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ASSESSMENT OF THE LATERAL STIFFNESS OF WALLS WITH OPENINGS

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Abstract. *Approximations obtained with different simplified methods to assess the elastic lateral stiffness of walls with openings are compared to results obtained with the finite element method using reasonably fine meshes. Multistory walls were studied, considering symmetric and asymmetric distribution of openings with respect to a vertical axis. From the obtained results it can be* observed *that for a single strip of asymmetric perforated walls, the best approximations are obtained using the equivalent frame method more commonly used in Mexico, the method proposed by Coull and Choudhury, as well as the method baptized as the equivalent no prismatic wide column analogy, particularly as the number of stories of the wall increases.*

1 INTRODUCTION

In displacement-based design is very important to estimate accurately the lateral displacements of a given structural system. Buildings with structural walls are very commonly used in seismic regions, mostly for reinforced concrete and masonry structures. Often, these structural walls have openings, particularly at the facades. Although regular opening patterns are often used, sometimes for architectural requirements (illumination, ventilation, aesthetics), complex opening patterns are not uncommon in seismic regions.

The impact of openings in the lateral stiffness of shear walls captured the attention of researchers and designers in the 1960s. In fact, many expressions, approximations and simplified analysis methods date back to those dates, because during that decade several midrise buildings with shear walls were built, which fostered practical research studies. Among other methods proposed at the times, one can highlight for their simplicity the methods proposed by Coull and Choudhury [1, 2] and the Equivalent Frame Method (EFM) proposed by Schwaighofer and Microys [3]. The approximations obtained with these methods were calibrated then with the results of few low rise walls with symmetrical openings patterns with respect to a vertical axis.

The practical modeling of walls with openings is a complex task that has not yet been fully addressed. In this case the use of the finite element method is essential if the aim is to obtain reliable results, both to estimate deformations, stiffnesses and stresses. In the absence of simple analytical methods that could estimate easy and reasonably the lateral stiffness matrix of walls with complex opening patterns (including asymmetries), Tena-Colunga [4-6] proposed the equivalent condensed beam method, a mixed method where the lateral stiffness matrix is obtained from equivalent condensed beams using the flexibility method and the finite element method. Although the method is accurate for practical purposes because, in essence, is a static condensation done numerically, it has the disadvantage that finite element meshes are required to define the equivalent condensed beams.

The development or improvement of simple models to analyze walls with openings has received relatively little attention in recent years, perhaps because the capabilities of personal computers since 1997 have facilitated the use of commercial finite element programs to model them reasonably. However, even today is still impractical modeling with the required rigor, complete medium to high rise buildings with several walls with openings using refined finite element meshes, especially if the distribution of the openings is irregular or complex in elevation. In many instances, practicing engineers do not have the time and experience needed to develop rigorous and detailed three-dimensional finite elements models that would allow them to analyze and design this type of structures, although they currently have access to the computer software (e.g. SAP 2000 or similar) and hardware to do it. Although the finite element method allow one to model such complex geometries, this method is not always practical for the analysis and design of complex and large buildings having walls with openings.

Therefore, approximate methods are still needed and used by structural engineers worldwide to estimate their lateral displacements using a reasonable number of degrees of freedom. Such methods would allow obtaining reasonable estimates of their corresponding dynamic characteristics (periods and mode shapes), lateral displacements, story drifts, etc. Once lateral deformations are reasonably assessed, relative distributions of forces and stresses can be obtained using also simple methods that would allow realizing suitable structural designs without the imperative need to perform more complex analyses with finite elements.

In this paper, the approximations obtained with different simplified methods to assess the lateral stiffness of walls with openings are compared to results obtained with the finite element method using reasonably fine meshes. Multistory walls were studied, considering symmetric and asymmetric distribution of openings with respect to a vertical axis.

2 SIMPLIFIED METHODS UNDER STUDY

2.1 Equivalent frame method

The equivalent frame method, which was originally proposed by Schwaighofer and Microys [3], consider a wall system with central openings (Figure 1), so it can be understood as an equivalent frame with two lines of walls or “wide-columns” connected by wide central beams. The general modeling assumptions were:

1. Centroidal axes for the cross sections of walls and connecting beams form an equivalent frame.
2. Cross section properties for all columns in the equivalent frame are identical to the corresponding sections for the walls.
3. Full cross section properties for the central beams are considered.
4. Rigid end zones in flexure are considered for the extreme sections of beams.

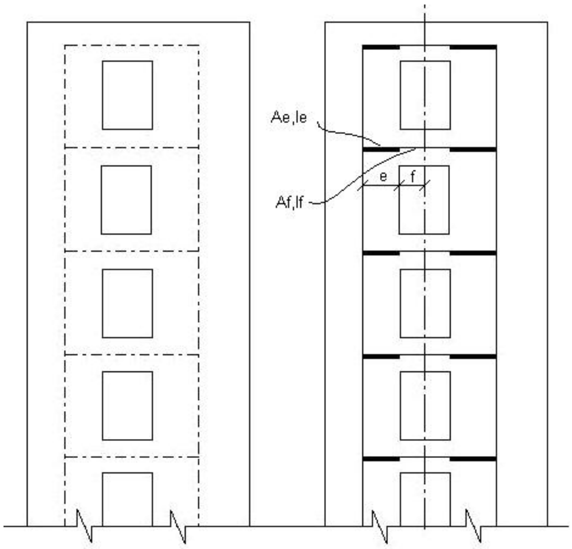


Figure 1. Equivalent frame proposed by Schwaighofer and Microys

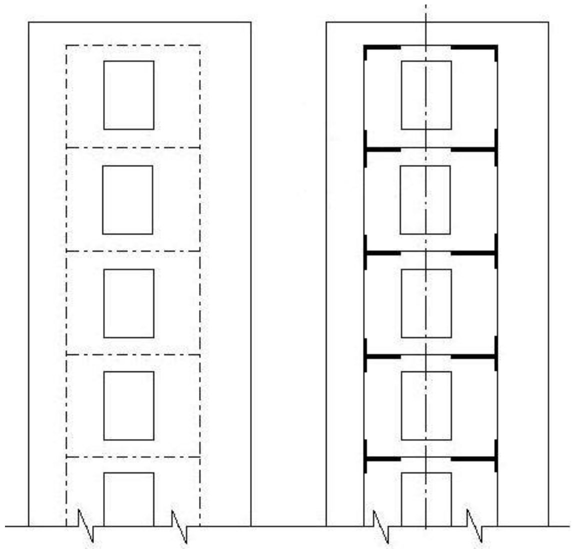


Figure 2. Equivalent frame under SM3 modeling

Schwaighofer and Microys [3] proposed equivalent areas (A_e) and moments of inertia (I_e) for the rigid-end zones, as function of the opening parameters e and f (Figure 1), where e is defined as the distance between the centroidal wall axis and the edge of wall/opening and f is the distance between the centroid of the opening and the edge of the wall/opening. These equivalent properties are computed as:

$$A_e = K_1 A_f \tag{1}$$

$$I_e = K_2 I_f \tag{2}$$

where the proposed values for constants K_1 and K_2 are identified in Table 1.

e/f	K_1	K_1
0.5	50	238
1.0	100	700
2.0	200	2600
3.0	300	6300
5.0	500	21500

Table 1. Constants proposed by Schwaighofer and Microys

From the data provided in Table 1, the following equivalent equations were obtained:

$$K_1 = 100\left(\frac{e}{f}\right) \tag{3}$$

$$K_2 = 0.0593\left(\frac{e}{f}\right)^4 + 99.348\left(\frac{e}{f}\right)^3 + 302.43\left(\frac{e}{f}\right)^2 + 296\left(\frac{e}{f}\right) + 1.7778 \tag{4}$$

Although the equivalent frame method is widely used for the modeling of walls with openings, it is uncommon that engineers would compute constants K_1 and K_2 as proposed by Schwaighofer and Microys. For example, in Mexico beam end zones are considered as infinitely rigid ends zones in flexure, and modeled as rigid links according to what it is described in the literature [4, 7]. Some other people consider that infinitely rigid end zones should be considered also for the equivalent wide columns that represent the walls (Figure 2). Therefore, three different modelings were evaluated for the equivalent frame method:

a) Model SM1. The equivalent frame method as originally proposed by Schwaighofer and Microys (computing A_e and I_e, Figure 1).
b) Model SM2. The equivalent frame method assuming perfectly rigid end zones at beam ends ($A_e \rightarrow \infty$ and $I_e \rightarrow \infty$, Figure 1).
c) Model SM3. The equivalent frame method assuming perfectly rigid end zones at beam and wall ends (Figure 2).

2.2 Method proposed by Coull and Choudhury

Coull and Choudhury [1, 2] proposed a simple method to estimate the peak lateral displacement of walls coupled by slabs or beams (Figure 3), where the lateral loads are resisted entirely by the walls. They proposed a simplified model where the coupling slabs or beams are replaced by an equivalent continuum shear media; that is, the coupling beams with individual stiffnesses EI_p are replaced by an equivalent continuum which stiffness per unit height (h) is EI_p/h. Therefore, they proposed to idealize the system as having two flexural members (walls) connected to a continuum shear media (Figure 3). Coull and Choudhury formulated the second order differential equation for such system and they were able to arrive to a close form solution.

The size of central openings has an important impact in the behavior of coupled shear walls. In the model proposed by Coull and Choudhury, openings are located in a regular, well defined vertical strip, as depicted in Figure 3. Coull and Choudhury proposed the parameter αH (where H is the total height of the structure) to evaluate the overall behavior of the coupled wall system, where:

$$\alpha = \sqrt{\beta \mu l} \tag{5}$$

$$\beta = \frac{12 I_p l}{h b^3 (I_1 + I_2)} \tag{6}$$

$$\mu = 1 + \frac{(A_1 + A_2)(I_1 + I_2)}{A_1 A_2 l^2} \tag{7}$$

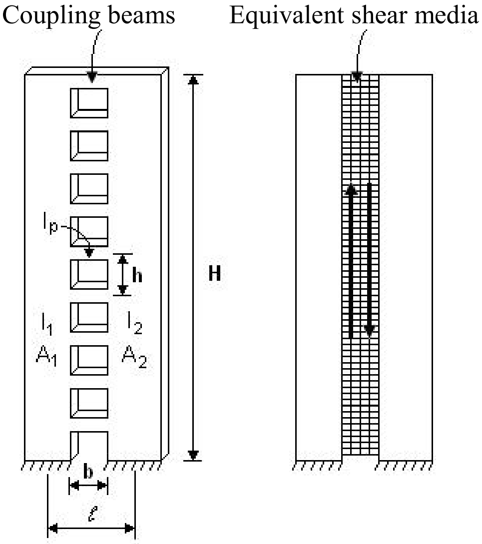


Figure 3. Model proposed by Coull and Choudhury

When αH>8, the coupling wall system tends to behave as a single cantilever wall, whereas when αH<4, the behavior is closer to an equivalent frame. Therefore, for a single vertical strip of openings, the impact of the openings in the lateral stiffness of the coupled wall system can be assessed by using the following stiffness parameters:

$$K_w = \frac{3EI_w}{H^3} \tag{8}$$

$$K_{wo} = \frac{3E(I_1 + I_2)}{H^3 K_4} \tag{9}$$

where K_w and K_{wo} are the lateral stiffnesses of the wall without or with openings respectively, E is Young´s modulus for the walls, I_w is the moment of inertia of the wall considering that there are no openings and:

$$K_4 = 1 - \frac{3}{\mu}\left(\frac{1}{3} + \frac{sinh(\alpha H)}{(\alpha H)^3 \cosh(\alpha H)} - \frac{1}{(\alpha H)^2} \right) \tag{10}$$

The peak top lateral displacement (Δ) for a given lateral load (F) is computed as:

$$\Delta = \frac{FH^3 K_4}{3E(I_1 + I_2)} \tag{11}$$

Coull and Choudhury only proposed how to assess the peak lateral displacement at the top of the building. Therefore, in order to estimate how displacements vary along the height for a given coordinate x along the height ($0 \leq x \leq H$), three simple functions were evaluated, as described lines below.

Model CC1:

$$\Delta(x) = \frac{Fx^3 K_4}{3E(I_1 + I_2)}; \quad 0 \leq x \leq H \tag{12}$$

Model CC2:

$$\Delta(x) = \frac{FH^{3/4} x^{7/4} K_4}{3E(I_1 + I_2)}; \quad 0 \leq x \leq H \tag{13}$$

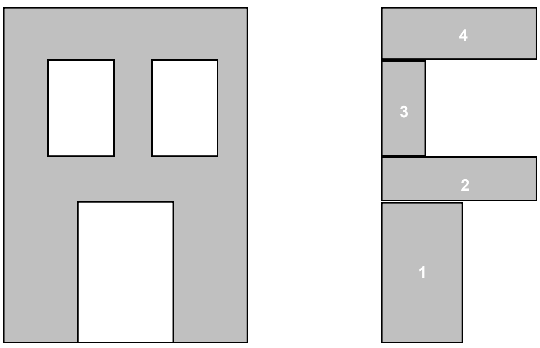
Model CC3:

$$\Delta(x) = \frac{FH^{3/4} x^{7/4} K_4}{3E(I_1 + I_2)}\left(1 - e^2\right); \quad 0 \leq x \leq H \tag{14}$$

where e is the opening eccentricity parameter described in section 3.2.

2.3 Equivalent non-prismatic wide column analogy

Another simple method that could be used to estimate lateral displacements of wall systems with openings is to represent such system along the height as a series of equivalent solid segments, ending with an equivalent non-prismatic wide column (Figure 4). With such discretization it is considered that the impact of openings is to diminish the effective cross section properties (area and inertia) of the wall in the segment that the opening is located. For example, the wall system depicted in Figure 4a can be substituted by the equivalent non-prismatic column with four segments depicted in Figure 4b. Therefore, once the effective cross section properties are defined for each segment, the lateral stiffness for the element can be assessed using methods for nonprismatic elements available in the literature [6, 8]. Alternatively, as equivalent prismatic cross sections are defined for the opening segments, available structural analysis software can be used to conduct the required analyses.



a) Wall with openings b) Tapered column

Figure 4. Discretization of a wall with openings as an equivalent nonprismatic wide column

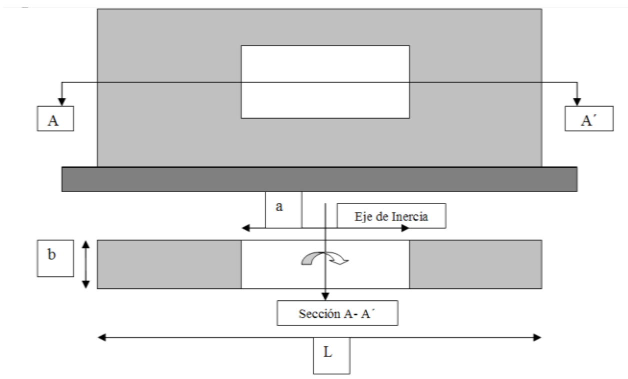


Figure 5. Cross section of the wall in the opening area

The key assumption for this simple method is how to assess the equivalent cross section properties (effective areas A_{eq} and effective moment of inertia I_{eq}) for the segments with openings. Three different assumptions were evaluated.

In Model CE1, it is assumed that in the opening area, the effective section is stiff enough to sustain its theoretical moment of inertia along the opening height. Therefore, taking as reference the section and dimensions depicted in Figure 5, the equivalent cross section properties under this assumption are:

$$I_{eq} = \frac{b}{12}\left(L^3 - a^3\right) \tag{15}$$

$$A_{eq} = b\left(L - a\right) \tag{16}$$

$$A_{ceq} = \frac{A_{eq}}{1.2} \tag{17}$$

In Model CE2, it is assumed that in the opening area, the effective section is basically a solid section which length is the sum of the two solid wall sections (in this case, from Figure 5 the total length would be: $L_{eq} = L_1 + L_2 = L\text{-}a$). Therefore, taking as reference the section and dimensions depicted in Figure 5, the effective areas are those identified in Eqs. 15 and 16 and the equivalent moment of inertia would be assessed as:

$$I_{eq} = \frac{b}{12}\left(L - a\right)^3 \tag{18}$$

In Model CE3, it is assumed that the flexibility in opening area is very important, so each wall section responds separately, so the effective cross section properties at that section are the sum of the properties of the individual solid segments. Under this assumption, and taking as a reference the wall segment depicted in Figure 5, assuming that the wall is completely symmetric, the effective areas are also those identified in Eqs. 15 and 16, but the equivalent moment of inertia would be computed as:

$$I_{eq} = 2\left[\frac{b}{12}\left(\frac{L - a}{2}\right)^3\right] \tag{19}$$

3 GEOMETRIC PARAMETERS FOR OPENINGS

3.1 Opening size parameter λ

The opening size parameter λ is defined, for a given story, as the ratio (in percentage) between the area (in elevation) of the opening and the gross area (in elevation) for the wall segment. Taking as reference the wall segment elevation depicted in Figure 6, this parameter is assessed as:

$$\lambda = 100\left(\frac{l_0 h_0}{Lh}\right) \tag{20}$$

3.2 Opening eccentricity parameter *e*

The opening eccentricity parameter *e* is defined, for a given story, as the normalized distance (in percentage) *d* between the centroidal axis of the opening and the centroidal axis of the wall with the available distance to the edge of the wall, as depicted in Figure 7. If the total length of the wall in Figure 7 is also *L* and the length of the opening is l_0 (Figure 6), then the opening eccentricity parameter is computed as:

$$e = 100\left(\frac{2d}{L - l_0}\right) \tag{21}$$

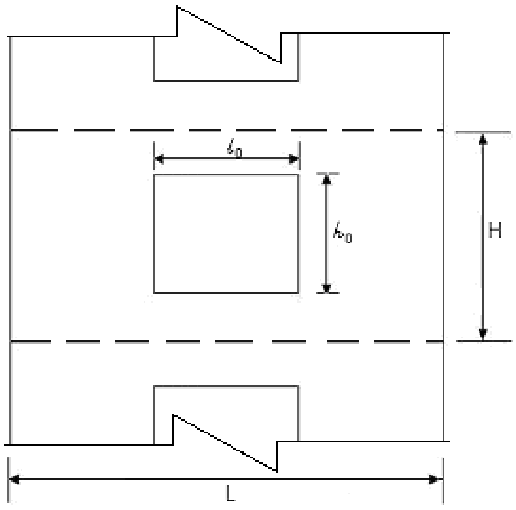


Figure 6. Definition of the opening size parameter λ

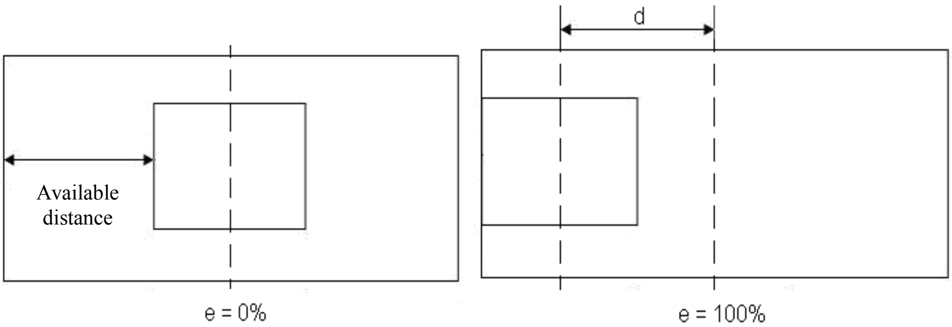


Figure 7. Definition of the opening eccentricity parameter *e*

4 EQUIVALENT OPENING FOR MULTIPERFORATED WALLS

For simplicity, most of the available simplified methods already proposed to assess the lateral stiffness of multi-story walls with openings have considered a well-defined central strip of openings (Figures 1 to 3). However, it is common to have perforated walls with various openings. In many instances, they are not regular (Figure 8). To try to extend such simple methods to more complicated opening geometries, Rivera [9] proposed a simple method to define an equivalent opening (Figure 8).

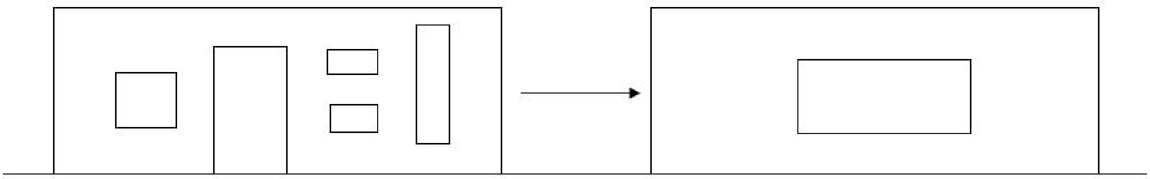


Figure 8. Simplification of multi-perforated walls into a wall with an equivalent single opening

The proposed procedure can be summarized as follows:

1. First, all openings are grouped together around the largest opening, to define a large rectangle with some rectangular appendices around (Figure 9).

2. The area for the equivalent rectangle, A_e, is simply the sum of all individual opening areas A_i.
3. The height to length proportion ratio for the equivalent opening is computed as a weighted average of the individual openings:

$$\frac{H_e'}{L_e'} = \frac{\sum_{i=1}^{n} H_i A_i}{\sum_{i=1}^{n} L_i A_i} \tag{22}$$

4. The dimensions of the equivalent opening are then computed as:

$$L_e = \sqrt{A_e\left(\frac{L_e'}{H_e'}\right)} \tag{23}$$

$$H_e = \sqrt{A_e\left(\frac{H_e'}{L_e'}\right)} \tag{24}$$

5. The centroidal axis for the equivalent opening is located in the centroidal axis of the original openings.

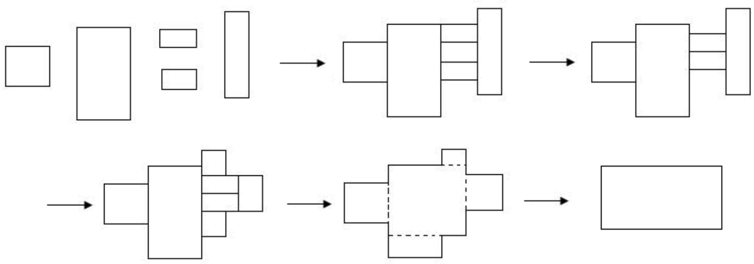
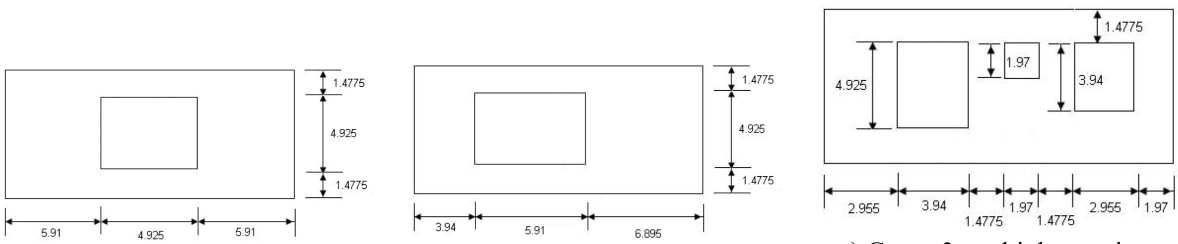


Figure 9. Example of the assessment of an equivalent opening

5 APPROXIMATIONS FOR MULTI-STORY PERFORATED WALLS

In order to assess the degree of approximation of the simple methods under evaluation, a series of multi-story walls with different opening layouts at each story were considered, as depicted in Figure 10. Walls with a single symmetric opening with an opening size parameter λ=18.4% were considered (Group 1, Figure 10a), as well a walls with a single asymmetric opening with λ=18.4% (Group 2, Figure 10b) and walls with multiple asymmetric openings with λ=26.5% (Group 3, Figure 10c). For the walls of Group 3, an equivalent opening as described in section 4 was defined for all the approximate methods under study.



a) Group 1, symmetric b) Group 2, asymmetric c) Group 3, multiple openings

Figure 10. Typical perforated wall configurations (at each story) under study (dimensions in ft)

For each group type, four different elevations were considered: 3, 6, 12 and 18 stories, as schematically depicted in Figure 11, where the walls for Group 1 are identified as A, B, C and D for the 3, 6, 12 and 18 stories respectively, E, F, G and H for Group 2 and I, J, K and L for Group 3. All stories have a story height *h*=7.88 ft and a length *L*=16.745 ft (Figure 10). Therefore, the global slenderness ratios for the studied walls vary from *H*/*L*=1.41 for the three-story models to *H*/*L*=8.47 for the eighteen-story models.

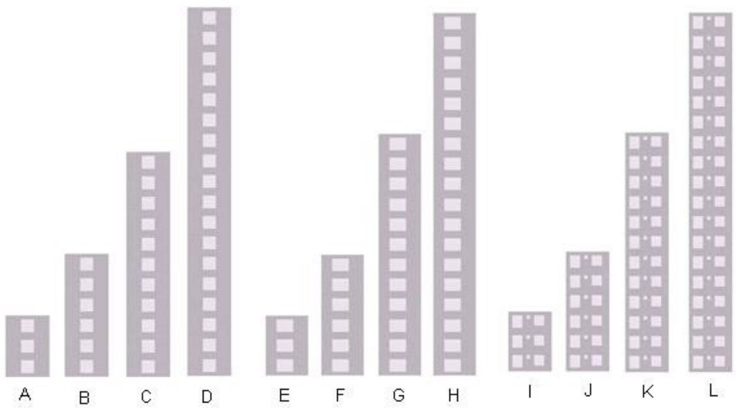


Figure 11. Elevations for the perforated wall models under study

All the described walls (Figures 10 and 11) were rigorously modeled in SAP-2000 using refined meshes with thick shell elements. The assumed elastic properties were E=1.728x10^8 lb/ft^2, ν=0.15 and a thickness *t*=0.7083 ft were considered for all walls. A total lateral load *F*=350,000 lb was uniformly distributed at the top of each walls. The average lateral displacements at the top of the walls computed from the results of SAP-2000 are reported and identified as "FEM" in Table 2. From the results reported in Table 2, it is observed that average lateral displacements are amplified in walls with asymmetric openings (Group 2) with respect to symmetric openings (Group 1), granted they have the same dimensions and the same opening areas.

Wall	FEM	CC	SM1	SM2	SM3	CE1	CE2	CE3
A (1)	1.115	0.577	1.106	0.968	0.841	0.556	0.629	0.973
B (2)	4.710	3.560	4.842	4.458	4.062	3.431	3.715	5.046
C (3)	28.477	26.170	28.969	28.084	26.376	25.408	26.524	31.760
D (4)	90.175	86.826	91.142	89.754	84.886	84.471	86.966	98.682
E (5)	1.314	0.703	1.254	1.142	0.974	0.567	0.674	1.012
F (6)	5.347	4.0188	5.384	5.066	4.558	3.456	3.868	5.177
G (7)	31.178	28.573	31.584	30.842	28.805	25.464	27.088	32.232
H (8)	97.684	94.0938	98.749	98.585	92.494	84.563	88.196	99.706
I (9)	1.299	0.870	1.532	1.446	1.1796	0.565	0.742	1.351
J (10)	5.916	4.519	5.966	5.730	5.0544	3.454	4.153	6.547
K (11)	38.262	30.848	33.134	32.624	30.562	25.474	28.247	37.741
L (12)	123.451	100.670	102.124	101.332	96.601	84.601	90.821	112.121

Table 2. Average lateral displacements (in) at the top of the walls with openings under study

Also, the results obtained with SAP-2000 are compared with the estimates obtained with the simplified methods under study: Coull and Choudhury (CC), the three described variations for the equivalent frame method (SM1, SM2 and SM3) and for the equivalent non-prismatic wide column analogy (CE1, CE2 and CE3). These approximations are also compared graphically in Figures 12 to 14, where numbers are used to identify the letters, equivalence that is identified in parenthesis in the column named "wall" in Table 2. In Figures 12 to 14, displacements are normalized with respect to the solution obtained with SAP-2000, which for practical purposes can be considered as “exact”; therefore 100% percent should be interpreted as the exact result.

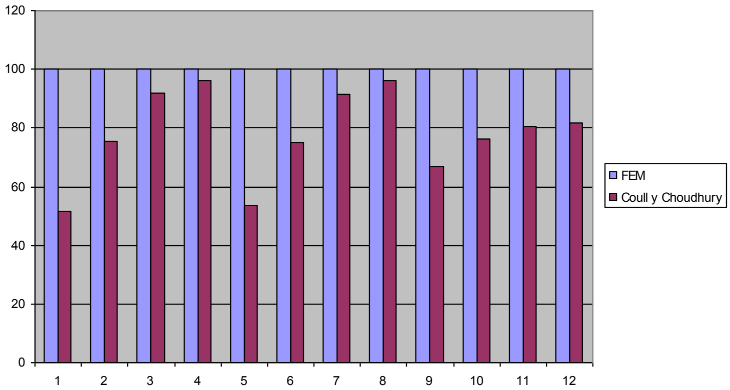


Figure 12. Approximations of the top lateral displacement between SAP-2000 (FEM) and the method of Coull and Choudhury. Displacements are normalized with respect to the FEM solution (100=exact)

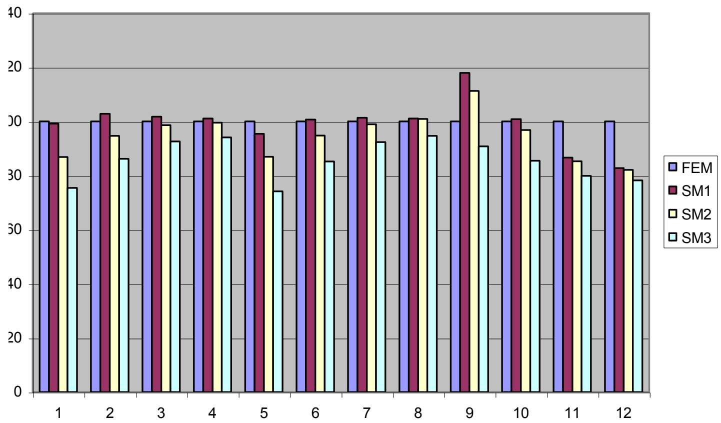


Figure 13. Approximations of the top lateral displacement between SAP-2000 (FEM) and variations of the equivalent frame method (SM1, SM2 and SM3). Displacements are normalized with respect to the FEM solution (100=exact)

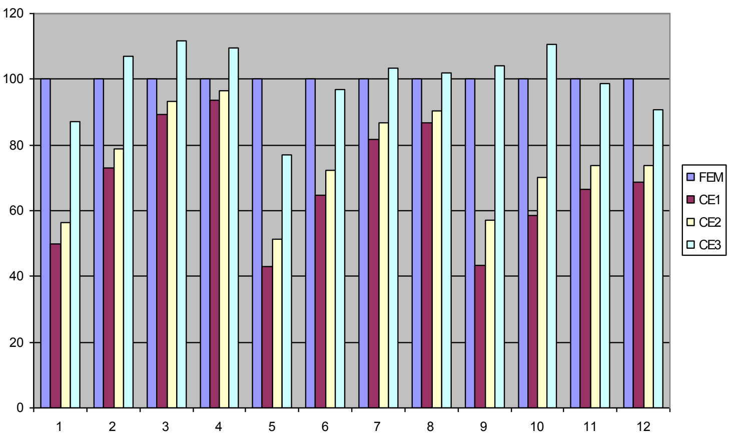


Figure 14. Approximations of the top lateral displacement between SAP-2000 (FEM) and variations of the equivalent non-prismatic wide column analogy (CE1, CE2 and CE3). Displacements are normalized with respect to the FEM solution (100=exact)

It is observed from Figures 12 to 14 that for all simplified methods, the degree of approximation increases as the number of stories increases, that is, as the global slenderness ratio H/L increases. Approximation are reasonable (higher than 70%) from the six story models, that is, when $H/L \geq 2.82$.

It can be observed from Figure 12 that, for the method proposed by Coull and Choudhury, the degree of approximation for a well-defined single opening layout is very similar independently if these openings are symmetric (Group 1, walls 1 to 4) or asymmetric (Group 2, walls 5 to 8). For multi-perforated walls (Group 3, walls 9 to 12), the approximation does not improve as much as the number of stories increases, but it is reasonable, taking into account the inherent simplicity in the definition of an equivalent opening.

It can be observed from Figure 13 that, for the equivalent frame method, the approximation is "exact" for practical purposes for multi-story walls with a single opening layout if the original proposal of Schwaighofer and Microys (SM1) is used, independently if the openings are symmetric (Group 1, walls 1 to 4) or asymmetric (Group 2, walls 5 to 8). However, for walls with multiple openings (Group 3, walls 9 to 12), the method losses accuracy as a consequence of using an equivalent opening. The degree of approximation is reduced when infinitely rigid end zones are considered for the beams (SM2). The worst approximations for this method are obtained when infinitely rigid en zones are also considered for the wall section (SM3).

Finally, it can be observed from Figure 14 that under the equivalent non-prismatic wide column analogy, best approximations are obtained when it is assumed that, in the opening region, each wall section responds separately (CE3); however, under such modeling average displacements can be overestimated with respect to finite element solutions. Under this modeling, the worst approximation was obtained when it is assumed that, in the opening area, the effective section is stiff enough to sustain its theoretical moment of inertia along the opening height (CE1), and as expected, much stiffer results were obtained.

5.1 Assessment of the impact of the opening eccentricity parameter *e*

From the results shown in the previous section it was observed that, for a single strip of openings along the height, average lateral displacements are affected by the asymmetry of the wall opening. Therefore, a parametric study was designed to assess the impact of the eccentricity of the wall opening in terms of the opening eccentricity parameter e described in section 3.2. For this purpose, perforated walls with typical length L=16.745 ft and story height h=7.88 ft with an opening size parameter λ=25% were considered. The considered values for e were e = 0% (symmetric), 20%, 40%, 60% and 80%. Perforated walls of 3, 6, 9, 12, 15 and 18 stories were considered, so their corresponding global slenderness ratio were H/L=1.41, 2.82, 4.24, 5.65, 7.06 and 8.74. In this study, average lateral displacements were computed for each story, so displacement profiles were obtained.

The described walls were rigorously modeled in SAP-2000 using refined meshes with thick shell elements (40 x 40 elements per story). The assumed elastic properties were E=1.728x10^8 lb/ft^2, ν=0.15 and a thickness t=0.7083 ft were considered for all walls. A total lateral load F=1x10^6 lb was uniformly distributed at the top of each walls. The average lateral displacement profiles are depicted in Figure 15, where it can be clearly observed that average lateral displacements increases as the opening eccentricity parameter increases, particularly when e>40%.

The average lateral displacements profiles obtained with the FEM and depicted in Figure 15 are compared with those obtained with the different approximate methods under study in Figures 16 to 21. In order to ease the comparison, for each model under study (number of stories and eccentricity parameter under consideration), average lateral displacements were normalized with respect to the average lateral displacement obtained at the top of each wall model under the finite element method (Δ_{FEM}). In fact, in each subplot the normalized average lateral displacement profile under the finite element modeling is presented as FEM (red continuous line with open square symbols).

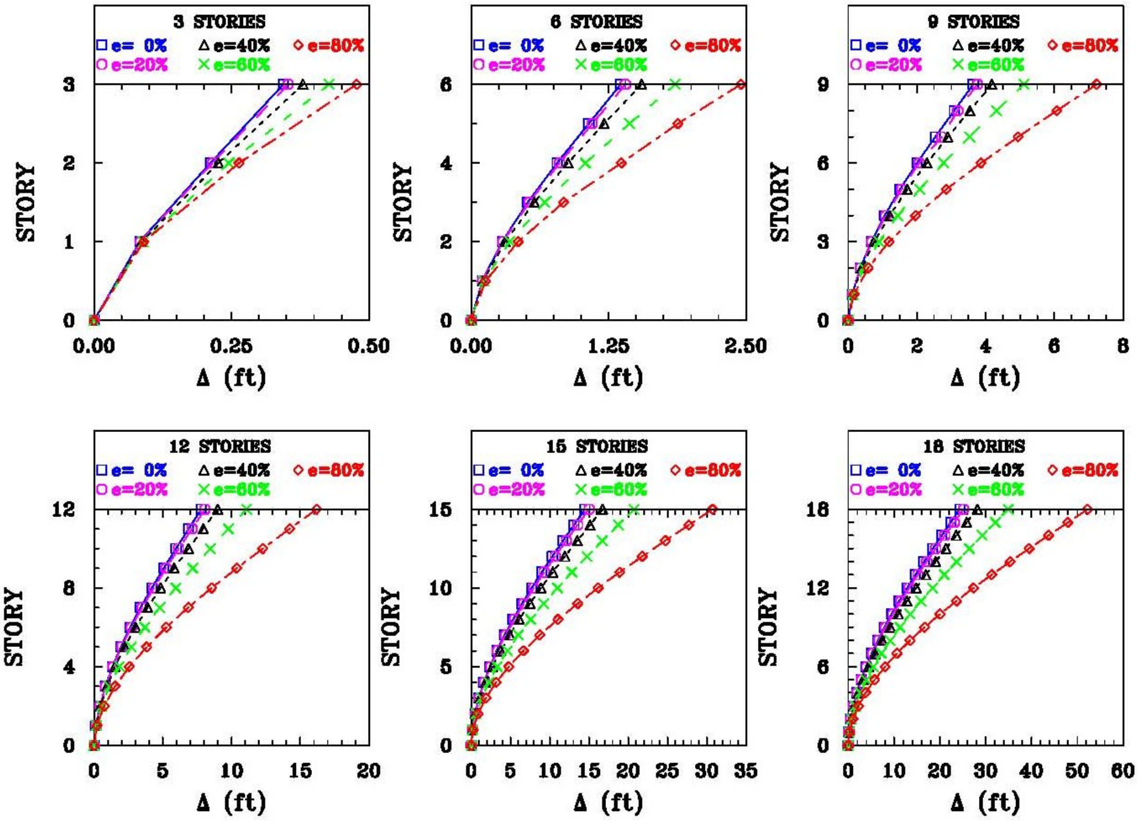


Figure 15. Average lateral displacement profiles for the walls with assymetric openings under study obtained from FEM analyses

It is observed from Figures 16 to 21 that approximations under the Coull and Choudhury method (CC1, CC2 and CC3) depend on the wall height or the global wall slenderness ratio. For example, it is observed from Figure 16 that approximations for the three-story walls (H/L=1.41) are poor (about 50% the average lateral displacement obtained with FEM models), even for walls with symmetric openings (e=0%). However, approximations improve as the number of stories increases, being greater than: a) 70% for the 6-story walls (H/L=2.82, Figure 17), b) 80% for the 9-story walls (H/L=4.24, Figure 18), c) 90% for the 12-story walls (H/L=5.65, Figure 19) and, d) 95% for the 15-story walls (H/L=7.06, Figure 20) and 18-story walls (H/L=8.74, Figure 21). It is worth noting that it was not observed an important reduction on the degree of approximation of this method when the opening eccentricity parameter e increases, that is, approximation obtained for e=20%, 40%, 60% and 80% were similar to those obtained for e=0% for each elevation under consideration (Figures 16 to 21). Regarding the proposed variation of displacement profiles along the height, it is observed from Figures 16 to 21, but particularly from Figures 19 to 21, that the assumption behind model CC2 (Eq. 13) is reasonable. In fact, there is not a significant difference with the assumption behind model CC3, where a correction to account for the eccentricity is proposed (Eq. 15). Therefore, in sake of keeping approximate methods as simple as possible, it is recommended to use the Coull and Choudhury method under the CC2 proposal for walls with global wall aspect ratios (total height H over base dimension L) $H/L \geq 4$, for any given opening eccentricity parameter (e), for walls with a single opening region well defined along the height of the wall, as approximations near or greater than 80% the average lateral displacements are obtained.

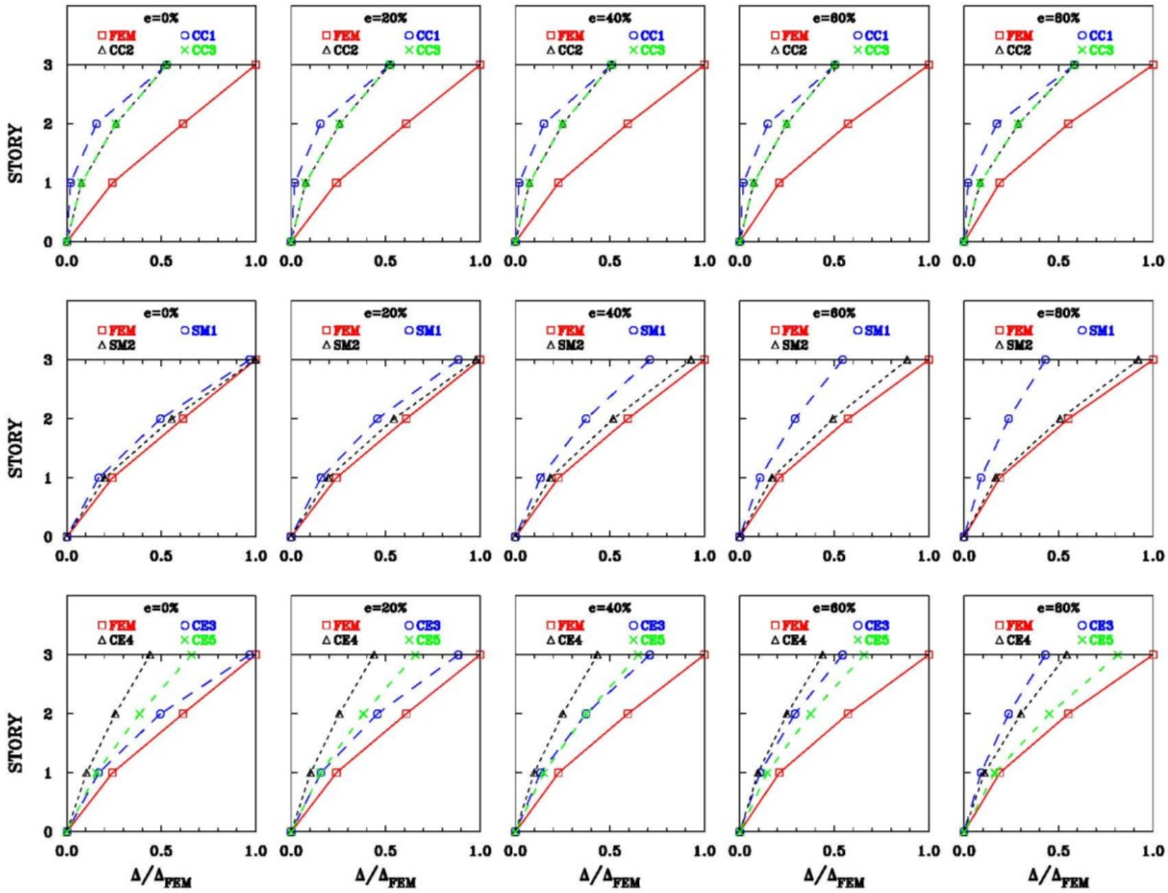


Figure 16. Comparison of normalized average lateral displacement profiles (approximate methods vs FEM) for the 3-story walls with assymetric openings under study

It is observed from Figures 16 to 21 that the original proposal of Schwaighofer and Microys for the equivalent frame method (SM1) is excellent for walls with a single symmetric opening along the height (e=0%), as average lateral displacements are just slightly underestimated or overestimated. It is observed from Figure 16 that for the 3-story models, average lateral displacements are underestimated less than 5% for model SM1 when e=0%; whereas from Figures 17 to 21, an overestimation around 10% is observed for model SM1 for walls from 6 to 18 stories when e=0%. In fact, approximations are still good enough for e=20% and perhaps for e=40%. However, it is also observed from Figures 16 to 21 that approximations under SM1 modeling are notably reduced as the opening eccentricity parameter e increases. This occurs because as the opening eccentricity parameter e increases, the e/f relationship proposed by Schwaighofer and Microys (Figure 1) also increases. Therefore, the proposed constants K_1 and particularly K_2 also increases (Table 1, Eqs.3 and 4) and, as consequence, the equivalent moment of inertia considerably augments (Eq. 2) and the resulting system is much more stiffer. From the observation of Table 1 and equations 1 to 4, it is clear that an increment in the e/f ratio increases more rapidly the bending stiffness (Eqs. 2 and 4) than the shear stiffness (Eqs. 1 and 3).

It is worth noting from Figures 16 to 21 that the equivalent frame method version which it is employed in Mexican design practice (SM2) has a good approximation for all the wall heights and opening eccentricity parameters under study. Nevertheless, it is also observed that the degree of approximation diminishes as the opening eccentricity parameter e increases. Nevertheless, it is also worth noting that in this research we considered only walls with a geometrically well-defined, single opening line along the height of wall, which is the key assumption for the method (Figure 2). It was shown before that such degree of approximation cannot be obtained for more complex opening geometries, even defining an equivalent regular opening.

Based upon the reported results, it can be concluded that the equivalent frame method version which is widely used in Mexican design practice (SM2) lead to better approximation than the original version proposed by Schwaighofer and Microys (SM1) for walls with a geometrical well-defined single line of openings along the height, particularly when the opening eccentricity parameter is 40% or greater (e≥40%). For e≤20%, both options (SM1 and SM2) lead to reasonable approximations.

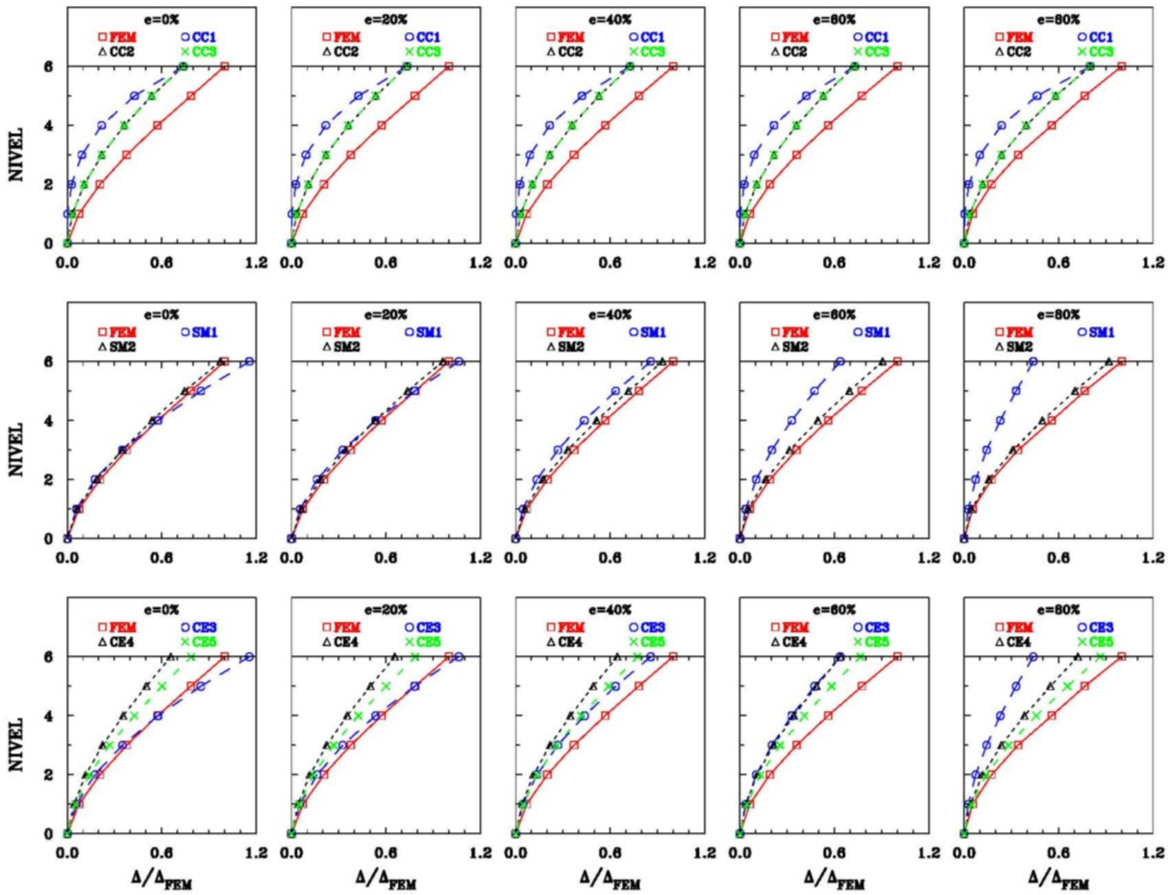


Figure 17. Comparison of normalized average lateral displacement profiles (approximate methods vs FEM) for the 6-story walls with assymetric openings under study

With respect to the equivalent non-prismatic wide column analogy, it is observed from Figures 16 to 21 that the proposed CE3 modeling lead to similar profiles to the equivalent frame method version originally proposed by Schwaighofer and Microys (SM1). Therefore, good approximations are obtained when openings are symmetric (e=0%) or the asymmetry is not very large (e=20%), whereas average lateral displacements are considerably underestimated as the opening eccentricity parameter e increases, as the equivalent moment of inertia increases cubically.

In order to try to improve the approximations for the proposed equivalent non-prismatic wide column analogy, other two options were evaluated. It was observed that although the CE1 model might be the easiest to apply, approximations were not good enough as it leads to very stiff approximations (Figure 14). Then, a simple improvement for the CE1 model was evaluated to take into account the opening eccentricity parameter e, so basically the equivalent moment of inertia (Eq. 15) is multiplied by $(1 - e^2)$ and this is identified as model CE4 in Figures 16 to 21. Finally, another simple expression was evaluated to correct the underestimation of displacements, particularly for low-rise walls, by multiplying CE4 models by $1/(1-1/N)$, where N is the number of stories. These models are identified as CE5 models in Figures 16 to 21.

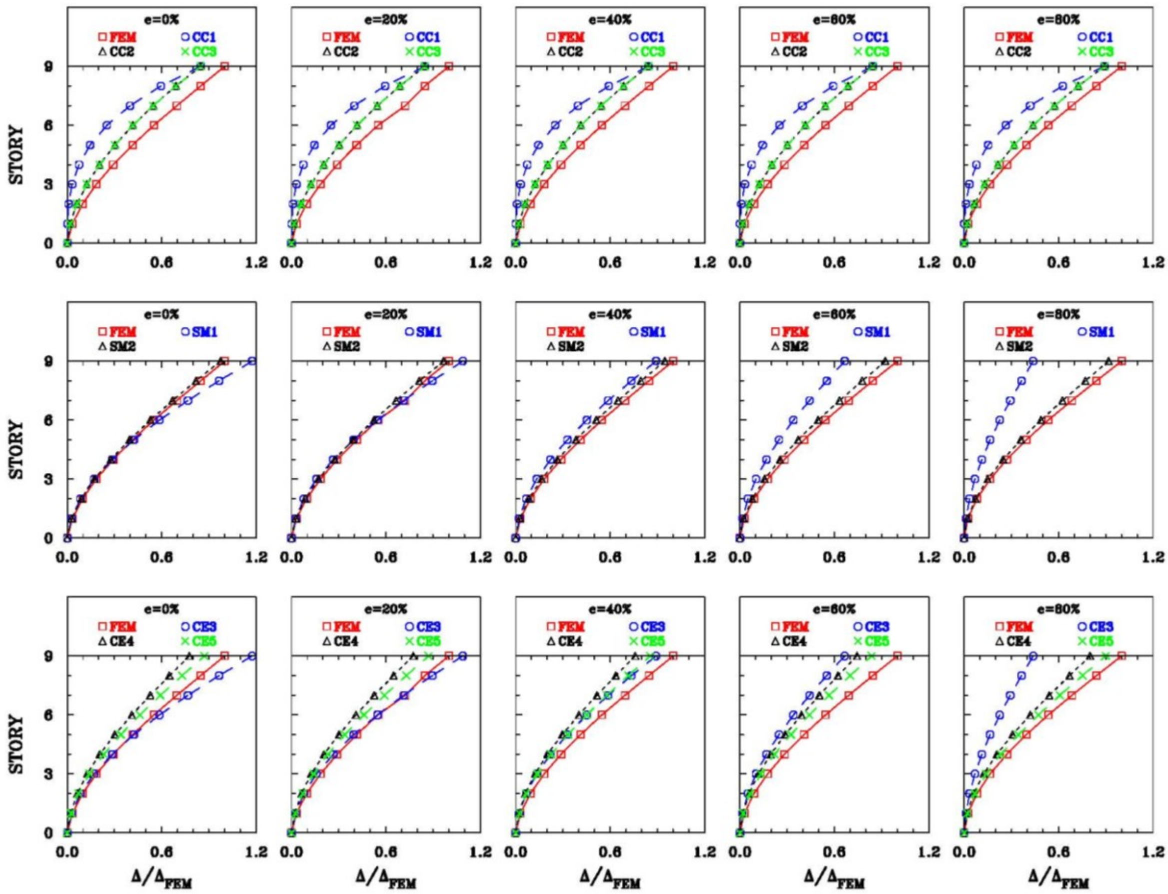


Figure 18. Comparison of normalized average lateral displacement profiles (approximate methods vs FEM) for the 9-story walls with assymetric openings under study

It is observed from Figures 16 to 19 that the proposed correction for eccentricity (models CE4) does not have an important impact on improving the obtained approximations for the simplified methods, independently of the number of stories and the opening eccentricity parameter e, that is, no improvement was observed in the approximations obtained when e=0% with respect to e=80%, except for 3-story models (Figure 16). However, when the opening eccentricity parameter and the proposed flexibility correction in terms of the number of stories are simultaneously accounted for (models CE5), the obtained approximations substantially improved. As expected, improvements were much more notorious for lowrise walls (Figures 16 to 17), where imprecisions were higher when only the correction for the opening eccentricity was accounted for (models CE4).

Based upon the described results, it is concluded that under the equivalent non-prismatic wide column analogy, still the most promising method is the CE3 modeling. However, as under such modeling, the lateral stiffness is considerably overestimated for lowrise walls (H/L<2), therefore, it seems reasonable to amplify the obtained results in terms of the number of stories and/or, more correctly, in terms of the global slenderness ratio for the wall. Therefore, for future studies, it is proposed to correct the approximations obtained with CE3 modeling using the following global slenderness correction factor F_{sld}:

$$F_{sld} = \frac{1}{1-\frac{1}{N}} = \frac{1}{1-\frac{0.47}{\frac{H}{L}}} \quad if \quad N>1 \quad or \quad \frac{H}{L}>0.47 \tag{25}$$

Also, a correction for the opening eccentricity parameter e is also important to consider, so the following opening eccentricity correction factor F_{ecc} should be used:

$$F_{ecc} = 1-e^2 \tag{26}$$

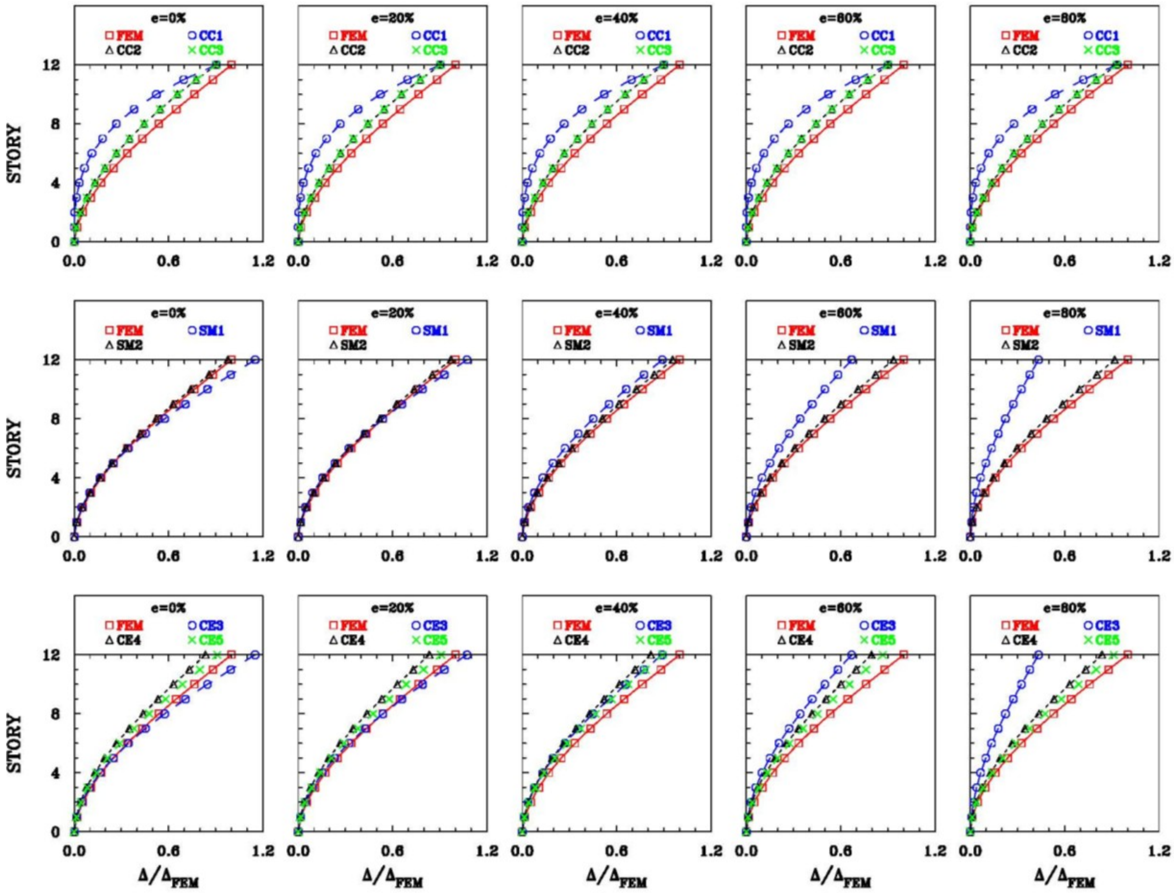


Figure 19. Comparison of normalized average lateral displacement profiles (approximate methods vs FEM) for the 12-story walls with assymetric openings under study

6 CONCLUDING REMARKS

The approximations obtained with different simplified methods to assess the lateral stiffness of walls with openings were compared to results obtained with the finite element method using reasonably fine meshes. Multistory walls were studied, considering symmetric and asymmetric distribution of openings with respect to a vertical axis.

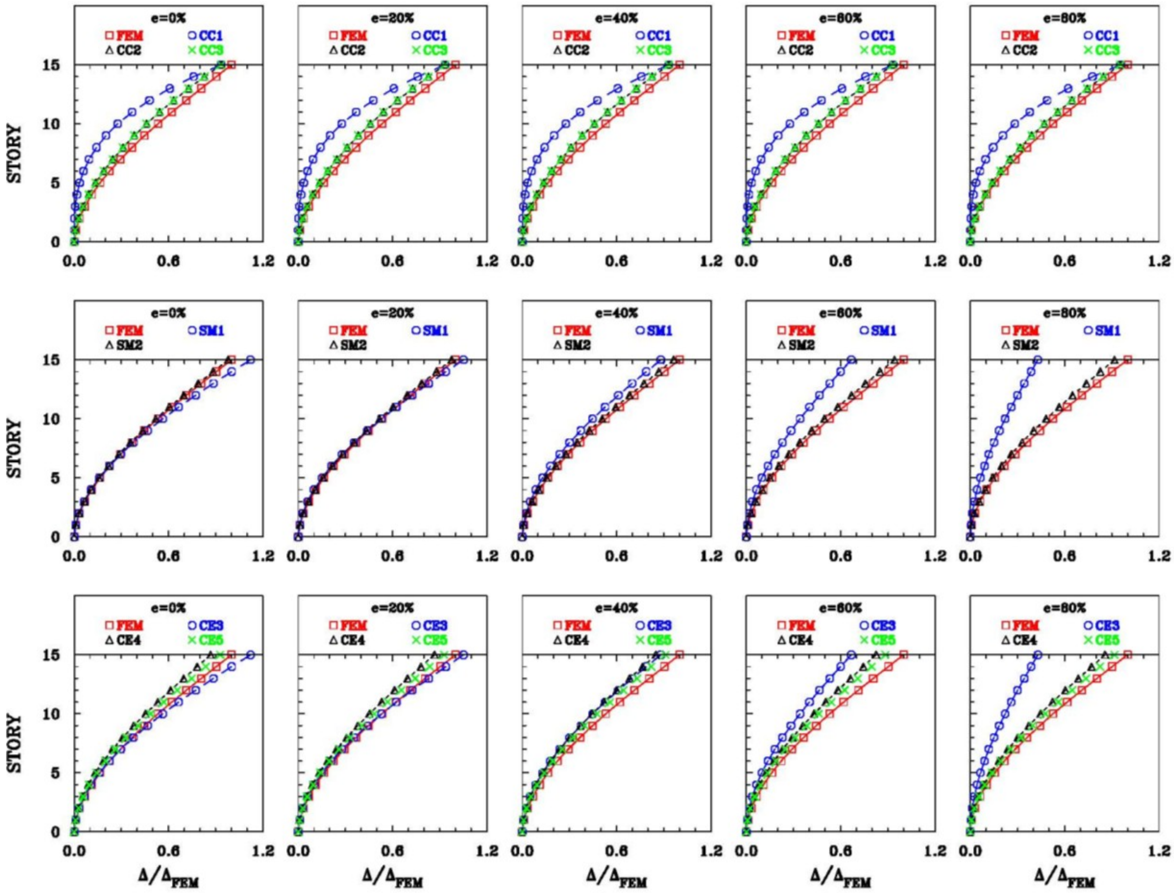


Figure 20. Comparison of normalized average lateral displacement profiles (approximate methods vs FEM) for the 3-story walls with assymetric openings under study

From the obtained results, the following observations can be done:

1. Approximations obtained with the method proposed by Coull and Choudhury depend on the global wall slenderness ratio H/B (wall total height divided by its width). Approximations are poor (around 50%) for relatively stocky walls ($H/L<2$) when compared to finite element results, even for walls with symmetric openings (e=0%). However, the accuracy improves as H/L increases, being higher than 80% for slender walls ($H/L<4$).
2. It was not observed an important reduction on the degree of approximation obtained with the method proposed by Coull and Choudhury when the opening eccentricity parameter e increases, that is, approximation obtained for e=20%, 40%, 60% and 80% were similar to those obtained for e=0%, for each elevation under consideration.
3. As expected, approximations obtained with the method proposed by Coull and Choudhury for multi-story, multi-perforated walls was much smaller when compared to those obtained for walls with a well-defined single opening distribution along the height, given both the original hypothesis for the method and the simple hypothesis explored to define an equivalent single opening.
4. The method proposed by Coull and Choudhury could be improved using the proposed expression (Eq. 13) to estimate how displacements vary along the height (CC2 modeling).
5. It can be recommended to use the method proposed by Coull and Choudhury when $H/L\geq4$ and for any value of the opening eccentricity parameter e, for walls with a well-

defined single layout of openings in elevation, as estimations of average lateral displacements are near or greater 80% when compared to FEM solutions.

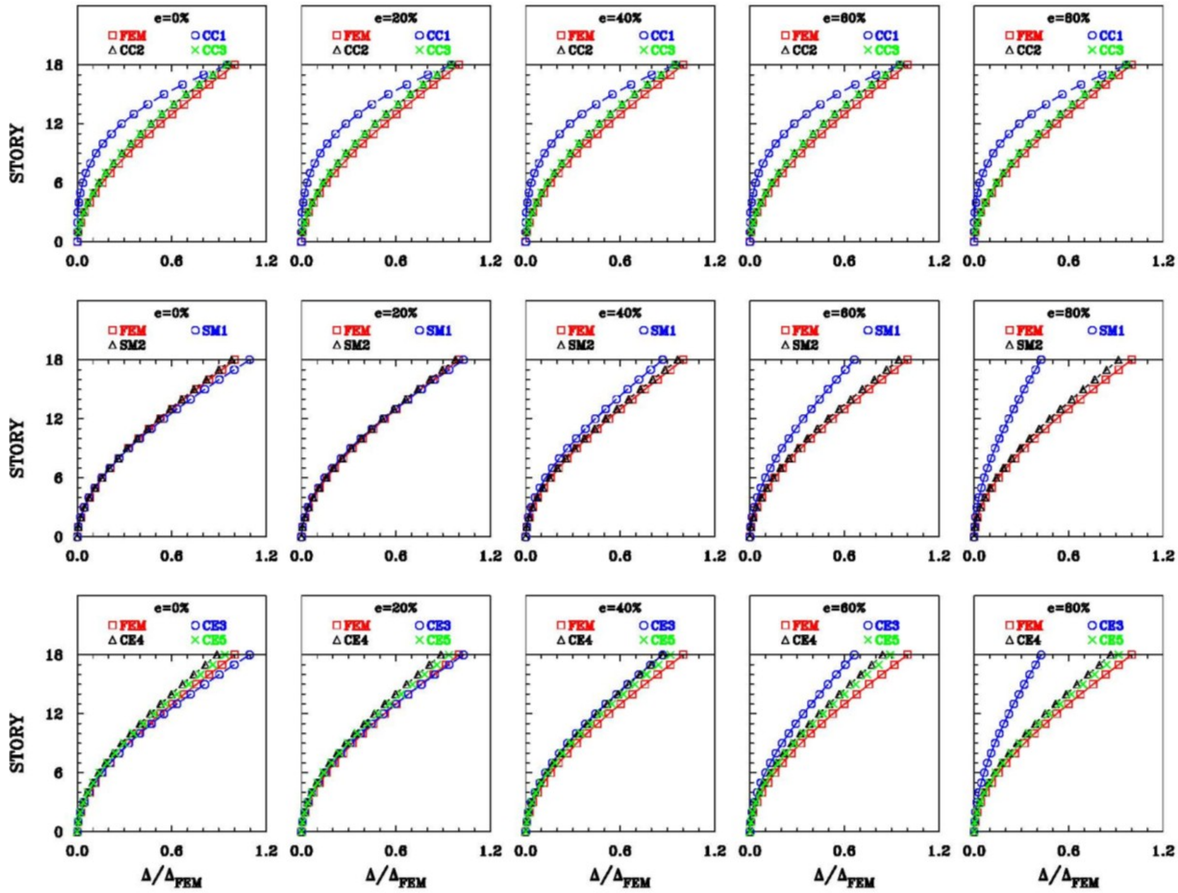


Figure 21. Comparison of normalized average lateral displacement profiles (approximate methods vs FEM) for the 3-story walls with assymetric openings under study

6. For walls with a well-defined single layout of openings in elevation, approximations obtained with the equivalent frame method as originally proposed by Schwaighofer and Microys (models SM1) can be considered exact for practical purposes. The accuracy increases as H/L increases when the opening is completely symmetric (e=0%), or the opening eccentricity parameter is not very large (e≤20%). However, the method notably looses accuracy when the opening eccentricity parameter is large (e>40%) or for complex multi-story, multi-perforated walls.
7. For walls with a well-defined single layout of openings in elevation, approximations obtained with the equivalent frame method most widely used in Mexican design practice (models SM2) can be considered exact for practical purposes, for all the wall heights and opening eccentricity parameters under study. The accuracy increases as H/L increases, but the degree of approximation diminishes as the opening eccentricity e parameter increases. As expected, the method notably looses accuracy for complex multi-story, multi-perforated walls, because key assumptions for the method do not consider such geometries.
8. From the results obtained in this study, it can be recommended to use the equivalent frame method most widely used in Mexican design practice (SM2) when H/L≥1.4 and for any value of the opening eccentricity parameter e, for walls with a well-defined single layout of openings in elevation. The original proposal of Schwaighofer and

Microys can also be used with confidence when $H/L \geq 1.4$ and $e \leq 20\%$, but its reliability diminished when $e > 40\%$.

9. The proposed equivalent non-prismatic wide column analogy lead to reasonable approximations when it is assumed that the flexibility in the opening area is very important, so each wall section responds separately and the effective cross section properties at that section are the sum of the properties of the individual solid segments (model CE3). Similarly to what it was observed in the equivalent frame method as originally proposed by Schwaighofer and Microys (models SM1), for walls with a well-defined single layout of openings in elevation, approximations under CE3 modeling increases as H/L increases, when openings are completely symmetric (e=0%) or the opening eccentricity parameter is not very large ($e \leq 20\%$). However, the method notably looses accuracy when the opening eccentricity parameter is large ($e > 40\%$).

It can be concluded that all the evaluated approximate methods (Coull and Choudhury, equivalent frame method and the equivalent non-prismatic wide column analogy) lead to reasonable approximations for walls with a well-defined single layout of openings in elevation when the perforated walls: a) are slender $H/L \geq 4$), b) have symmetric openings or the opening eccentricity parameter is not large ($e \leq 20\%$) and, c) openings are reasonably large (opening size parameter $\lambda > 18\%$). With the exception of the version of the equivalent frame method most widely used in Mexican design practice (SM2), the accuracy of these methods is reduced as the opening eccentricity parameter (e) increases, particularly for relatively stocky walls ($H/L < 2$) when $e \geq 40\%$.

All evaluated approximate methods looses accuracy for multi-story, multi-perforated walls, particularly if opening patterns are complex along and across the wall, even with the definition of a single equivalent opening. Therefore, in order to try to expand the applicability of these simple methods, it seems important to keep exploring simple expressions to account for the following parameters: a) the impact of the opening eccentricity parameter e in the amplification of average lateral displacements, b) explore other simple forms to obtain an equivalent opening for multi-perforated walls that may improve approximations and, c) the impact of the stiffness of different floors systems on the lateral displacement profiles of walls with openings. Such parameters should be considered in future research studies, where multi-perforated wall opening patterns commonly used in urban buildings may also be evaluated.

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