analytical fragility curves [23], which demonstrated that N2 [15], when compared to nonlinear dynamic analysis, provides accurate results for low and high intensity levels.

Pushover analyses were thus performed for each bridge of the three considered classes applying a load pattern proportional to the first transverse mode and the pier top closest to the deck center of mass as controlling point. The corresponding curves exhibited considerable dispersion, something that strengthens the motivation of modelling a set of bridges with the inclusion of geometrical and material properties variability. Some specificities of the N2 method are the use of inelastic spectra and the fact that an elasto-perfectly plastic force-displacement relationship for the linearization procedure is assumed. In addition, the definition of the target displacement distinguishes between short-period and medium/long period based on the corner period ($T_s$), according to Figure 3.

![Figure 3: Design response spectrum [27].](image)

Even though the corner period $T_s$ is not easily predictable in a spectrum directly derived from real records, the latter were nevertheless considered for consistency with seismic hazard outcomes and observations regarding record variability. The corner period was identified according to Equations (3) and (4), where $S_{D_S}$, the design spectral response acceleration parameter at short periods, was assumed as the highest spectral acceleration obtained from each real scaled record. $S_{D_1}$ is the design spectral response acceleration parameter at 1-second period.

$$T_0 = 0.2S_{D_1}/S_{D_S}$$

$$T_s = S_{D_1}/S_{D_S}$$

For each bridge of each period class (0.2, 0.3, 0.75), the steps of the conventional N2 method were followed, transforming the correspondent pushover curve, referred to a multi degree of freedom (MDOF) system, in the equivalent curve of the associated single degree of freedom (SDOF) system. The bilinerization was conducted assuming as the ultimate and yielding force the 0.85 of the peak value, which ensured a more reliable approximation of the first branch of the curve when an automatize procedure is used. The performance points could be identified along the SDOF first and MDOF after, according to the equations proposed in Eurocode 8 [13]. The entire procedure was repeated for each of the 30 selected records and for 15 intensity measure levels.

7 FRAGILITY CURVES FOR BRIDGE POPULATIONS

Once all the performance points are known and compared to the selected damage limit states, a methodology for the derivation of the fragility curve needs to be selected. Among the different approaches that can be found in literature, the multiple stripes analysis [28] tech-
nique was considered the most appropriate. Such procedure is the most suitable when different ground motions at different intensity measure levels are used, which typically occurs when the conditional spectrum method is used: the representative ground motions change at each intensity measure level because the target properties follow the same trend. The main advantage of such approach is that there is no need for reaching the intensity measure level for which structural collapse occurs for all records. The most suitable fitting technique is the maximum likelihood method [29-31], however, also the alternative procedure used for estimating fragility function parameters of multiple stripes analysis, which consists in the minimization of the sum of the squared errors (SSE), was taken into account.

The aforementioned procedure typically considers an individual structure subjected to a certain number of ground motions and, at each intensity measure level, counts the number of collapses out of the total number of ground motions. Given that the objective of the present study was to define a fragility curve referred to a set of bridges of a specific class, the procedure was adapted to fit such need. In fact, the fractions of each damage limit state were obtained counting, for each record of each IML, the number of bridges that attained or exceeded the considered damage limit state out of the total number of bridges of the considered class. Assuming that the observations of the achievement of a certain damage limit state or not for each bridge and for each spectrum is independent of the observations from other bridges and ground motions, the probability attaining or the exceeding the considered damage limit state is given by a binomial distribution, as in Equation 5.

$$P(e_{ij} \text{ exceedances DLS of } n_{ij} \text{ bridges for record } i_j) = \left( \frac{n_{ij}}{e_{ij}} \right) p_{ij}^{e_{ij}} (1 - p_{ij})^{n_{ij} - e_{ij}}$$ (5)

Where \( p_i \) is the probability that the ground motion \( i \) at intensity level \( j \) will cause the attainment or exceedance of damage limit state DLS of the considered structure.

In order to get the highest probability of attaining or exceeding the considered damage limit state obtained from structural analysis, the product of the binomial probabilities of each record at each IML is considered to get the likelihood of the entire data set, which need to be maximize in order to obtained fragility function parameters. More easily, the logarithm of the likelihood function of Equation (6) can be maximized according to Equation (9).

$$\text{Likelihood} = \prod_{i=1}^{ij} \left( \frac{n_k}{e_k} \right) p_k^{e_k} (1 - p_k)^{n_k - e_k}$$ (6)

The likelihood function takes \( ij \) values, considering each record at each intensity measure and \( \prod \) indicates the product over all \( ij \) levels.

Considering that the lognormal cumulative distribution of Equation (7) is commonly used to express fragility functions, when combined with Equation (6), it takes the form of Equation (8), where \( P(\text{DS} \mid \text{IM} = x) \) is the probability that a certain ground motion with \( \text{IM} = x \) will cause in the structure the attainment or exceedance of the considered damage state (DS), \( \Phi(\ldots) \) is the standard normal cumulative distribution function (CDF), and \( \theta \) and \( \beta \) are the median of the fragility function and the standard deviation of \( \ln(\text{IM}) \) respectively.

$$P(\text{DS} \mid \text{IM} = x) = \Phi \left( \frac{\ln(x/\theta)}{\beta} \right)$$ (7)

$$\text{Likelihood} = \prod_{i=1}^{ij} \left( \frac{n_k}{e_k} \right) \Phi \left( \frac{\ln(x_k/\theta)}{\beta} \right)^{e_k} \left( 1 - \Phi \left( \frac{\ln(x_k/\theta)}{\beta} \right) \right)^{n_k - e_k}$$ (8)
\[
\{\hat{\theta}, \hat{\beta}\} = \arg\max_{\theta, \beta} \sum_{k=1}^{ij} \left\{ \ln \left( \frac{n_k}{e_k} \right) + e_k \ln \Phi \left( \frac{\ln(x_k/\theta)}{\beta} \right) + (n_k - e_k) \ln \left( 1 - \Phi \left( \frac{\ln(x_k/\theta)}{\beta} \right) \right) \right\}
\]  

(9)

\(\hat{\theta}\) and \(\hat{\beta}\) denotes the estimates of fragility function parameters, \(\theta\) and \(\beta\) respectively.

According to the previous variable definitions, the alternative method adopted, SSE, is given by Equation (10).

\[
\{\hat{\theta}, \hat{\beta}\} = \arg\min_{\theta, \beta} \sum_{k=1}^{ij} \left( \frac{e_k - \Phi \left( \frac{\ln(x_k/\theta)}{\beta} \right)}{n_k} \right)^2
\]

(10)

8 RESULTS

8.1 Fragility curves

Fragility function estimates were derived according to either the maximum likelihood algorithm or the minimization of the sum of the squared errors, as described in §7, referring to global displacement ductility as the highest among the piers of each bridge.

Different curves were derived for bridge classes of 0.2, 0.3, 0.75s for five different IMs (Iv, PGV, PGA, PGV, vRMS, Sa) and are illustrated in Figure 4 and 5 for both MLE and SSE fitting algorithms together with fractions, obtained considering each record and counting the number of bridges of a specified class attaining or exceeding the correspondent damage limit state out of the total number of bridges.

Figure 4: Fragility curves in terms of Iv and PGA for period classes 0.2s, 0.3s and 0.75s.
Figure 5: Fragility curves in terms of PGV, vRMS and $S_a$ for period classes 0.2s, 0.3s and 0.75s.

8.2 Measures of dispersion: fragility curves goodness-of-fit

In order to understand which IM-based fragility curve corresponds to lower scatter among the different bridges hence to higher accuracy in representing the bridge class, an analysis of the goodness-of-fit around each IM-based group of curves is carried out.

Statistical analysis of fragility curves has been conducted in the past [32] by defining a specific approach for performing goodness-of-fit tests. The outcome of such tests is the statistical p-value, a function of the observed sample results, typically used for testing statistical hypothesis and compared with a significance level, commonly known as $\alpha$ for claiming if the hypothesis of following a certain model can be accepted or rejected. However, in the order to rapidly compare results with a simple scalar that can provide an immediate idea of the level of dispersion, the coefficient of determination, $R^2$, was considered for defining the level of dispersion.
persion of the structural analysis data and the fitted model along y-axis. On the other hand, the
dispersion along the x-axis, which represents the quality of the fit curve with respect to the
total set of bridges and records, was analyzed by evaluating the root-mean-square deviation
in Equation (11), normalized with respect the difference between the observed maximum
\(x_{\text{max}}\) and minimum values \(x_{\text{min}}\), obtaining the normalized root-mean-square deviation
of Equation (12), parameters which allows the comparison between datasets with different scales.

\[
RMSD = \sqrt{\frac{\sum_{i=1}^{n}(x_{\text{obs},i}-x_{\text{model},i})^2}{n}} \tag{11}
\]

\(x_{\text{obs},i}\) and \(x_{\text{model},i}\) are the observed values and the fitted values, given the same probability of
exceedance for each level of observed probability of exceedance and \(n\) is the sample size.

\[
NRMSD = \frac{RMSD}{(x_{\text{max}}-x_{\text{min}})} \tag{12}
\]

The results in terms of \(R^2\) and NRMSD do not exhibit relevant differences whether MLE
or SSE fitting procedures are employed. Accordingly Figure 6 illustrates \(R^2\) values and Figure
7 depicts NRMSD values, both using MLE fitting for each damage state and each class of
bridges.

Figure 6: \(R^2\) values associated to MLE fitting for each damage state and each class of bridges.

A lower scatter (\(R^2\) closer to one) means that the model fits better the structural analysis data
and, from a bridge population point of view, the IMs which shows less dispersion are Fajfar
index (Iv), peak ground velocity (PGV), and root mean square velocity (vRMS). It is worth
noting that for such intensity measures the level of accuracy in terms of coefficient of disper-
sion is essentially constant for all damage states and bridge classes, whereas for peak ground
acceleration (PGA) and spectral acceleration (S\(_a\)), which denote lower levels of accuracy, the
dispersion increases with the damage state i.e. demand level.

Figure 7: NRMSD values associated to MLE fitting for each damage state and each class of bridges.

When the dispersion along the x-axis is examined, considering that a smaller value of
NRMSD indicates a lower dispersion around the different bridges, again, Iv, PGV and vRMS
perform better with respect PGA and S\(_a\) which, moreover, vary with both bridge class and
damage state level.
9 CONCLUSIONS AND FUTURE DEVELOPMENTS

The present study was focused on the development of nonlinear static analytical fragility curves of bridge populations based on traditional and innovative intensity measures, a field of research that has been mainly dedicated to buildings, by means of a statistically sound comparison between fitting and error measures. The adopted structural population was obtained considering a relevant number of structural configurations, randomly generated using Latin Hypercube sampling and grouped in three classes, defined by a central period of 0.2s, 0.3s and 0.75s. To assess the performance of such structural configurations, thirty response spectra, compatible with the seismic hazard of the region of Molise, in Italy, for fifteen levels of intensity measures were employed. The multi stripe analysis method was selected as the most suitable for estimating fragility parameters, through the fitting of the structural dataset.

The results have shown that Fajfar index ($I_v$), peak ground velocity (PGV), and root mean square velocity (vRMS) lead to less dispersion around the fitted fragility curves, in agreement with the findings of previous studies, which demonstrated that the same IMs exhibited better performance in terms of efficiency, practicality and proficiency in prediction of multiple engineering demand parameters when dealing with bridge populations.

Future developments of the present study will address the validation of the results obtained with nonlinear static analysis with the nonlinear dynamic counterpart. Moreover, the employed nonlinear static procedure can be further scrutinized by adopting different load patterns to better assess the performance of irregular configurations.

REFERENCES


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