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DEM & FEM/DEM MODELS FOR LATERALLY LOADED MASONRY WALLS

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Abstract. *The wide amount of historic masonry constructions in Italy and other European countries makes of paramount importance the development of reliable tools for the evaluation of their structural safety. Masonry is a heterogeneous structural material obtained by composition of blocks connected by dry or mortar joints. The use of refined models for investigating the in-plane nonlinear behavior of periodic brickwork is an active field of research. The mechanical properties of joints are usually considerably lower than those of blocks, therefore it can be assumed that damages occur more frequently along joints. Thus, a key aspect is represented by the evaluation of the effective behavior of joints and its reliable description into numerical models. With this purpose, in this contribution, different models are defined to simulate, with an appropriate accuracy, the behavior of masonry: Discrete Element Model (DEM) and a combined Finite Element and Discrete Element Model (FEM/DEM). Models are based on rigid block hypothesis and joints modeled as Mohr-Coulomb interfaces. These assumptions may be suitable for historical masonry, in which block stiffness is larger than joint stiffness, allowing to assume blocks as rigid bodies, moreover joint thickness is negligible if compared with block size. Analysis is performed in the nonlinear field to investigate the behavior of masonry walls subject to lateral loads, in order to simulate their seismic response, with particular attention to the determination of limit load multipliers.*

1 INTRODUCTION

Masonry is an heterogeneous structural material obtained by the juxtaposition of natural or artificial bricks joined by mortar layers. In the last years several models have been developed based on the micromechanical analysis of the masonry. In the literature [1] different constitutive masonry models have been proposed according to some assumptions: arrangement of the masonry; model of the bricks; models of the mortar; macroscopic model obtained by homogenization or other identification procedures. In this contribution, masonry panels subject to in plane actions are analyzed by means of both FEM/DEM and DEM models, taking into account their nonlinear behavior.

The FEM/DEM i.e. the combination of discrete elements with finite elements allows studying both linear and non-linear masonry behavior. The model is here based on the assumption of both rigid and elastic block and mortar joints modeled as zero thickness linear elastic and Mohr-Coulomb interfaces. Blocks are modeled by means of finite elements while interfaces are modeled as discrete elements. The FEM/DEM analysis is carried out by using an open source code, Y2D, developed by prof. A. Munjiza [2] and implemented by Toronto Group [3,4], who is one of the pioneers of this coupling technique.

Instead in the DEM, a discrete model is considered: the model is based on the assumption of rigid block and mortar joints modeled as zero thickness elastic-plastic interfaces, adopting a Mohr-Coulomb yield criterion. Hence masonry is seen as a “skeleton” [5] in which the interactions between the rigid blocks are represented by forces and moments depending on their relative displacements and rotations and that are limited by the yield criterion chosen. The analysis is performed in this case by extending to more general interfaces an ad-hoc code originally developed by the authors [6,7].

Both models considered fall in the field of discrete or distinct element methods, introduced by the research group of Lemos [8,9] and frequently adopted in the field of rock mechanics.

Recently, a comparison between FEM/DEM and DEM has been carried out for studying masonry linear behavior [10]; in this contribution, such comparison is extended to the nonlinear field. Masonry panels having different height-to-width ratios, subject to self-weight and lateral forces, are considered and incremental analyses are performed for determining the limit load multiplier of lateral forces with respect to self-weight and the corresponding collapse mechanism.

2 GEOMETRIC MODEL

The blocks which form the masonry wall are modeled as rigid bodies connected by interfaces (mortar or dry thin joints). A standard running bond periodic masonry is considered, with block dimensions *a* (height), *b* (width) and *t* (thickness); hence each block has six neighboring blocks (Figure 1).

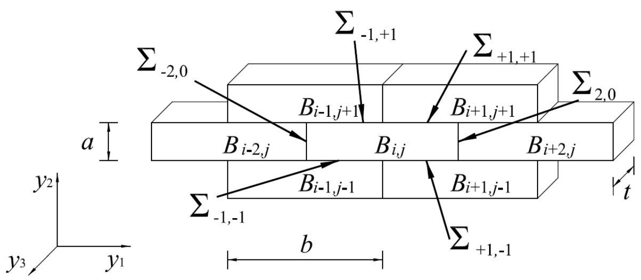

Figure 1: Geometric model adopted.

Let $\mathbf{y}^{i,j}$ denote the position of the center of the generic $B_{i,j}$ block (see Fig. 1), in the Euclidean space; it should be noticed that j can actually take arbitrary values while i is such that i+j is always an even number. Each block exhibits a rigid body motion given by:

$$\mathbf{u}^{i,j}(\mathbf{y}) = \mathbf{u}^{i,j} + \mathbf{\Omega}^{i,j}(\mathbf{y} - \mathbf{y}^{i,j}) \tag{1}$$

where $\mathbf{u}^{i,j}$ is the translation vector of the block characterized by two components and $\mathbf{\Omega}^{i,j}$ is its rotation skew tensor, characterized by one component $\omega_3^{i,j}$, representing block rotation with respect to its center. Due to the regularity of the structure, each block is surrounded by six blocks by means of six interfaces or joints Σ_{k_1,k_2}, with k_1, k_2 = ± 1 for horizontal interfaces and k_1 = ± 2, k_2 = 0, for vertical interfaces. For example, the interfaces of the $B_{0,0}$ block are:

$$\begin{aligned}
&\Sigma_{-1,-1}, \Sigma_{-1,+1} = \{-b/2 \le y_1 \le 0,\ y_2 = \pm a/2, -s/2 \le y_3 \le s/2\},\\
&\Sigma_{+1,+1}, \Sigma_{+1,-1} = \{0 \le y_1 \le b/2,\ y_2 = \pm a/2, -s/2 \le y_3 \le s/2\},\\
&\Sigma_{2,0}, \Sigma_{-2,0} = \{y_1 = \pm b/2, -a/2 \le y_2 \le a/2, -s/2 \le y_3 \le s/2\}.
\end{aligned} \tag{2}$$

3 INTERFACE CONSTITUTIVE MODEL

Interfaces between blocks are modeled following an elastoplastic costitutive law, based on a Mohr-Coulomb yield criterion.

3.1 Elastic interface and isotropic joint

If the mortar joint is modeled as an elastic interface [11] the deformation between two blocks may be written as a function of the displacement jump across the interface [[**u**]]. The constitutive prescription for the contact is a linear relation between the tractions on the block surfaces and the jump of the displacement field: $\boldsymbol{\sigma}\mathbf{n} = \mathbf{K}[[\mathbf{u}]]$ on Σ_{k_1,k_2}, where $\boldsymbol{\sigma}$ is the stress tensor, **n** is the unit normal to the interface and the stiffness matrix of the interface, **K**, is given by:

$$\mathbf{K} = \frac{1}{e}\left[\frac{E^M}{2(1+\nu^M)}\left(\mathbf{I} + \frac{1}{(1-2\nu^M)}(\mathbf{n}\otimes\mathbf{n})\right)\right] \tag{3}$$

where E^M and ν^M are the Young's modulus and the Poisson's ratio of mortar, and **I** is the identity tensor. Note that tensor **K** has, in this case, a diagonal form.

3.2 Interface with Mohr-Coulomb response

If the mortar joint is modeled as a Mohr-Coulomb interface, the yield criterion depends on cohesion $c \ge 0$ and friction angle $0 < \phi < \pi/2$. Adopting a statically admissible approach, the interfacial failure condition can be expressed as

$$f(\sigma_n, \sigma_t) = |\sigma_t| + \sigma_n \tan\phi - c \le 0, \tag{4}$$

where σ_n and σ_t denote the normal and shear component of the stress vector acting on the interface Σ. Adopting a kinematically admissible approach, for any point along the interface, the Mohr-Coulomb yield criterion is expressed by

$$\pi(\mathbf{n}, [[\dot{\mathbf{u}}]]) = \begin{cases} c \cdot \mathrm{ctg}\phi\, [[\dot{\mathbf{u}}]]\cdot\mathbf{n} & \text{if } [[\dot{\mathbf{u}}]]\cdot\mathbf{n} \ge |[[\dot{\mathbf{u}}]]|\sin\phi \\ +\infty & \text{otherwise} \end{cases} \tag{5}$$

where $[[\dot{\mathbf{u}}]]$ denotes the velocity jump across the interface Σ when following the normal **n** to the Σ interface. The first case may be also expressed as $u^{\perp} \geq u^{\parallel} \cdot \tan\phi$.

4 DISCRETE MODEL

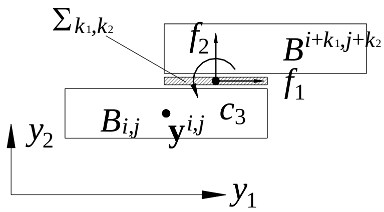

Figure 2: Interface between adjacent blocks and interfacial actions.

The discrete model presented here is based on the implementation of a numerical method already formulated in the case of regular periodic masonry [6]. The interactions between the blocks $B_{i,j}$ and $B_{i+k_1,j+k_2}$ through the interface are represented by unknown distribution of actions $\mathbf{f} = \{ f_1^{k_1,k_2} \quad f_2^{k_1,k_2} \quad c_3^{k_1,k_2} \}^T$ (Fig. 2), that are related to the relative displacements and rotations between adjacent blocks by means of the constitutive relation is $\mathbf{f} = \mathbf{K}[[\mathbf{u}]]$, where $[[\mathbf{u}]] = \{ d_1^{k_1,k_2} \quad d_2^{k_1,k_2} \quad d_3^{k_1,k_2} \}^T$ collects the relative displacements or displacement jumps. The degrees of freedom of block $B_{i,j}$ are globally denoted by $\mathbf{q}^{i,j} = \{ u_1^{i,j}, u_2^{i,j}, \omega_3^{i,j} \}^T$ and **q** collects the degrees of freedom of the entire masonry panel.

The equation of motion of the discrete system is:

$$\mathbf{M}(\partial^2 \mathbf{q} / \partial t^2) + \mathbf{F}_{int} = \mathbf{F}_{ext}. \tag{6}$$

Where $\mathbf{F}_{ext}$ is the vector of the applied in plane actions, **M** is the (diagonal) mass matrix of the panel collecting block mass and polar inertia and $\mathbf{F}_{int}$ is the vector of internal forces. Such vector is equal to $\mathbf{K}_{panel}(\mathbf{q} + \mu \partial \mathbf{q} / dt)$ if the masonry assemblage lies in the elastic field, where $\mathbf{K}_{panel}$ is the in plane stiffness matrix and μ is the damping coefficient (neglected for simplicity). The equilibrium equation above may be solved adopting a molecular dynamics solution method [14,15]. Starting from an external load $\mathbf{F}_{ext}$ characterized by forces and/or couples applied to block centers, the equation of motion may be solved adopting a the predictor-corrector algorithm GEAR of order 2 [6] without determining explicitly panel matrices. At time *t*, for a given increment $\{\delta d_n \quad \delta d_t \quad \delta d_3\}^T$ in the time increment δt, with $n = 1$, $t = 2$ for a vertical interface or $n = 2$, $t = 1$ for a horizontal one, the new interfacial actions are computed by evaluating for first the elastic contribution:

$$\begin{aligned} f_n^{el} &= f_n(t) + K_n \, \delta d_n, \\ f_t^{el} &= f_t(t) + K_t \, \delta d_t, \; . \\ c_3^{el} &= c_3(t) + K_m \, \delta d_3. \end{aligned} \tag{7}$$

Where $\{K_n \quad K_t \quad K_m\}$ are normal, tangential and rotational stiffness of the interface. The elastic guess is correct and $\{ f_n(t+\delta t) \quad f_t(t+\delta t) \quad c_3(t+\delta t)\}^T = \{ f_n^{el} \quad f_t^{el} \quad c_3^{el} \}^T$ if interfacial forces $\{ f_n^{el} \quad f_t^{el} \quad c_3^{el} \}^T$ satisfy the following Mohr-Coulomb conditions, that represent, respectively, the detachment, sliding and rotational failure modes:

$$\begin{aligned} f_n &= \mathbf{f} \cdot \mathbf{e}_{\perp} \leq f_c, \\ | f_t | &= | \mathbf{f} \cdot \mathbf{e}_{\parallel} | \leq (f_t - f_n) \tan \phi, \\ | c_3 | &\leq (f_c - f_n) l_c. \end{aligned} \tag{8}$$

where $f_c = c\, S/\tan\phi$ is the tensile strength of the interface, with $S = S_v = at$ or $S_h = bt/2$ and l_c is the characteristic length of the interface, represented by the distance of the interfacial normal force with respect to block center.

If $f_n^{el} > f_c$, then $\{ f_n(t+\delta t) \quad f_t(t+\delta t) \quad c_3(t+\delta t) \}^T = \{ f_c \quad 0 \quad 0 \}^T$, otherwise the normal projection according to the tangential force criterion is done:

$$\begin{aligned} & f_n' = f_n^{el} - \lambda_t K_n \tan\varphi\, d_n, \\ & f_t(t+\delta t) = f_t^{el} - [sign(f_t^{el})] \lambda_t K_t d_t, \\ & \lambda_t = [| f_t^{el} | + (f_n^{el} - f_c) \tan\phi] / [K_t + K_n \tan^2\phi] \end{aligned} \tag{9}$$

Then $\{ f_n' \quad f_t(t+\delta t) \quad m^{el} \}^T$ is projected according to the moment criterion obtaining:

$$\begin{aligned} & f_n(t+\delta t) = f_n' - \lambda_m K_n l_c, \\ & c_3(t+\delta t) = c_3^{el} - [sign(c_3^{el})] \lambda_m K_m, \\ & \lambda_m = [| c_3^{el} | + (f_n' - f_c) l_c] / [K_m + K_n l_c^2]. \end{aligned} \tag{10}$$

Eq. 6 may be also solved by determining explicitly the stiffness matrix of the entire panel and solving as usual the system of equations: $\mathbf{q} = (\mathbf{K}_{panel})^{-1} \mathbf{F}_{ext}$, moreover panel stiffness matrix may be updated taking into account Mohr-Coulomb yield criterion. Panel stiffness matrix was determined in [16] for performing modal analysis of masonry structures and further details may be found in [17].

5 FEM/DEM MODEL

In the early 1990s FEM and DEM have been combined and the resulting method was termed the combined FEM/DEM [2]. It is in essence a discrete element method with individual elements meshed into finite elements. Finite elements allow to model elastic deformation (if any), while discrete element algorithms allow to model interaction, fracture and fragmentation processes. The FEM/DEM method provides a consistent procedure to study masonry structures [10,18] thanks to the possibility of creating models made of separated blocks. In particular, these models can properly represent the behavior of historical masonry constructions, which could be considered as made of dry stone blocks exhibiting a periodic pattern. The combination of DEM and FEM provided by the open source code Y2D, developed by prof. A. Munjiza [2] and implemented by Geo Group of Toronto University [3,4] allows to further extend the study to both linear and nonlinear masonry behavior. In particular blocks can be assumed to behave (differently from DEM described above) as elastic bodies while mortar joints might be idealized as elastic or elastic-plastic zero-thickness Mohr-Coulomb interfaces. In the present case blocks have been modeled by means of finite elements while interfaces are modeled as discrete elements.

6 FEM/DEM AND DEM MODEL PROCEDURES

DEM and FEM/DEM models are different for several aspects.

The structural model of DEM consists of rigid blocks, with loads and restraints applied at block centers and joint between blocks are modeled as interfaces having translational and rotational stiffness. The parameters involved in the DEM are block geometry, joint stiffness (translational and rotational) values, that depend on mortar elastic modulus E^M, Poisson ratio and joint dimensions (thickness, area and inertia). As described in the 4th paragraph, joints are modelled as Mohr-Coulomb cohesive interfaces. If dry joints are considered, fictitious mortar stiffness values may be adopted with very low value of cohesion. DEM is characterized by a small number of degrees of freedom involved in numerical analysis, hence it requires a relatively small computational effort with respect to models based on finite elements.

The structural model of the FEM/DEM consists of a mesh of Finite Elements, made with triangular elements, hence loads and restraints are applied at nodes. The height of each element is $h = a/2$, thus each block has been meshed with 16 elements. In FEM/DEM, the interfaces are made by specific “crack elements”, dedicated four nodes cohesive elements which are embedded between the hedges of all adjacent triangular elements pairs since the beginning of simulation [19]. The potential crack path can open everywhere in the mesh, here in order to simulate the behavior of historic masonry panels, in which cracks usually occur mainly in the mortar joints [20,21], two different joints have been used: one inside the blocks and the other between adjacent blocks. The former has a very high cohesion value in order to avoid the breaking of blocks, while the latter has a very low cohesion value, in order to model dry joints. The parameters involved in the FEM/DEM are block mechanical properties and joint properties: cohesion, tensile strength, friction ratio and fracture energy. In order to emulate the behavior of rigid blocks, the adopted Young modulus is $E^B = 1000\ E^M$, while the Poisson ratio has been set $\nu^B = 0$.

In order to calibrate the two models, cohesion and friction adopted for joints are the same, whereas the fracture energy in FEM/DEM [19] joints has been evaluated on the base of mortar elastic modulus adopted in DEM. As previously stated, in the DEM, due to rigid block hypothesis, forces are applied at block centers, whereas in the FEM/DEM, forces are lumped at the inner nodes of each block subdivision (Fig. 3).

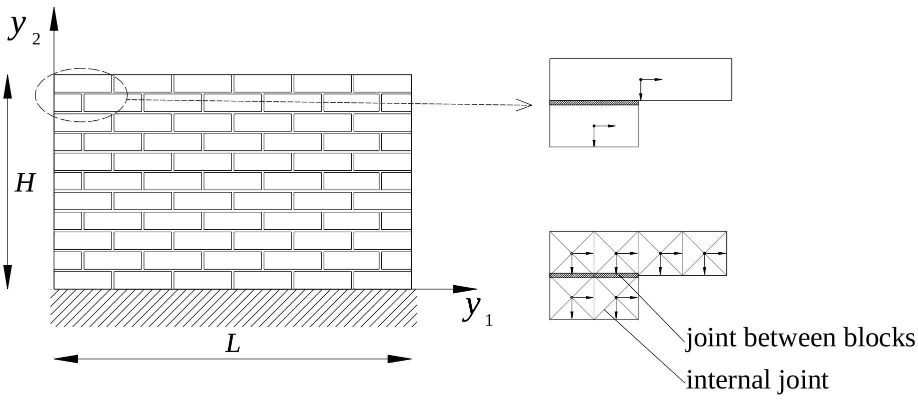

Figure 3: Generic masonry panel modelled with DEM and FEM/DEM.

7 STRUCTURAL ANALYSIS

The stability problem of a masonry panel subject to its own weight and horizontal body forces may be solved analytically by adopting a homogenization procedure based on the definition of a homogeneous material, equivalent to the heterogeneous one in its geometry and in the properties of its constituent materials [22]. The representative elementary volume (REV) adopted for the homogenization is one half of the assemblage in Fig. 1.

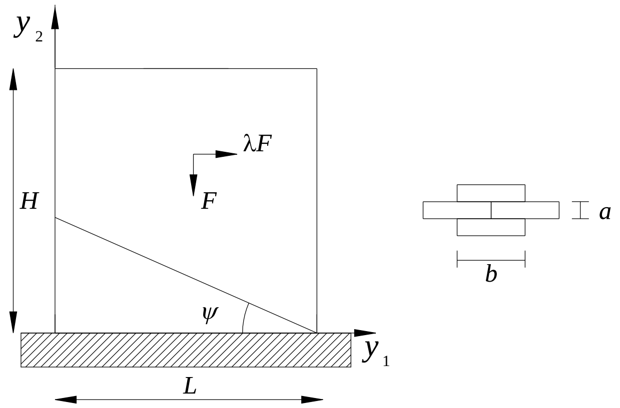

Figure 4: Homogenized masonry panel with rigid block failure and detail of the REV considered.

Considering the hypotheses of rigid blocks and Mohr-Coulomb interfaces, following De Buhan and De Felice [12], the support function $\pi(\varepsilon)$ and the yield criterion $G(\mathbf{y})$ on the REV are defined as:

$$\pi(\varepsilon) = sup\{\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}},\ \boldsymbol{\sigma} \in G(\mathbf{y})\} \quad \Leftrightarrow \quad G(\mathbf{y}) = \{\boldsymbol{\sigma} \mid \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} \le \pi(\varepsilon), \forall\ \varepsilon\}\,. \tag{11}$$

Where ε denotes a second-order strain rate tensor. The Mohr-Coulomb failure condition at the interfaces provides:

$$< \pi(\mathrm{grad}^s(\dot{\mathbf{u}}) >= \begin{cases} \dfrac{1}{ab}\displaystyle\int_\Sigma \left(\dfrac{c}{tan\phi}\right)[[\dot{\mathbf{u}}]]\cdot\mathbf{n} & \text{if } [[\dot{\mathbf{u}}]]\cdot\mathbf{n} \ge |\,[[\dot{\mathbf{u}}]]\,|\sin\phi \\ +\infty & \text{otherwise} \end{cases}. \tag{12}$$

Details of the macroscopic yield criterion may be found in the works of De Buhan and De Felice, Cecchi and Vanin [12,13].

Then, a homogeneous rectangular panel width L, height H and thickness t is considered (Fig. 4). Vertical forces depend on the specific weight of the panel and horizontal forces are denoted by the multiplier λ that is gradually increased. An upper-bound estimate of the ultimate value of λ, corresponding to the collapse of the panel, is obtained by applying a yield design kinematic approach to the homogenized structure [12]. Following the procedure adopted by Cecchi and Vanin [13], at the collapse, when joints exhibit only frictional strength properties, the free mechanism condition is estimated through the collapse multiplier λ, that is a function of $m = 2a/b$, ϕ and $r = H/b$. In case of shear failure the collapse multiplier is:

$$\lambda = \tan\phi \tag{13}$$

In the case of flexural failure, a rigid block mechanism is considered in which the part of the panel above the line having inclination ψ is given a virtual rigid body motion. Then, a relation between the rotation angle ψ in the homogenized panel, representing the highest slope of the fracture line, and ϕ in the elementary cell is established by equating the power spent in the macroscopic homogenized panel and in microscopic structure:

$$\tan\psi \le \left(\frac{m}{\tan\phi}\right)^{-1/2} \tag{14}$$

Therefore the flexural failure depends both on the mechanical properties and the geometry of the micro-structure and on the geometry of panel and collapse multiplier is given by:

$$\lambda = \begin{cases} \dfrac{1}{2}\left(\dfrac{m}{\tan\phi}\right)^{-1/2} & \text{if } r = \dfrac{H}{L} \le \left(\dfrac{m}{\tan\phi}\right)^{1/2} \\ \dfrac{3r - 2\left(\dfrac{m}{\tan\phi}\right)^{-1/2}}{3r^2 - \left(\dfrac{m}{\tan\phi}\right)} & \text{otherwise} \end{cases} \tag{15}$$

In particular the first expression of Eq. 15 represents the load multiplier for a local crisis characterized by a flexural failure depending only on m and ϕ parameters, while the second expression of Eq. 15 represents the load multiplier for a global crisis of the panel (flexural failure depends not only on the m and ϕ parameters, but also on the $r = H/b$ ratio).

The values of λ provided by the equations 13 and 15 are adopted as reference solutions for the analyses performed with DEM and FEM/DEM.

8 NUMERICAL TESTS

A set of base supported panels having height H, base L and thickness t is considered. Block dimensions are: b = 240 *mm*, a = 60 *mm*, t = 120 *mm*, 6 blocks along panel width are considered and three different panel height-to-width ratios are taken into account (H/L equal to 0.5, 1.0 and 2.0, Fig. 5). As stated in the 6th paragraph, negligible cohesion c is considered, whereas a friction ratio $\tan\phi$ = 0.6 is assumed, corresponding to a friction angle of about 30°.

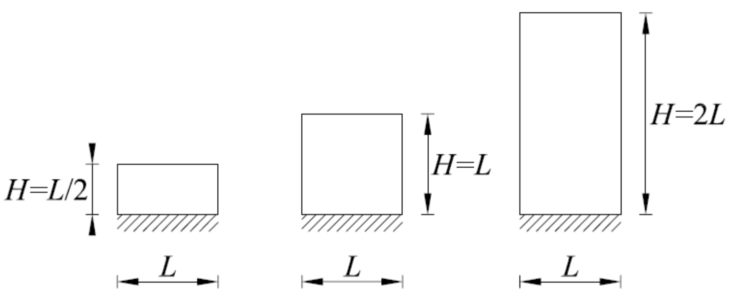

Figure 5: Case studies considered.

Each panel is subject to a uniform vertical load representing its self-weight and to a horizontal increasing force representing a lateral acceleration statically applied. For representing this loading condition in the DEM, due to rigid block hypothesis, each block is subject to a vertical force $F_{2,i}$ and a horizontal one $F_{1,i} = \lambda F_{2,i}$, whereas in the FEM/DEM, horizontal and vertical forces are lumped at the inner nodes of each block subdivision (Fig. 6).

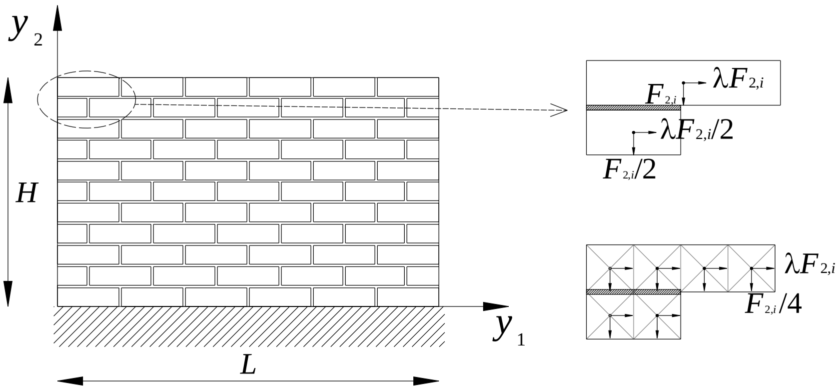

Figure 6: Generic masonry panel subject to self-weight and proportional lateral loads modelled with DEM and FEM/DEM.

Nonlinear incremental analyses of the panels considered are performed in order to determine their ultimate load multiplier (λ_{DEM} and $\lambda_{FEM/DEM}$) and the corresponding collapse mechanisms. Dry joints are taken into account, hence analytic solutions determined in the 7th paragraph are taken as reference (λ_{REF}) and evaluated for increasing *H*/*L* ratio and constant 2*a*/*b* ratio based on block dimensions (Fig. 7).

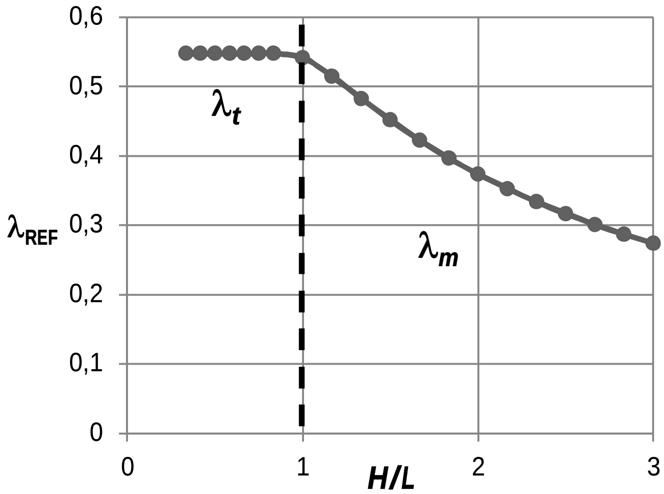

Figure 7: Ultimate load multipliers for masonry panels and increasing *H*/*L* ratio.

Fig. 8 shows load multiplier values versus displacement values for the three panels considered and obtained with DEM and FEM/DEM, together with the ultimate load multiplier obtained analytically. Displacements are evaluated at the upper-right corner of the panel. Moreover, limit load multipliers are collected in Tab. 1.

Results are in good agreement between the reference values of multipliers obtained analytically and the ones obtained by DEM and FEM/DEM. Instead some differences may be found in the values of displacements obtained by the two models, due to the different aspects of the two methods previously highlighted in the 6th paragraph. In particular, displacements in DEM are influenced by the Young modulus of joints E^M which is not taken into account in FEM/DEM. The two models have been calibrated calculating the fracture energy adopted in joints in FEM/DEM on the base of mechanical parameters adopted in DEM: Young modulus and cohesion. But, being the joints assumed as dry, so without cohesion, the behavior of joints is mainly related to friction ratio $\tan\phi$, thus the differences in the displacements obtained by the two models are sensible. In the next step of this research it will be taken into account cohesion and the differences in the displacements should decrease.

Fig. 9 shows collapse mechanisms for the three panels considered obtained with DEM and FEM/DEM.

Results show that both models are suitable to describe the global behavior of masonry walls, and in particular they are able catch the non-linear behavior of masonry panels subjected to lateral loads. Panels with a ratio $H/L<1$ collapse with a sliding mechanism, while panels with a ratio $H/L>1$ collapse with a overturning mechanism, as shown by the mechanisms obtained by the two models, which are in a good agreement. Small differences may be found in panels with a ratio $H/L=1$, for which the mechanism can be either sliding that overturning. Crack patterns obtained by the two models are very close, but DEM provides a mechanism in which the upper part of the panel exhibits a small rotation, while the mechanism obtained by FEM/DEM shows sliding in the upper part of the panel.

H/L	$\lambda_{FEM/DEM}$	λ_{DEM}	λ_{REF}
0.5	0.550	0.550	0.548
1.0	0.450	0.510	0.542
2.0	0.350	0.360	0.374

Table 1: Comparison between ultimate load multipliers.

9 CONCLUSIONS

- The DEM and FEM/DEM models represent a simple and effective tool for the study of non-linear behavior of masonry structures, in particular with reference to masonry panels subjected to lateral loads.
- The DEM and FEM/DEM models are able to take into account the real texture of masonry walls, thus they are able to describe with accuracy the real cracks pattern that may develop in masonry walls and to catch the potential mechanisms of collapse.
- The next step of the research will involve the adoption of real mechanical parameters of mortar joints, in particular with reference to cohesion, and will consider different texture and geometry of panels.
- The behavior of masonry panels will be compared with experimental results, with the purpose of the evaluation of in-plane and out-of-plane behavior of masonry walls.

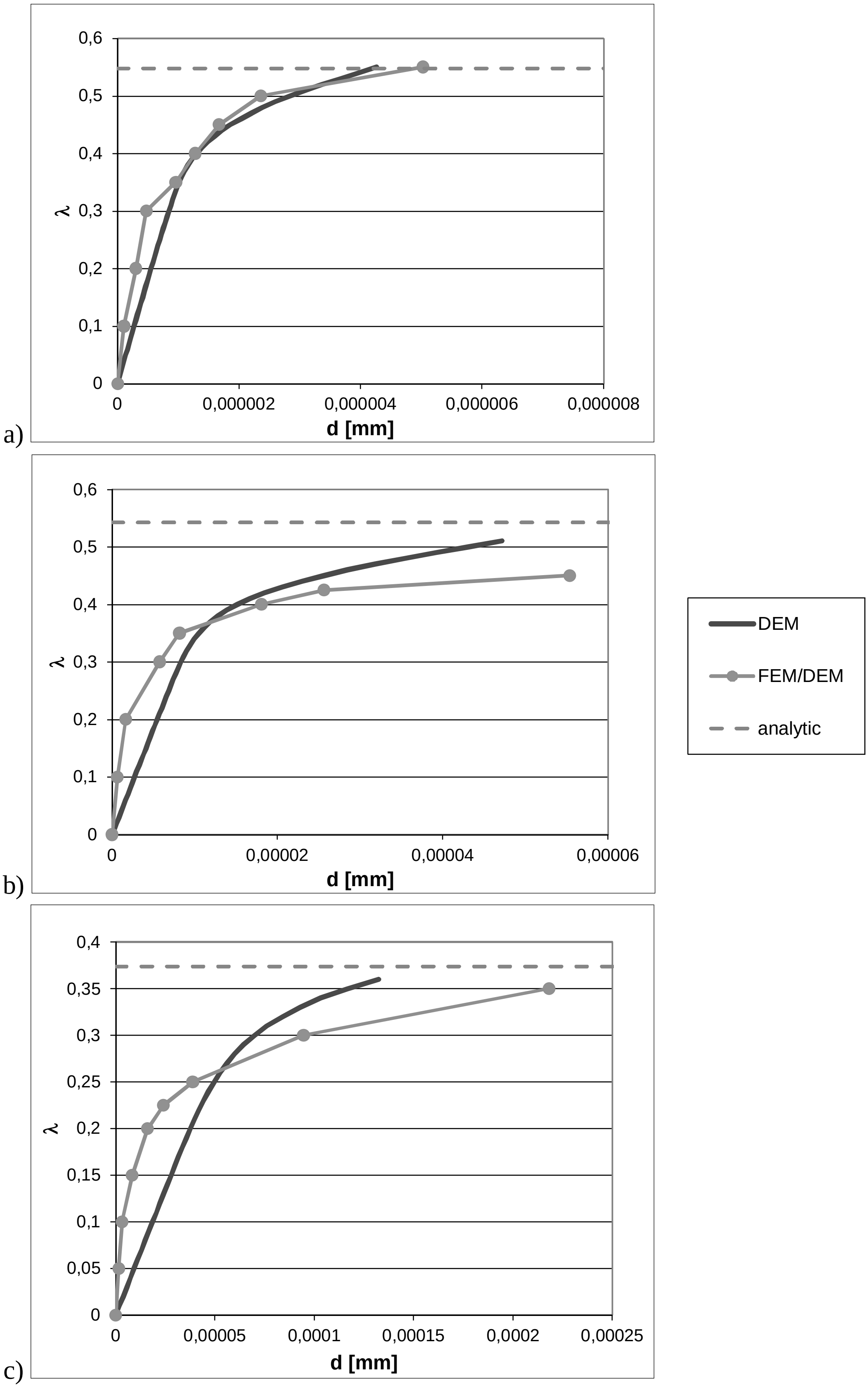

Figure 8: Incremental analyses, load multiplier vs. displacement at the upper-right corner of the panel. (a) *H*/*L* = 0.5, (b) *H*/*L* = 1, (c) *H*/*L* = 2.

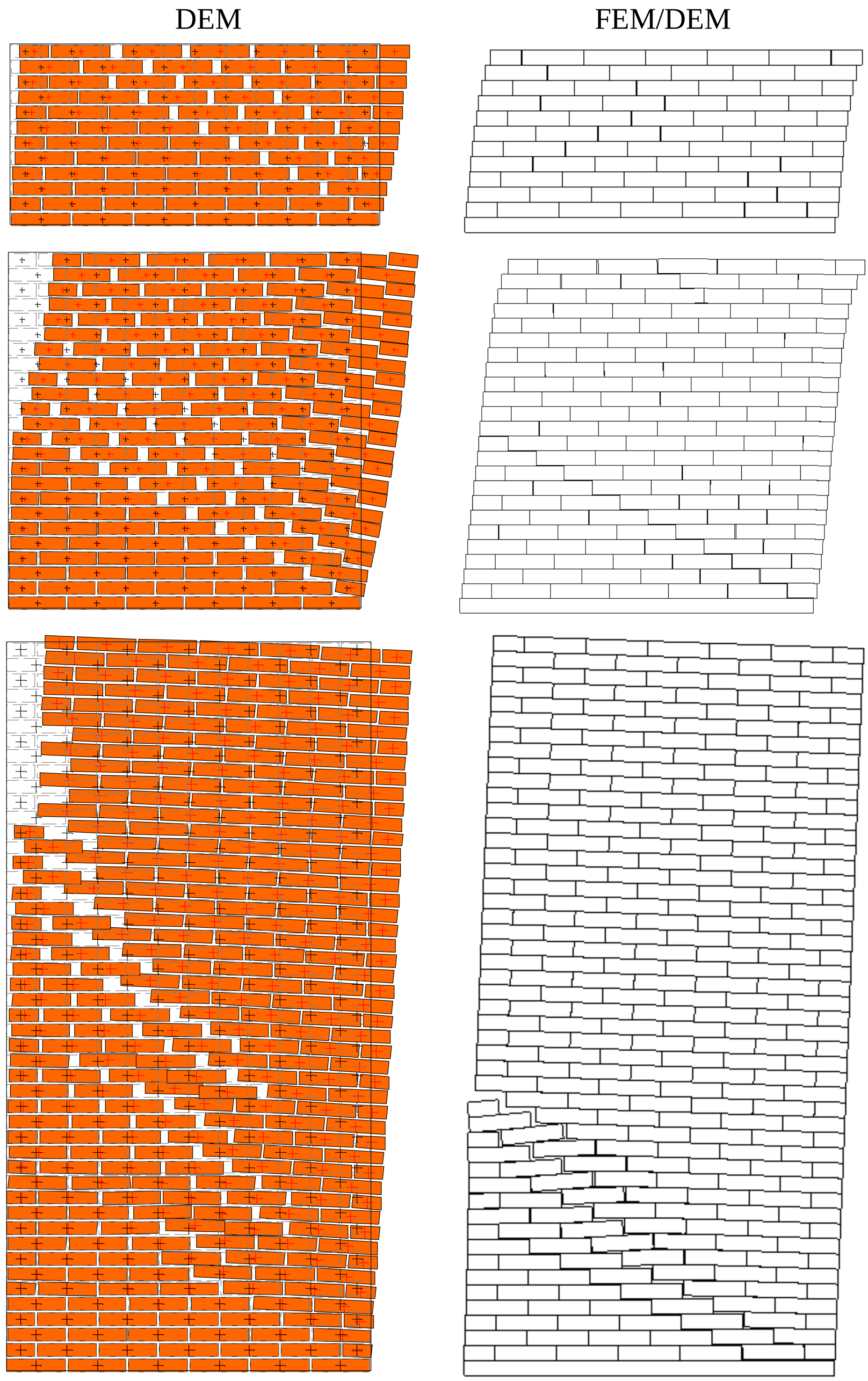

Figure 9: Collapse mechanisms of masonry panels.

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