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FROM THE NOTIONS OF NONLINEARITY TOLERANCES TOWARDS A DEFICIENCY IN COMMERCIAL TRANSIENT ANALYSIS SOFTWARES AND ITS SOLUTION

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Abstract. *The notions of nonlinearity tolerances in static and dynamic analyses are reviewed. A deficiency in conventional nonlinear dynamic analysis, also existing in the transient analysis commercial soft wares, is recognized. A simple approach to practically eliminate the deficiency is proposed, and its adequacy is demonstrated numerically.*

1 INTRODUCTION

The true behavior of structural systems is nonlinear and dynamic. For nonlinear dynamic analysis of structural systems, the broadly accepted approach is to define the structural and mathematical models, discretize the mathematical models in space, and solve the resulting ordinary initial value problems in time. In view of the typical initial value problem defining the structural motion, stated below [1-3]:

$$\begin{aligned}
&\mathbf{M}\ddot{\mathbf{u}}(t)+\mathbf{f}_{\text{int}}(t)=\mathbf{f}(t) \qquad 0\le t<t_{end}\\
&\mathbf{u}(t=0)=\mathbf{u}_0\\
&\dot{\mathbf{u}}(t=0)=\dot{\mathbf{u}}_0\\
&\mathbf{f}_{\text{int}}(t=0)=\mathbf{f}_{\text{int}_0}\\
&\mathbf{Q}\le 0
\end{aligned} \tag{1}$$

(t and t_{end} imply the time and the duration of the dynamic behavior; $\mathbf{M}$ is the mass matrix; $\mathbf{f}_{\text{int}}$ and $\mathbf{f}(t)$ stand for the vectors of internal force and excitation; $\mathbf{u}(t)$, $\dot{\mathbf{u}}(t)$, and $\ddot{\mathbf{u}}(t)$ denote the vectors of displacement, velocity, and acceleration; $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, and $\mathbf{f}_{\text{int}_0}$ define the initial status of the model (regarding the essentiality of considering $\mathbf{f}_{\text{int}_0}$ in Eqs. (1), also see [4]); and finally, $\mathbf{Q}$ represents some restricting conditions, in problems involved in nonlinearity; see [5, 6]), the most versatile analysis method is time integration [7, 8]. The process of time integration is schematically displayed in Figure 1, and in addition, for nonlinear analyses, at time stations, where nonlinearities are detected, some iterative computation, for localizing the nonlinearities, is strongly recommended [3, 9-12]. Before each nonlinearity iteration, conditions not to implement the iteration are being checked, and lead to either stop of the iteration and continuation of the analysis, repeat of the iterative computation, or stop of the iteration and halt of the analysis, i.e. (a) stop of the iteration after k iterations and continuation of the analysis, expressed as:

$$\begin{aligned}
i=1,2,3\ldots k-1 &: \quad \left\|{}^{i}\delta\right\|>\bar{\delta}\\
i=k<K &: \quad \left\|{}^{i}\delta\right\|\le\bar{\delta}
\end{aligned} \tag{2}$$

(b) Another iteration, formulated as

$$\begin{aligned}
&i=1,2,3\ldots k \;:\quad \left\|{}^{i}\delta\right\|>\bar{\delta}\\
&k<K
\end{aligned} \tag{3}$$

and (c) Stop of the iteration and halt of the analysis, stated as

$$\begin{aligned}
&i=1,2,3\ldots k \;:\quad \left\|{}^{i}\delta\right\|>\bar{\delta}\\
&k=K
\end{aligned} \tag{4}$$

In Eqs. (2)-(4), i is an indicator for the nonlinearity iteration, k stands for the number of iterations carried out, equivalently introducing the last or current iteration, ${}^{i}\delta$ implies the residual of a balance equation in terms of force, displacement, energy, momentum, etc. at the i^{th} iteration, $\bar{\delta}$ is the tolerance introducing the maximum residual acceptable, and finally, K denotes the maximum number of iterations acceptable, at each detection of nonlinearity, essential to prevent useless unending iterations in presence of dominating round off errors.

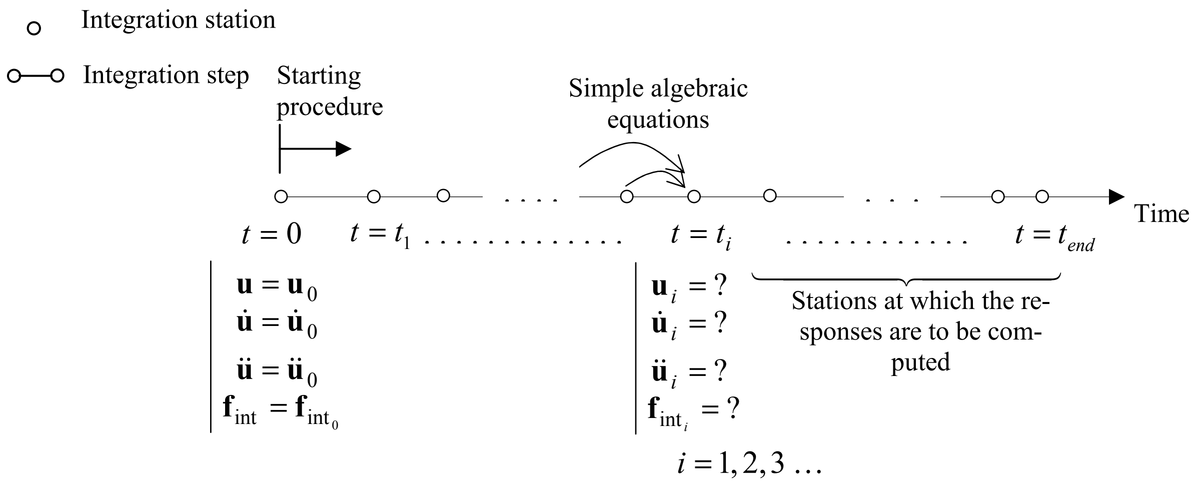

Figure 1: A schematic illustration of the time integration process.

In static nonlinear analyses, Eq. (1) is replaced with

$$\begin{aligned} &\mathbf{f}_{\text{int}} = \mathbf{f} \\ &\mathbf{Q} \le 0 \end{aligned} \tag{5}$$

and the step-by-step consideration of the **f** at sequential time instant throughout the time interval $[0 \quad t_{end}]$ is being replaced with incremental implementation of **f**, i.e.

$$\begin{aligned} &{}_0\mathbf{f} = \overline{\mathbf{O}} \\ &{}_j\mathbf{f} = {}_{j-1}\mathbf{f} + \Delta\mathbf{f} \quad , \quad j = 1, 2, \ldots n \\ &\Delta\mathbf{f} = \frac{\mathbf{f}}{n} \end{aligned} \tag{6}$$

where, n denotes the total number of the increments and a left subscript implies the amount and value of the argument. Nevertheless, Eqs. (2)-(6) remain unchanged and valid in both static and dynamic analyses (see [9-11, 13]). Therefore, it is reasonable to expect differences between the notions of the parameters in Eqs. (2)-(4), in static and dynamic analyses. This paper is dedicated to this purpose, based on which, a deficiency in the current analysis soft wares is also introduced and suggestions for overcoming the deficiency are stated.

2 NOTIONS OF THE TOLERANCES IN STATIC AND DYNAMIC ANALYSES

In a static analysis, the analysis in each increment is exact, unless some nonlinearity is detected. To say better, at the j^{th} increment,

$$\begin{aligned} &\mathbf{K} \; {}_j\Delta\mathbf{u} = {}_j\Delta\mathbf{f} \\ &{}_j\mathbf{u} = {}_{j-1}\mathbf{u} + {}_j\Delta\mathbf{u} \\ &\qquad\qquad j = 1, 2, 3, \ldots \end{aligned} \tag{7}$$

where, ${}_j\mathbf{K}$ is a constant matrix representing the stiffness, at the application of the j^{th} increment of loading

$${}_j\Delta\mathbf{f} = \Delta\mathbf{f} \tag{8}$$

Accordingly, ${}_j\Delta\mathbf{u}$ is available exactly, and hence, when detecting nonlinearities, the residuals are representations of deviations from the exact solutions. Therefore, the tolerances stand for the maximum acceptable inaccuracy, at the detections of nonlinearities.

In the dynamic case, Eq. (7) is replaced with

$$\begin{aligned} {}_j\mathbf{M}\ddot{\mathbf{u}}+{}_j\mathbf{C}\dot{\mathbf{u}}+{}_j\mathbf{K}\mathbf{u}={}_j\mathbf{f} \quad &, \quad t_{j-1} \le t < t_j \\ \mathbf{u}(t=t_{j-1})=\mathbf{u}_{j-1} & \\ \dot{\mathbf{u}}(t=t_{j-1})=\dot{\mathbf{u}}_{j-1} & \end{aligned} \tag{9}$$

to be generally and conventionally solved for $\mathbf{u}_j$ and $\dot{\mathbf{u}}_j$, by a time integration scheme. Time integration methods generally result in approximate responses [4, 14, 15]. Consequently, different from static analyses, where, ${}_j\mathbf{u}$ can be computed exactly, in dynamic analyses, $\mathbf{u}_j$ and $\dot{\mathbf{u}}_j$ are available approximately (even in totally linear analyses). Therefore, the residuals can not represent deviations from exact solutions, but only, stand for the differences between the carried out analyses and the ideal analyses, for which, throughout the analyses,

$$\delta = 0 \tag{10}$$

Accordingly, in dynamic analyses, tolerances are merely upper bound controls for the above mentioned differences, and do not necessarily control the inaccuracy of the computations at the detected nonlinearities. In other words, different from the tolerances in static analyses, the tolerances in dynamic analyses do not represent upper limits for the computational errors, and hence, disregarding them would not necessarily deteriorate the accuracy. This is specifically true, when the contribution of the errors because of the approximate formulation of time integration is considerable compared to the errors originated in nonlinearity.

3 A DEFIECIENCY IN TRANSIENT ANALYSIS COMMERCIAL SOFTWARES AND A SIMPLE SOLUTION FOR IT

As implied in the last lines of Section 2, in nonlinear dynamic analyses, no accuracy can be guaranteed, by nonlinearity iterations and considering specific tolerances. Consequently, Eq. (4), and the halt of the analyses, via Eq. (4), loses its essentiality.

Currently, the commercial soft wares, dedicated to structural dynamic analysis, consider Eq. (4) and stop the analysis when incapable of arriving at residuals not larger than the tolerances, in a maximum number of iterations (K in Eq. (4)). This halt of analyses is in fact a deficiency causing the analyzer/operator to repeat the analyses with different values of the parameters, e.g. integration step size, nonlinearity tolerance, K, nonlinearity iterative method, etc. entailing additional computational cost.

Since, with attention to the explanation in Section 2, the source of the above mentioned halt is independent of the analysis accuracy, as a solution to the deficiency above, even when the residuals are not small enough after K iterations, the analyses can be continued. In simpler words, in nonlinear dynamic analyses, we can replace Eqs. (2)-(4) and: (a) stop the iterations and continue the analysis, when

$$i = 1,2,3\ldots k-1 \; : \; \left\|{}^i\delta\right\| > \bar{\delta} \quad \text{and} \quad \left(\left\|{}^k\delta\right\| \le \bar{\delta} \quad \text{or} \quad k = K\right) \tag{11}$$

and (b) implement another repetition when

$$i = 1,2,3\ldots k \; : \; \left\|{}^i\delta\right\| > \bar{\delta} \quad \text{and} \quad k < K \tag{12}$$

In other words, we claim that replacing Eqs. (2)-(4) with Eqs. (11) and (12), leads to non-stop analysis of nonlinear structural dynamic problems (even when the residuals are not necessarily smaller than or equal to the tolerances), with no significant effects or at least practically acceptable effects on the accuracy of the analyses. The implementation of this idea is simple and merely needs disregarding the control of residuals, when $k = K$; see [4, 16]. In view of the conventional approach, i.e. Eqs. (2)-(4), the approach proposed above can be also considered as temporarily changing the tolerance to infinity, in order to prevent the halt of the analysis; see [17].

4 NUMERICAL STUDY

Consider the two shear buildings in Figure 2(a), with the structural members introduced in Table 1, subjected to the ground acceleration displayed in Figure 2(b). Obviously, the behavior is potentially nonlinear, because of the linear-elastic/perfectly-plastic characteristics, and meanwhile the possibility of poundings. The behavior is actually nonlinear in view of the responses depicted in Figure 3, where, the superposition principle [18] is violated. Implementing the average acceleration time integration [19], integration step sizes equal to the steps of the ground motion's record, the fractional time stepping nonlinearity solution method [20-22], and nonlinearity iterations over displacements, considering $K = 5$, and $\bar{\delta} = 10^{-6}$, lead to the

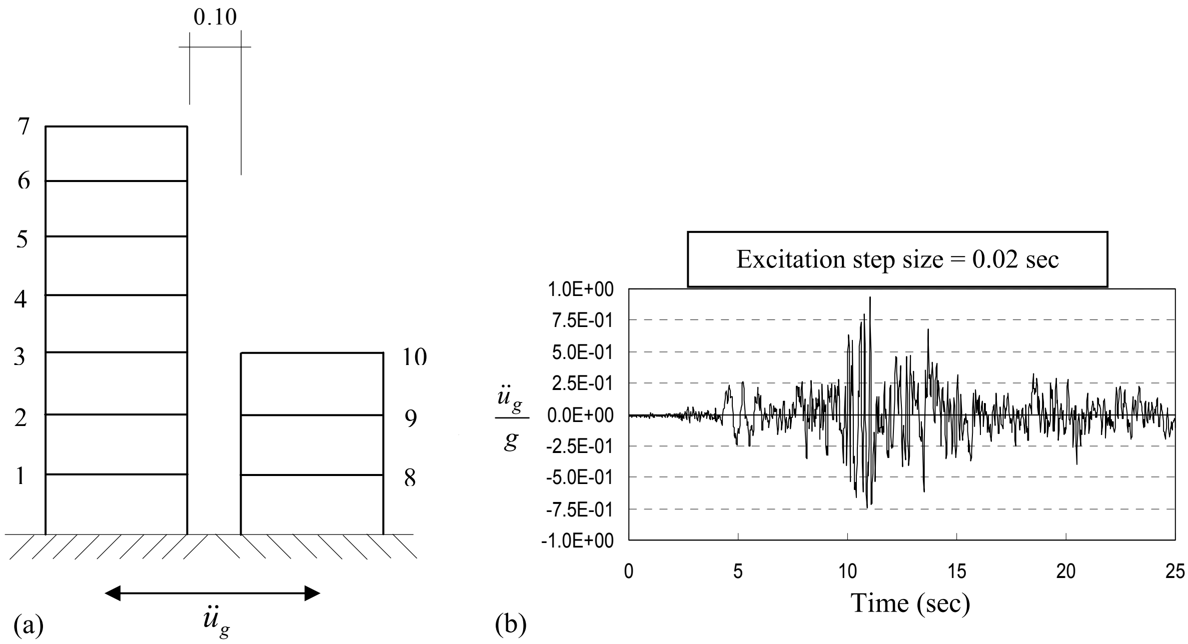

Figure 2: The structural system in the second example: (a) configuration; (b) excitation.

Floor	1	2	3	4	5	6	7	8	9	10
Mass $\times 10^{-3}$	2068	2064	2060	2056	2052	2048	2044	2052	2048	2044
Stiffness$\times 10^{-6}$	840	820	700	680	660	640	620	660	640	620
Yielding displacement $\times 10^{3}$	18.5	19	20	20.5	21	21.5	22	21	21.5	22
Damping	Negligible (considered zero)									
Restitution factor	1.0									

Table 1: The characteristics of the system introduced in Figure 2.

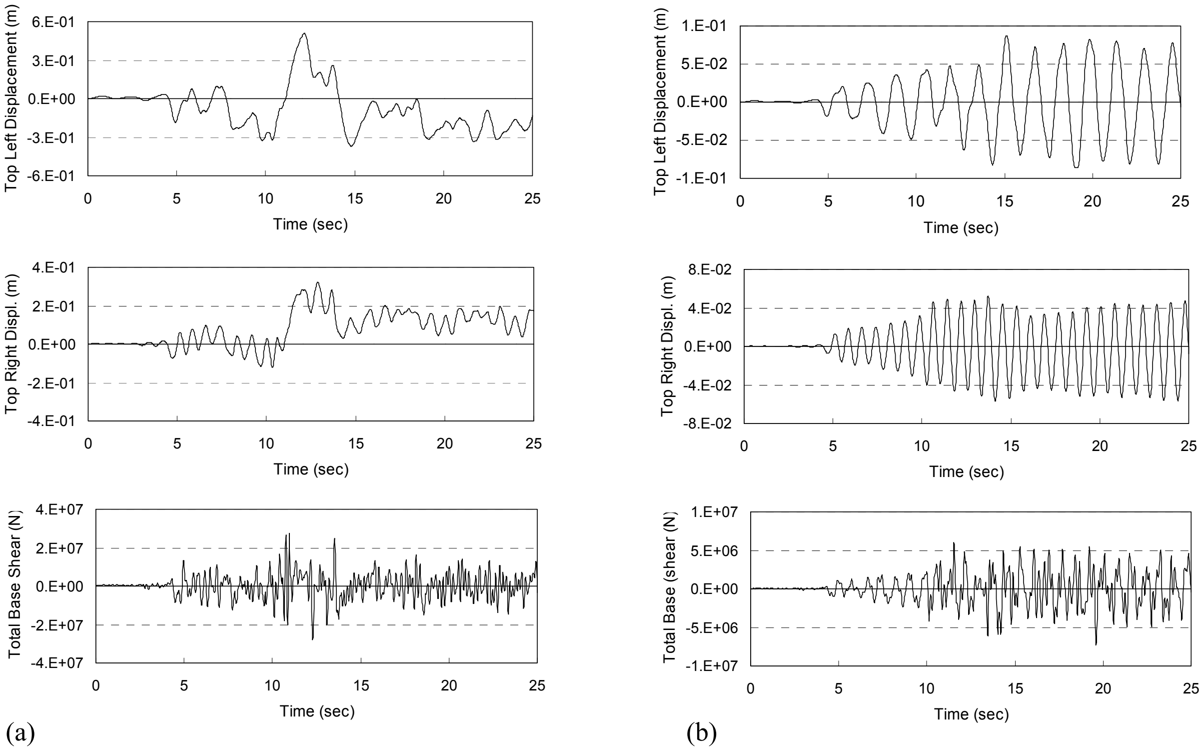

Figure 3: Highly precise responses histories for: (a) the system introduced in Figure 2 and Table 1, (b) the system introduced in Figure 2 and Table 1 after dividing the excitation to ten.

responses displayed in Figure 4. Apparently, by using Eqs. (11) and (12), we have arrived at responses histories throughout the total integration interval (different from the conventional analyses, where the analysis is stopped at $t = 4.58$ sec). Comparing these responses (reported in Figure 4(b)), with the highly precise responses in Figures 3(a), reveals the acceptable accuracy provided by the proposed approach, specifically, in practical areas like earthquake engineering. The study is repeated, as below:

(a) After changing the nonlinearity tolerance twice to 10^{-8} and 10^{-4}.
(b) After omitting one of the two sources of nonlinearity, and once considering only the piece-wise linear/perfectly plastic behavior and once considering only the pounding.
(c) after changing the time integration method, once to the Wilson-θ ($\theta = 1.42$) method [23-25], and once to the Generalized-α ($\rho = 0.8$) method [26].
(d) After changing the nonlinearity solution method.
(e) After adding nonzero classical damping equal to two and five percent.
(f) After scaling the excitation and causing nonlinear behaviors with different severities.
(g) After changing the excitation to excitation different in peak acceleration and frequency content.
(h) After changing the structural system to systems with different complexities.

The results were conceptually similar, revealing the validity of the claims in Section 2, not reported here for the sake of brevity.

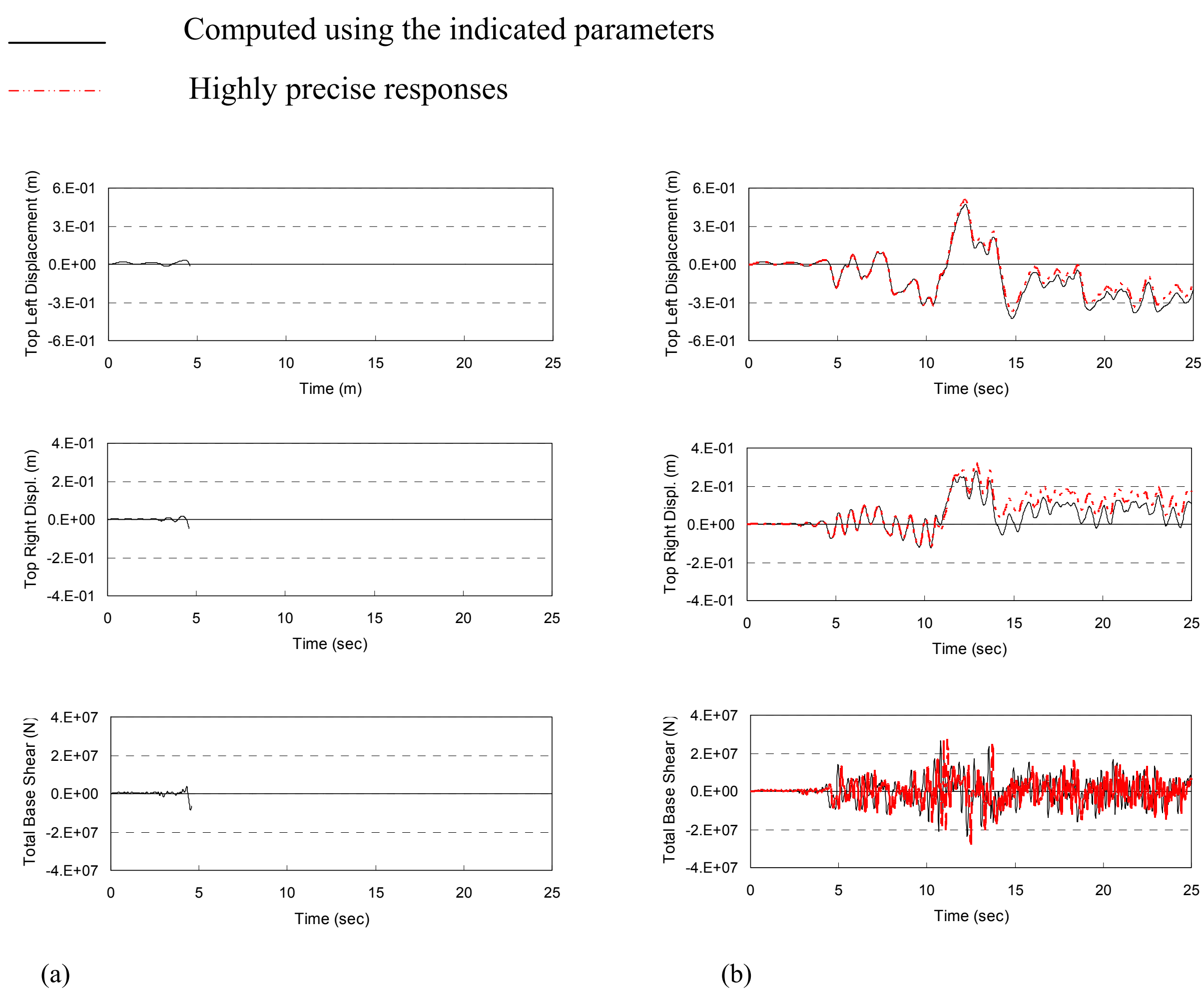

Figure 4: Responses histories computed by average acceleration for the system in Figure 2 and Table 1: (a) conventional, i.e. using Eqs. (2)-(4), (b) when implementing Eqs. (11) and (12) instead of Eqs. (2)-(4).

5 CONCLUSION

Concentrating on the notions of nonlinearity residuals and tolerances in static and dynamic analyses, we in this paper demonstrated that, different from static analyses, in dynamic analyses, nonlinearity tolerances do not necessarily imply upper bounds on the computational errors at nonlinearity detections/localizations. Accordingly, no analysis needs to be stopped when the nonlinearity residuals are not small enough after the maximum acceptable number of nonlinearity iterations. This is both computationally possible, and also leads to accuracies practically acceptable. The claims are discussed theoretically and demonstrated numerically.

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