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ON THE USE OF MECHANICAL METAMATERIALS FOR INNOVATIVE SEISMIC ISOLATIONS SYSTEMS

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Abstract. *We design novel versions of pentamode materials whose bulk and shear moduli are controlled by adjusting the dimensions of the struts and nodal junctions, and the material properties. We show that layers of such metamaterials can be effectively employed as deformable pads of novel base isolation systems. Benchmark examples highlight that a commercial rubber bearing can be effectively replaced by innovative isolators formed by pentamode pads confined between steel plates. Future research lines concerned with seismic applications of tunable pentamode materials are presented and discussed.*

1 INTRODUCTION

The nascent, rapidly growing field of mechanical and acoustic metamaterials is attracting increased attention from many research sectors, due to the great technological potential of unconventional materials with properties mainly derived by their geometrical structure, rather than their chemical composition (refer, e.g., to review papers [1,2]). Mechanical and acoustic metamaterials exhibit a variety of unusual behaviors that are not found in natural materials. These may include: exceptional strength- and stiffness-to-weight ratios; excellent strain recoverability; very soft and/or very stiff deformation modes; auxetic behavior; phononic band-gaps; sound control ability; negative effective mass density; negative effective stiffness; negative effective refraction index; superlens behavior; and/or localized confined waves, to name some examples (cf. [1-14] and references therein). The category of “extremal materials” has been introduced in [3] to define materials that simultaneously show very soft and very stiff deformation modes (unimode, bimode, trimode, quadramode and pentamode materials, depending on the number of soft modes). This definition applies to a special class of mechanical metamaterials – composite materials, structural foams, cellular materials, etc. – which feature special mechanical properties. Rapid prototyping techniques for the manufacture of materials with near-pentamode behavior have been recently presented in [8] (macroscale) and [9] (microscale). Pentamode materials have been proposed for transformation acoustics and elasto-mechanical cloak (refer, e.g., to the recent paper [10] and the references therein), but their potential in different engineering fields is still only partially explored.

The present study with the use of pentamode lattices as tunable seismic isolation devices, making profit from the control of the soft modes of such materials through the tuning of the bending moduli of members and junctions [8-11]. An illustrative numerical example shows a practical implementation of a pentamode lattice as a base isolation device, establishing a comparison of the elastic moduli and the force-displacement response of such a system with those exhibited by a conventional rubber bearing.

2 ELASTIC MODULI OF LATTICE MATERIALS

A recent study by A. N. Norris [11] presents a discrete-to-continuum approach to the elastic moduli of different periodic structures, including pentamode, simple cubic, BCC, FCC octet truss and tetrakaidecahedral lattices. We hereafter summarize the main results of such a study, with reference to the unit cell of a *d*-dimensional lattice that is composed of a number *Z* of elastic members (rods) and occupies a volume *V*.

With reference to the unit cells examined in [11], it is easily shown that the elastic stiffness tensor that characterize the lattice response at the mesoscopic scale shows cubic symmetry with independent stiffness coefficients C_{11}, C_{12} and C_{44} (Voigt notation). Let us introduce the following notations

$$M_i = \int_0^{R_i} \frac{dx}{E_i A_i}, \quad N_i = \int_0^{R_i} \frac{x^2 dx}{E_i I_i} \tag{1}$$

for the axial and bending compliances of the lattice members, where x is an axial coordinate with origin at the junction of the rods; while R_i, E_i, A_i and I_i respectively indicate the reference length, the Young modulus of the material, the cross-sectional area and the moment of inertia of the cross-section of *i*-th rod, assuming polar-symmetry of such a cross-section (*i = 1, …, Z*). By discarding the member forces induced by the node compliances, we obtain the results presented in Table 1, where $K = (C_{11} + 2C_{12})/3$ denotes the bulk modulus of the lattice; $G=\mu_1=C_{44}$ denotes the shear modulus; and we set: $\mu_2=\left(C_{11}-C_{12}\right)/2$. All the lattices examined in Table 1, with exception to the case $Z=14$ (*tetrakaidecahedral lattice*), are characterized by members with equal values of R_i, M_i, and N_i (respectively denoted by R, M, and N). The tetrakaidecahedral unit cell can be regarded as the summation of the simple cubic unite cell ($Z=6$) and a scaled version of the BCC cell ($Z=8$). It is obtained by truncating the BCC members with an octahedron featuring 14 faces and 36 edges of equal length a. In this case, the members of the parent simple cubic cell have equal length $R_1=\sqrt{2}a$ and equal compliances M_1 and N_1; while the members of the parent BCC cell have equal length $R_2=\sqrt{\frac{3}{2}}a$ and equal compliances M_2 and N_2. It is worth noting that, in the stretch-dominated limit ($N_i\to\infty$), the pentamode unit cell ($Z=4$) has zero shear moduli: ($G=\mu_2=0$) and nonzero bulk modulus K. This means that such a lattice material can be regarded as a "solid water" [3,8,9], which is well suited for the manufacturing of next generation seismic isolation devices.

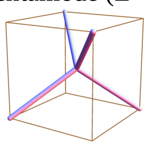
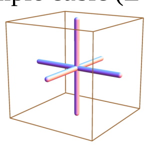
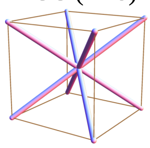
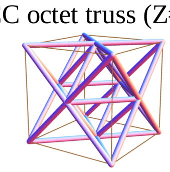
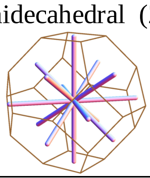
unit cell	K	G/K	μ_2/K	V
pentamode (Z=4) 	$\frac{4R^2}{9MV}$	$\frac{9M}{4M+2N}$	$\frac{3M}{2N}$	$\frac{64}{3\sqrt{3}}R^3$
simple cubic (Z=6) 	$\frac{2R^2}{3MV}$	$\frac{3M}{2N}$	3/2	$8R^3$
BCC (Z=8) 	$\frac{8R^2}{9MV}$	$1+\frac{M}{2N}$	$\frac{3M}{2N}$	$\frac{32}{3\sqrt{3}}R^3$
FCC octet truss (Z=12) 	$\frac{4R^2}{3MV}$	$\frac{3}{4}+\frac{3M}{4N}$	$\frac{3}{8}+\frac{9M}{8N}$	$4\sqrt{2}R^3$
tetrakaidecahedral (Z=14) 	$\frac{1}{9V}\left[\frac{6R_1^2}{M_1}+\frac{8R_2^2}{M_2}\right]$	$\frac{3}{2}\frac{1/M_1+1/N_2}{1/M_1+1/M_2}$	$\frac{1/N_2+2/M_2+3/N_1}{2(1/M_1+1/M_2)}$	$8\sqrt{2}a^3$

Table 1: Unit cells and elastic properties of three-dimensional lattice materials [11].

3 USE OF PENTAMODE LATTICES FOR BASE ISOLATION DEVICES

Let us refer to the representative pentamode element shown in Fig. 1, which is composed of four unit cells endowed with rods composed of two truncated bi-cones. The latter show a small-size diameter d at the extremities of the rods, and a larger diameter D at the mid-span of the rods. The lattice constant a is related to the rod length R through

$$a = \frac{4R}{\sqrt{3}} \tag{2}$$

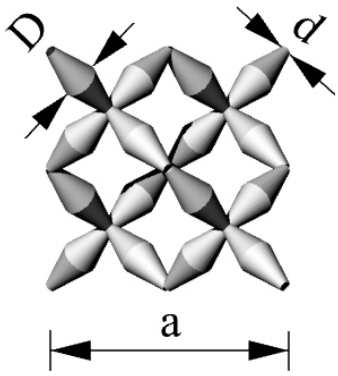


Figure 1: Geometry of a pentamode module formed by four unit cells.

The law of variation the generic rod radius r with the abscissa x is written as

$$r\,(x) = r_{\min} + \left(\frac{r_{\max} - r_{\min}}{R/2}\right)x, \quad 0 \le x < \frac{R}{2} \tag{3}$$

$$r\,(x) = r_{\max} - \left(\frac{r_{\max} - r_{\min}}{R/2}\right)\left(x - \frac{R}{2}\right), \quad \frac{R}{2} \le x \le R \tag{4}$$

where $r_{\min} = d/2$ and $r_{\max} = D/2$. On using the above relationships, and taking into account that the cross-section area and the moment of inertia of the rods are respectively given by $A\,(x) = \pi\left[r\,(x)\right]^2$ and $I\,(x) = \pi\frac{\left[r\,(x)\right]^4}{4}$, we finally obtain the following expressions of the axial and bending compliances of the generic rod

$$M = \frac{R}{\pi E\, r_{\max} r_{\min}}, \quad N = \frac{R^3\left(2r_{\max}^2 + r_{\max}r_{\min} + r_{\min}^2\right)}{3\pi E\, r_{\max}^3 r_{\min}^3} \tag{5}$$

where E denotes the Young modulus of the rod material. Making use of Eqns. (5) into the expressions of K and G/K ratio given in Tab. 1 ($Z = 4$), we finally obtain

$$K = \frac{4R^2}{9VM} = \frac{\pi EdD}{64\sqrt{3}R^2}, \quad G = \frac{9\sqrt{3}\pi\left(dD\right)^3 E}{768\left(dDR\right)^2 + 512\left(d^2 + 2D^2 + dD\right)R^4} \tag{6}$$

$$\frac{G}{K} = \left[\frac{4}{9} + \frac{8\left(2D + dD + d^2\right)R^2}{27d^2D^2}\right]^{-1} \tag{7}$$

It is worth comparing the analytic prediction of the G/K ratio provided by Eqn. (7), with the numerical estimate of the same ratio given in Kadic et al. [9], which has been obtained by fitting the results of finite element simulations of pentamode lattices under hydrostatic pressure and simple-shear loading

$$\frac{G}{K} = \left[\left(\frac{R}{d}\right)^2 \sqrt{\frac{R}{D}}\right]^{-1} \tag{8}$$

Fig. 2 compares the predictions of Eqns. (7) and (8) for the G/K ratio, assuming the lattice constant $a = 15$ mm ($R = 6.5$ mm), different D/a ratios ($D/a = 0.174, 0.087, 0.044$), and letting the d/D ratio vary from zero to one. The limiting cases with $d/D = 0$ and $d/D = 1$ respectively correspond to a stretch-dominated pentamode lattice (no bending effects), and a pentamode lattice showing rods with constant cross-section (cylindrical rods).

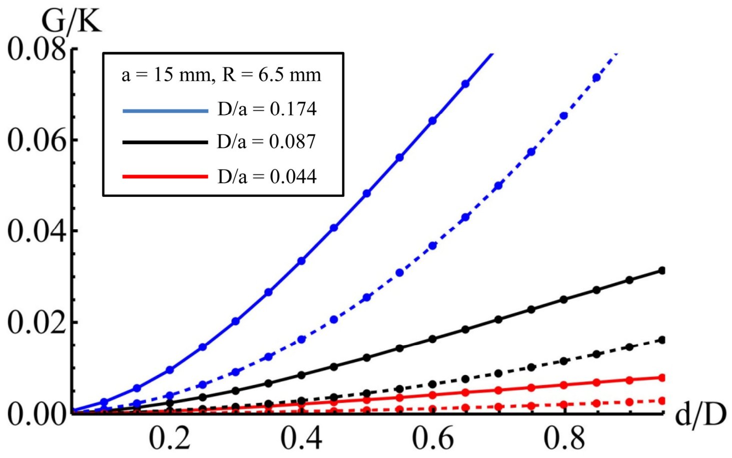


Figure 2 Curves plotting the G/K ratio of pentamode lattices against the d/D ratio, for $a = 15$ mm and different values of the D/a ratio. The solid curves correspond to the analytic predictions of Eqn. (7) [11], while the dashed curves correspond to the numerical estimates of Eqn. (8) [9].

The mismatches between the analytic and numerical predictions of the G/K ratio in Fig. 1 are explained by the fact that the analytic formula (7) is based on a beam model for the rods (1D theory), while the numerical estimate (8) accounts for a 3D elastic response of such elements (through fine meshes of tetrahedral finite elements [9]). Both predictions highlight that the shear modulus of pentamode lattices can be finely tuned by properly designing the diameters d and D of the struts [8,9]. The following section illustrates practical implementations of pentamode lattices for the construction of innovative seismic isolation devices.

3.1 Practical examples

Let us compare the mechanical response of pentamode lattices with that of a commercial rubber bearing with diameter $\varnothing 33.5"\left(0.8509\,m\right)$ and height $13.85"\left(0.3518\,m\right)$, which is composed of 29 rubber layers of $7\,mm$ each; 28 steel shims (spacers) of $3.04\,mm$ each; two terminal rubber layers of $31.8\,mm$ each and covers, without lead plug (isolator Type E [RB-800] produced by Dynamic Isolation Systems, Inc, McCarran, NV, USA). Figs. 3 and 4 illustrate the hysteretic (low-damping) response of such an isolator in terms of cyclic shear stress (τ) vs. shear strain (γ) curves, and normal stress (σ) vs. axial strain (ε) curves (with preload of $\sigma = 0.31 MPa$) at constant velocity (courtesy of Caltrans Testing Facility, University of California, San Diego). Such experimental results provide an average shear modulus $\bar{G} = 0.791 MPa$, and an average compression modulus of the rubber-steel composite $\bar{E}_c = 330\, MPa$ [15].

Table 2 illustrates the geometrical properties of different pentamode pads, which can be confined between steel plates to form innovative pentamode bearings (PMBs). Such pads are obtained by replicating the pentamode module shown in Fig. 1 $n_a \times n_a$ times in the plane of the pad, and one single time along the thickness ($n_v = 1$), implying pad thickness $t \equiv a$. The lattice struts are made of steel (SPMB1 and SPMB2: $E = 206 GPa$) or the "FullCure850 VeroGray" polymeric material supported by Stratasys® 3D printers (PPMB1 and PPMB2: $E = 1.4 GPa$ [8]), and show different values of the geometric variables a, D and d. Making use of Eqn. (6) for the shear modulus of the pentamode pad, and the following formula for the composite, steel-pentamode compression modulus [15]

$$E_c = 6.73 G\, S_1^2 \tag{9}$$

where $S_1 = \frac{L}{4t} = \frac{n_a}{4n_v}$, we predict $G = \bar{G} = 0.791 MPa$, $E_c = 341 MPa$ for all the PMBs based on the pads described in Table 2, i.e., shear and compression moduli almost equal to those of the comparative rubber bearing. Figures 3 and 4 provide comparisons between the experimental stress-strain curves of the analyzed rubber bearing and the linear stress-strain responses of the PMBs in Table 2 (assuming absence of dissipation in the latter, to a first, simplifying, approximation).

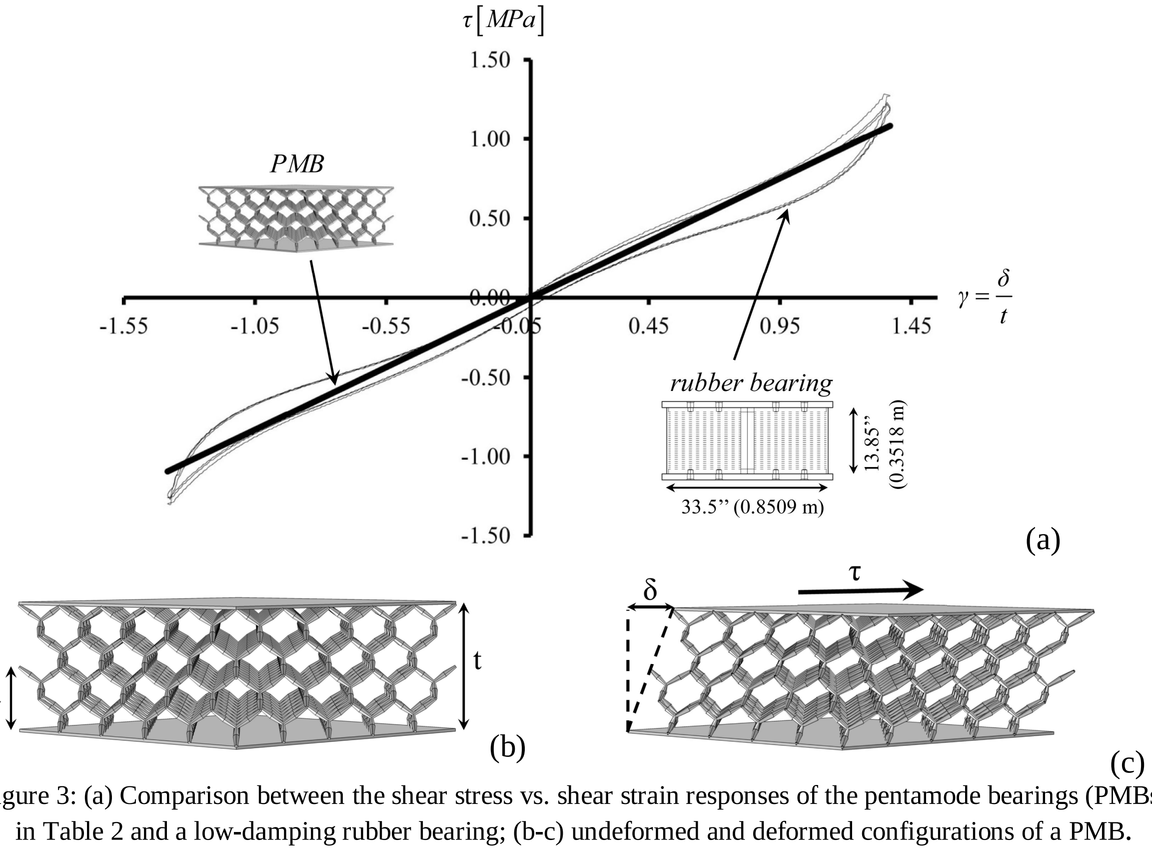


Figure 3: (a) Comparison between the shear stress vs. shear strain responses of the pentamode bearings (PMBs) in Table 2 and a low-damping rubber bearing; (b-c) undeformed and deformed configurations of a PMB.

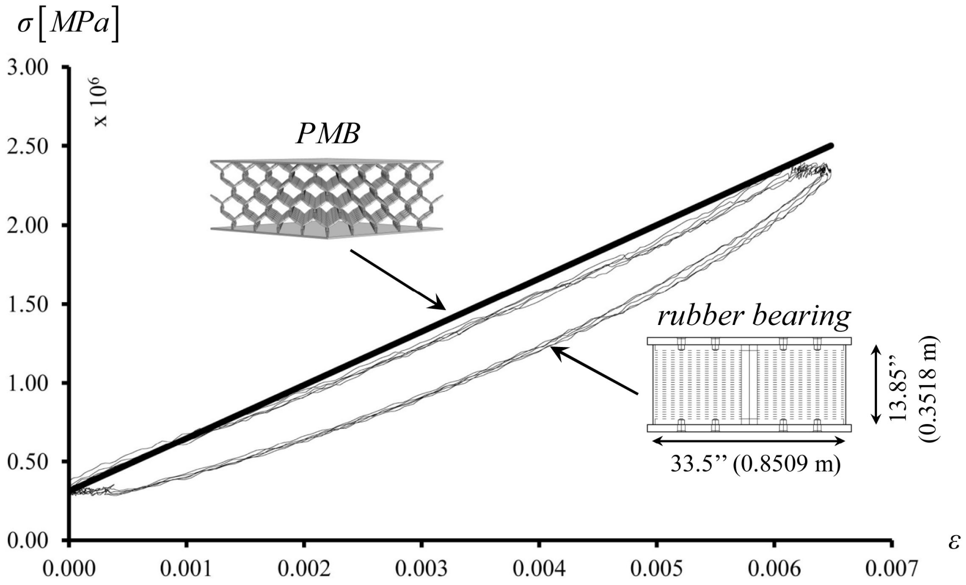


Figure 4 Comparison between the axial stress vs. axial strain responses of the pentamode bearings (PMBs) in Table 2 and a low-damping rubber bearing.

Case	$t \equiv a[mm]$	$R[mm]$	n_a	$L = n_a\, a[m]$	$d = \frac{D}{2}[mm]$
SPMB1	5	2.17	32	0.16	0.19
SPMB2	10	4.33	32	0.32	0.37
PPMB1	5	2.17	32	0.16	0.66
PPMB2	10	4.33	32	0.32	1.32

Table 2: Geometric properties of pentamode pads to be employed to form seismic isolators.

4 CONCLUSIONS

We have presented novel versions of pentamode materials: artificial structural crystals showing shear moduli markedly smaller than the bulk modulus [3,8,9]. Innovative seismic isolators based on pentamode lattices confined between steel plates have been designed. The soft modes of such materials have been controlled by tuning of the bending moduli and the geometry of members and junctions. We have shown that such novel isolators can exhibit mechanical response similar to that of conventional rubber bearings, provided that the material and the geometry of the lattice are suitably designed.

We may conclude that the main advantages of pentamode bearings over traditional structural bearings [16-18] are the following:

- the mechanical properties of the soft layers forming such devices mainly depend on the geometry of the pentamode lattices, more than on the chemical nature of the employed materials (metallic, ceramic, polymeric, etc.);

- it is easy to adjust the mechanical properties of pentamode bearings to those of the structure to be isolated, by playing with the lattice geometry and the nature of the material (see Eqn. (6) and Table 2), as opposed to rubber bearings, where instead the achievement of very low shear moduli implies marked reductions of the vertical load carrying capacity, making such devices not particularly convenient in the case of structures with very high fundamental periods of vibrations (such as, e.g., very tall buildings; highly compliant structures; very soft soils; etc.) [16-18];
- the dissipation of pentamode bearings can be conveniently designed through an accurate choice of the material to be used for the pentamode lattices, and inserting, - when necessary, an additional dissipative element within the device (such as, e.g., a lead core);
- the possibility to design and fabricate laminated composite bearings showing layers with different materials, geometries and properties: such a design approach is instead much less effective in the state-of-the-art laminated rubber bearings, where the only lamination variable consists of the type of rubber to be employed for the soft pads (natural rubber or synthetic rubber);
- the freedom in the choice of the materials of the pentamode lattices, by keeping the elastic properties of the device essentially unchanged, allows the designer to adapt the energy dissipation capacity and the life span (i.e., the durability) of the device to the actual use conditions [17-18];
- the possibility to replace the fluid components of the structural bearings and energy absorbing devices currently available on the market (such as, e.g., viscous fluid dampers and tuned mass dampers) with pentamode lattices: such a replacement would lead to significantly reduce the technical issues related to fluid leaking and frequent maintenance, which currently affect the state-of-the-art devices involving fluid materials [17-18];
- the mechanical properties of pentamode bearings can be dynamically adjusted and measured, by equipping selected struts of the pentamode lattices with sensors and/or actuators;
- pentamode bearings directly manufactured from computer-aided design data outputted by a computational material design phase, employing advanced and fast additive manufacturing techniques at different scales and single or multiple materials (metals, polymers, etc.).

Several aspects of the present work pave the way to relevant further investigations and generalizations that we address to future work. First, mechanical models for composite rubber-steel bearings [15] need to be generalized to pentamode-steel bearings, accounting for the peculiar deformation models of such systems. Second, physical models of pentamode isolators need to be constructed, employing, e.g., additive manufacturing techniques [8,9], and laboratory tested as seismic base-isolation devices [19], in order to experimentally asses their isolation and dissipation capabilities arising, e.g., from inelastic response and/or material fracture [20]. Another relevant generalization of the present research regards the design of dynamically tunable systems based on the insertion of prestressed cables and/or curved rods within pentamode lattices, with the aim of designing novel metamaterials and bio-inspired lattices tunable by local and global prestress [12-14, 21-28]. Future studies will also address the experimentation of pentamode materials as components of new-generation seismic dampers.

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