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CALIBRATION AND VALIDATION OF A SIMPLIFIED MODEL OF NONLINEAR SOIL-STRUCTURE INTERACTION FOR SHALLOW FOUNDATIONS

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Abstract. *Modeling of soil-structure interaction for shallow foundations entails three sources of nonlinearities: foundation uplifting, sliding along the soil-footing interface and irreversible displacement due to soil plasticity. Foundation macroelements allow reducing computational efforts in the resolution of seismic response including nonlinear soil-structure interaction. This is achieved by replacing the soil domain and the foundation by a 2-noded element with a sophisticated nonlinear constitutive law reproducing the aforementioned nonlinearities.*

Definition of uplift and soil plasticity models and of the coupling between the two require a set of parameters that depend on the soil characteristics and the foundation geometry. For practical applications, calibration of these parameters is required.

In this paper, calibration tables have been produced for the parameters describing the foundation uplift behavior. In the case of a rectangular foundation (that has been calibrated for the first time), two extra parameters are introduced with respect to the strip and circular footing: the footing aspect ratio and the direction angle of the resultant overturning moment. In addition, a standardized methodology has been proposed to calibrate the parameters describing soil plastic behavior.

The macroelement validation procedure has been carried out in the case of a bridge pier founded on a strip footing. The discrepancy between macroelement calculations and detailed finite element modeling has been evaluated for twenty earthquake records. Prediction of the superstructure maximum displacements (mean error < 10 %, standardized deviation < 20%) and efforts (mean error < 20 %, standardized deviation < 10%) is validated. Macroelement limitations concern the evaluation of foundation residual displacements, suggesting that further development should be focused on the improvement of plasticity model for soil irreversible behavior.

1 INTRODUCTION

Macroelement theory aims at evaluating the nonlinear seismic response of structures founded on shallow foundations. It models the foundation uplift and sliding together with soil plasticity, which represent the three sources of nonlinearity arising at the foundation level in dynamic soil-structure interaction.

In their current state of development macroelements can be used for preliminary seismic design. Further improvements of the macroelement calibration and validation procedures may provide sufficient guarantees for their use as a final justification of a structure. This objective motivates the work presented in sections 3 and 4.

2 THE CONCEPT OF FOUNDATION MACROELEMENT

2.1 Motivations for developing foundation macroelements

The foundation macroelement is a 2-noded linear segment, with a sophisticated constitutive law so as to grasp the nonlinear soil-structure interaction behavior. As illustrated in Figure 1, the advantage of macroelement modeling with respect to finite element modeling is that there is no need to represent explicitly the foundation and soil domain.

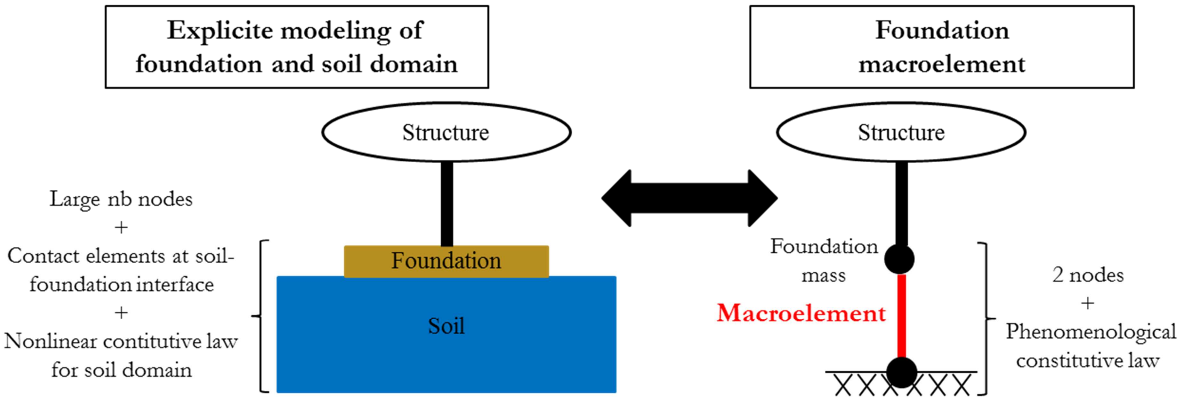


Figure 1: The concept of foundation macroelement

The drastic reduction of the number of degrees-of-freedoms limits significantly computational effort. As a result, two main applications can be mentioned for engineers and researchers:

a) parametric studies, requiring to perform a large number of calculations,
b) and real-time hybrid tests (ref. [8]), when the numerical substructure is the soil and foundation domains.

2.2 Theoretical background

Three hypotheses define the context in which the foundation macroelement can be used: the shallow foundation should be rigid, without embedment and supporting an isolated structure (structure-soil-structure interaction cannot be modeled).

The hypothesis of a rigid foundation allows the description of the foundation kinematics via the displacement and rotation of a single arbitrary point, for example its center of gravity. Figure 2 introduces the notations characterizing the foundation kinematics and load parameters, with B the foundation characteristic dimension and $N_{\max}$ the maximum vertical force supported by the foundation.

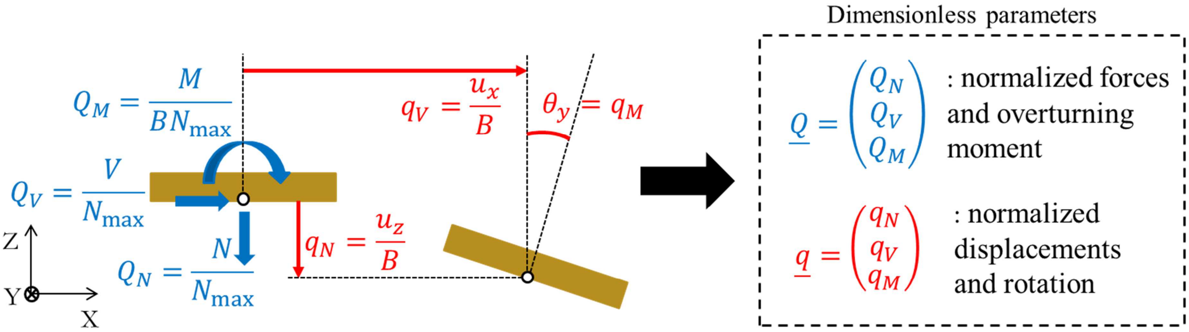


Figure 2: Macroelement formulation - Definition of force and displacement parameters

The main idea for deriving the macroelement constitutive law is the dissociation between reversible and irreversible phenomena. Reversible phenomena regroup the soil elastic behavior and the foundation uplift. Irreversible phenomena entail soil plastic deformations and foundation sliding.

Such a distinction is established by splitting the foundation displacement increment vector $\underline{\dot{q}}$ into an elastic and a plastic component, which are computed at each time step by separate algorithms. Reference [2] details the formulation of the tangent normalized stiffness matrix $\underline{\underline{K}}$ and of plastic modulus $\underline{\underline{H}}$ which are introduced in Figure 3.

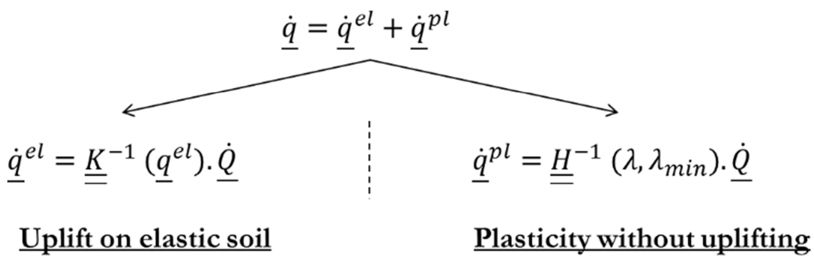


Figure 3: Decomposition of displacement increments

Even though the foundation uplift and soil plasticity are conceptually dealt separately, the numerical implementation of the macroelement associates these two phenomena at each step of calculation of the foundation displacement increment vector $\underline{\dot{q}}$. This operation is performed through an iterative process detailed in ref. [3].

3 MACROELEMENT CALIBRATION

3.1 Objectives

The macroelement theory, as developed in references [2], [3] and [5], introduces six parameters to characterize the formulation of the normalized stiffness matrix $\underline{\underline{K}}$ and the plastic modulus $\underline{\underline{H}}$. Table 1 provides an overview of their definition. Further details are provided in section 3.3 and 3.4.

Footing uplift	p_1^{el}	normalized stiffness matrix $\underline{\underline{K}}$	uplift initiation on elastic soil
	p_2^{el}		overturning moment – rotation diagram
	p_3^{el}		settlement – rotation diagram
Soil plasticity	p_1^{pl}	plastic modulus $\underline{\underline{H}}$	initial loading
	p_2^{pl}		reloading
Uplift-plasticity coupling	p_1^{c}	uplift initiation on plastic soil	

Table 1: Summary of the macroelement parameters requiring specific calibration

The following aims have been addressed:

- Concerning elastic parameters p_1^{el}, p_2^{el} and p_3^{el}: to produce calibration tables for the newly studied case of rectangular footings.
- Concerning plastic parameters p_1^{pl} and p_2^{pl}: to implement the proposals of standardized calibration procedures mentioned in references [2], [3], [5], [6] and [11], so as to provide guidelines to select which is the most appropriate for a given calculation.

3.2 Reference models

The macroelement calibration procedures requires a reference model describing accurately the footing uplift and soil plasticity phenomena. This task is carried out thanks to detailed finite element modeling of the soil and the foundation, implemented on DYNAFLOW software (ref. [12]).

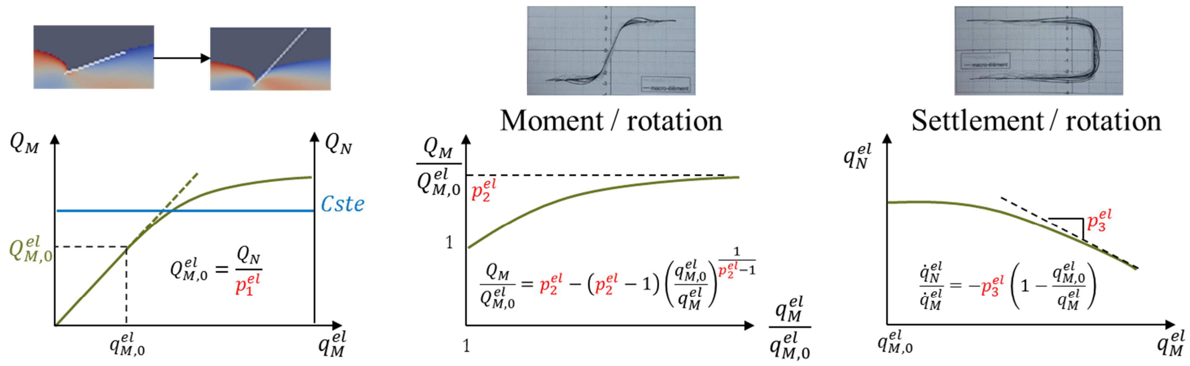
Each calibration objective requires to define a specific DYNAFLOW reference model. Table 2 summarizes the main common features and differences that can be encountered in these models.

Calibration procedure	$\boldsymbol{p_i^{el}}$	$\boldsymbol{p_i^{pl}}$	$\boldsymbol{p_1^{c}}$
Soil domain boundary conditions	identical		
Soil-footing interface	uplift activated	no uplift	uplift activated
Soil constitutive law properties	elastic	plastic	plastic
Loading input data	Q_N = constant q_M linear increase	cyclic loading	Q_N = constant q_M linear increase
Numerical algorithm	Newton-Raphson type		

Table 2: Overview of the reference models used for calibration

3.3 Elastic parameters

Figure 4 illustrates the physical meanings of p_1^{el}, p_2^{el} and p_3^{el} thanks to an approach based on the phenomenological constitutive laws of the footing after uplift initiation. The mathematical equations defining these laws provide a practical means for calibration.



a) Uplift initiation b) Phenomenological constitutive laws after uplift initiation

Figure 4 : Elastic parameters definition

With:

- $Q^{el}_{M,0}$ the normalized overturning moment Q_M of uplift initiation on an elastic soil.
- $q^{el}_{M,0}$ the footing rotational angle of uplift initiation on an elastic soil.

The calibration of the elastic parameters for a rectangular footing requires a significant number of calculations since these parameters depend on:

- The footing rotation axis characterized by the angle ψ.
- The footing aspect ratio R.

Figure 5 details the definition of these parameters.

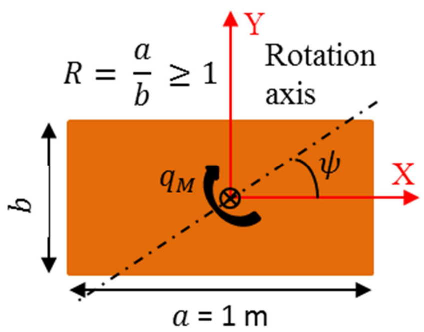


Figure 5: Definition of parameters R and ψ

$R \setminus \psi$	0°	15°	30°	45°	60°	75°	90°
1	4,5	4,7	5,1	5,3	5,1	4,7	4,5
2	8,6	8,8	8,1	6,9	5,8	5,0	4,6
3	12,7	11,9	9,4	7,3	5,9	5,0	4,7
4	16,5	14,3	10,2	7,5	5,9	5,1	4,7

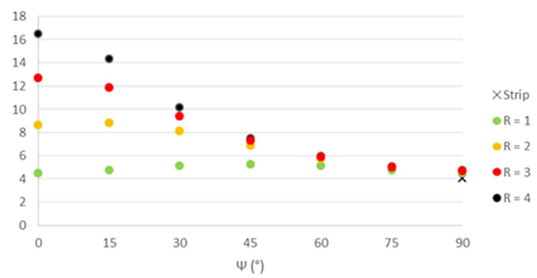


Table 3 & Figure 6: Elastic parameter p^{el}_1 calibration

R* \ *ψ	**0°**	**15°**	**30°**	**45°**	**60°**	**75°**	**90°**
1	2,5	3,1	4,9	6,9	4,9	3,1	2,5
2	2,5	3,7	7,3	6,1	3,8	2,9	2,6
3	2,5	4,2	6,9	4,9	3,5	2,7	2,5
4	2,4	5,1	6,4	4,4	3,3	2,7	2,5

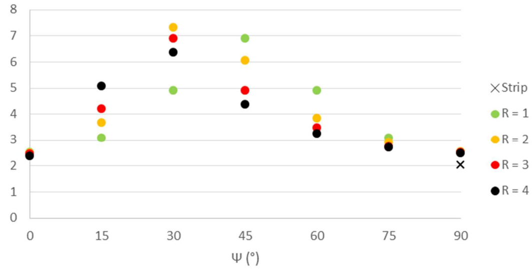


Table 4 & Figure 7: Elastic parameter p_2^{el} calibration

R* \ *ψ	**0°**	**15°**	**30°**	**45°**	**60°**	**75°**	**90°**
1	0,46	0,43	0,40	0,38	0,40	0,43	0,46
2	0,23	0,22	0,24	0,30	0,37	0,42	0,44
3	0,16	0,16	0,21	0,29	0,37	0,42	0,44
4	0,12	0,13	0,20	0,29	0,37	0,42	0,44

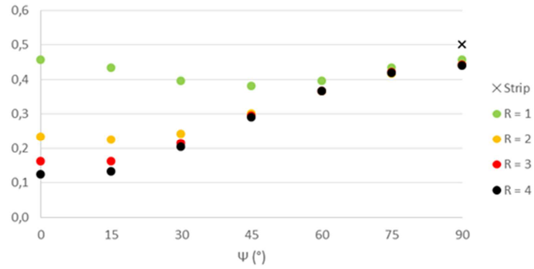


Table 5 & Figure 8: Elastic parameter p_3^{el} calibration

The elastic parameters for the rectangular footing are calibrated considering the projection of the overturning moment on the rotational axis, as these two directions no longer match for ψ different from 0° and 90°.

3.4 Plastic parameters

Plastic parameters p_1^{pl} and p_2^{pl} characterize the macroelement hypoplastic model. The following physical meanings can be given to these parameters:

- p_1^{pl} characterizes initial loading, in the sense that the plastic increment $\underline{\dot{q}}^{pl}$ linearly increases with $\frac{1}{p_1^{pl}}$.
- p_2^{pl} characterizes the evolution of plastic behavior during the reloading phase. Plastic displacements as in initial loading are developed when $p_2^{pl} = 0$. Elastic response is retrieved when $p_2^{pl} \to \infty$.

Plastic parameters are calibrated to best fit force-displacement and moment-rotation curves between DYNAFLOW reference model and the macroelement. Four methodologies characterized by different loading paths are considered:

I. Cyclic vertical loading.
II. Initial vertical loading followed by horizontal cyclic loading.
III. Initial vertical loading followed by overturning moment cyclic loading.
IV. A combination of the methodologies II and III: initial vertical loading followed by horizontal and overturning moment cyclic loadings such as $M = hV$, with h the superstructure's characteristic height.

Figure 9 illustrates these loading paths in the space of dimensionless force parameters in which the hypoplastic bounding surface is defined.

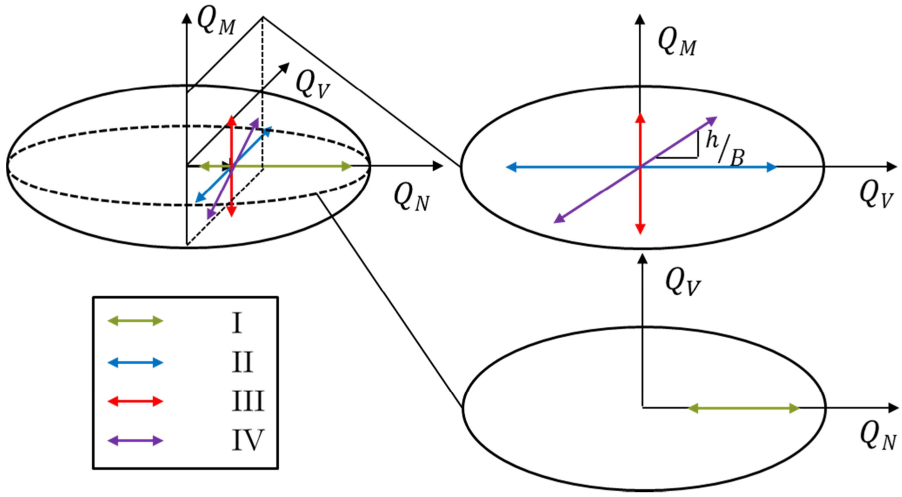


Figure 9: Loading paths considered for plastic parameters calibration methodologies

The four calibration methodologies are carried out for the strip footing case relying on a semi-infinite purely cohesive soil domain. Table 6 summarizes the results obtained under the assumptions of a kinematic and isotropic hardening law for the soil.

Methodology	p_1^{pl}		p_2^{pl}	
	Kinematic	*Isotropic*	*Kinematic*	*Isotropic*
I	1,33	1,33	∞	∞
II	2,29	2,42	0	0,39
III	1,66	1,84	0	2,48
IV	1,82	2,03	0	0,89

Table 6: Plastic parameters calibration

The following points should be noted:

a) The striking variation of p_2^{pl} calibration between the isotropic and kinematic assumptions for the soil hardening law. This observation has no repercussions because separate calibration tables could be considered.

b) The important variation observed between the four calibration methodologies questions the macroelement ability to grasp the soil plastic behavior for all types of loadings with only two parameters.

c) Parameter p_1^{pl} calibration according to methodology IV is predicted by the following linear relation involving the calibration results according to the methodologies II and III:

$$p_{1,\mathrm{IV}}^{pl} = \frac{1}{1+\left(\frac{h}{B}\right)^2} p_{1,\mathrm{II}}^{pl} + \frac{\left(\frac{h}{B}\right)^2}{1+\left(\frac{h}{B}\right)^2} p_{1,\mathrm{III}}^{pl}$$

The coefficient $\alpha_V = \frac{1}{1+\left(\frac{h}{B}\right)^2}$ and $\alpha_M = \frac{\left(\frac{h}{B}\right)^2}{1+\left(\frac{h}{B}\right)^2}$ respectively correspond to the horizontal force and overturning moment proportion of the total normalized load $\sqrt{Q_V^2 + Q_M^2}$.

Knowing that seismic excitation leads to loading histories close to methodology IV, the main conclusion of this section is that methodology III offers the best option to perform plastic parameters calibration for common cases with $\frac{h}{B} > 2$. In that respect, methodology III could be implemented to produce calibration tables for plastic parameters.

4 MACROELEMENT VALIDATION

4.1 General methodology

This section aims at assessing the foundation macroelement qualitative behavior and accuracy when subject to seismic loading. The analysis is carried out for a strip footing. The assumption of a kinematic hardening law for the soil is assessed.

The typical case of a bridge pier founded on a cohesive soil is examined, as presented in Figure 10.

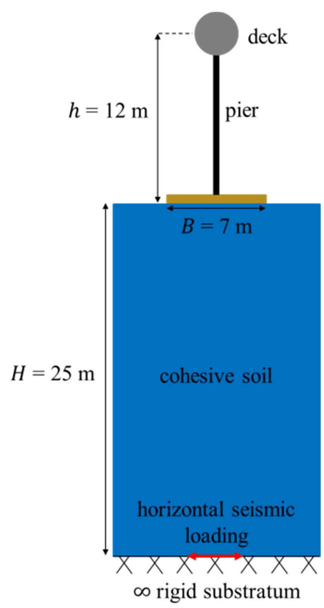


Figure 10: Definition of earthquake engineering context for the validation procedure

Twenty acceleration time histories selected from Project DARE (ref. [4]) are examined. They correspond to a horizontal acceleration imposed at the bedrock level. This selection intends to cover a wide range of frequencies and seismic intensity levels.

The following criteria are considered so as to assess the macroelement precision:

a) The maximum shear force $V_{\max}$ and bending moment $M_{\max}$ developed at the base of the pier.
b) The maximum horizontal displacement $u_{x,\max}^{\text{deck}}$ and rotation $\theta_{y,\max}^{\text{deck}}$ of the deck.
c) The foundation residual displacements and rotation $u_{x,\text{res}}$, $u_{z,\text{res}}$ and $\theta_{y,\text{res}}$.

4.2 Reference model

Footing uplift was modeled in DYNAFLOW (ref. [12]) using contact elements, previously introduced for the calibration of the elastic and uplift-plasticity coupling parameters.

It should be noted that footing sliding is not considered at this stage, i.e. no constitutive law such as Mohr-Coulomb is introduced at the interface to limit the tangential force transferred by the footing to the soil domain. This choice ensures that the footing uplift is not inhibited by the sliding phenomenon.

4.3 Macroelement input data

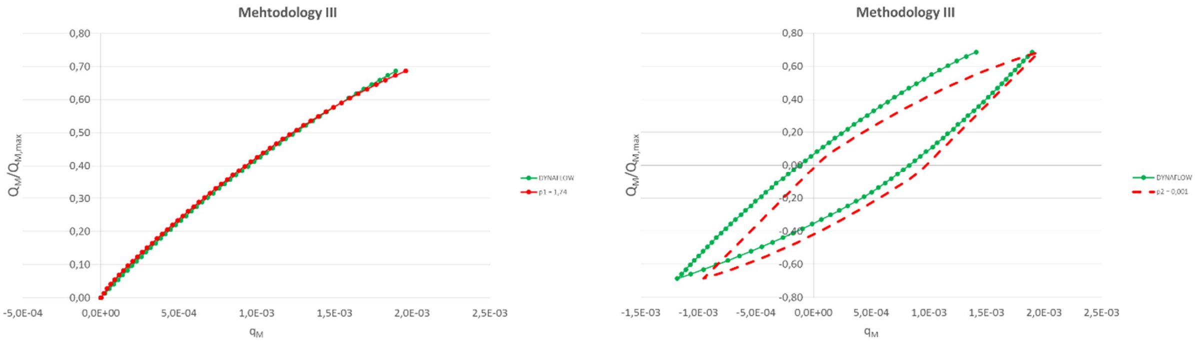
Table 7 details the values of the numerical parameters used in the study.

<table>
<tr><th>Type of parameter</th><th>Symbol</th><th colspan="2">Definition</th><th>Unit</th><th>Value</th><th>Source</th></tr>
<tr><td rowspan="2">Dimensional</td><td>B</td><td colspan="2">Footing characteristic dimension</td><td>m</td><td>7,00</td><td>Design</td></tr>
<tr><td>$N_{\max}$</td><td colspan="2">Maximum centered vertical load</td><td>$MN.ml^{-1}$</td><td>5,397</td><td>EC 7</td></tr>
<tr><td rowspan="6">Viscoelastic</td><td>K_{NN}^0</td><td colspan="2" rowspan="3">Footing normalized dynamic impedance</td><td>ml^{-1}</td><td>209,3</td><td rowspan="6">Gazetas' analytical expressions (ref. [9])</td></tr>
<tr><td>K_{VV}^0</td><td>ml^{-1}</td><td>202,9</td></tr>
<tr><td>K_{MM}^0</td><td>$rad^{-1}.ml^{-1}$</td><td>77,8</td></tr>
<tr><td>A_{NN}^0</td><td colspan="2" rowspan="3">Footing normalized dashpots</td><td>$s.ml^{-1}$</td><td>5,7</td></tr>
<tr><td>A_{VV}^0</td><td>$s.ml^{-1}$</td><td>3,7</td></tr>
<tr><td>A_{MM}^0</td><td>$s.ml^{-1}$</td><td>0,068</td></tr>
<tr><td rowspan="3">Footing uplift</td><td>p_1^{el}</td><td colspan="2">Uplift initiation</td><td>-</td><td>4</td><td rowspan="3">§3.3</td></tr>
<tr><td>p_2^{el}</td><td rowspan="2">Constitutive law</td><td>Q_M - q_M</td><td>-</td><td>2</td></tr>
<tr><td>p_3^{el}</td><td>q_N - q_M</td><td>-</td><td>0,5</td></tr>
<tr><td rowspan="5">Soil plasticity</td><td>$Q_{V,\max}$</td><td colspan="2" rowspan="2">Normalized maximum horizontal force and overturning moment</td><td>-</td><td>0,195</td><td rowspan="2">EC 7</td></tr>
<tr><td>$Q_{M,\max}$</td><td>-</td><td>0,111</td></tr>
<tr><td>p_1^{pl}</td><td rowspan="2">Plasticity modulus</td><td>initial loading</td><td>-</td><td>1,74</td><td rowspan="2">§3.4</td></tr>
<tr><td>p_2^{pl}</td><td>reloading</td><td>-</td><td>0,00</td></tr>
<tr><td>n_g</td><td colspan="2">Non associativity plastic law</td><td>-</td><td>1,00</td><td>Von Mises law</td></tr>
<tr><td>Uplift & plasticity coupling</td><td>p_1^c</td><td colspan="2">Footing uplift on plastic soil</td><td>-</td><td>0,90</td><td>§3.1</td></tr>
<tr><td>Foundation sliding</td><td>φ</td><td colspan="2">Soil friction angle</td><td>°</td><td>0,00</td><td>Cohesive soil</td></tr>
</table>

Table 7: Macroelement calibration for the validation procedure

The fictitious zero value attributed to the soil friction angle is a macroelement convention to express that the soil is cohesive.

In compliance with section 3.4, methodology III is implemented to calibrate the plastic parameters for the macroelement validation procedure. Figure 11 shows that the calibration leads to a good matching of moment – rotation curves. However, the macroelement significantly minimizes the foundation settlement produced during the bending moment cyclic loading
(-77 %). As mentioned is section 4.4, this observation is useful to recalibrate macroelement prediction of settlement.



a) p_1^{pl} calibration (initial loading)

b) p_2^{pl} calibration (cyclic loading)

Figure 11: Validation procedure – Plastic parameters calibration curves

4.4 Validation results

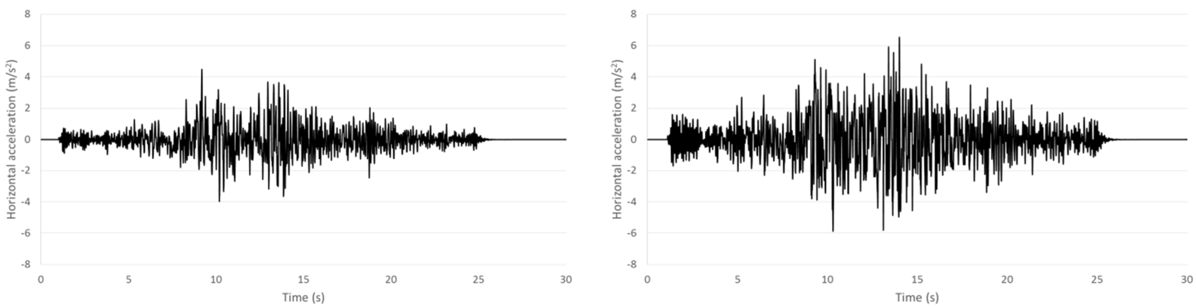
Table 8 establishes the conclusion of the validation procedure.

Criterion	Quantitative assessment	Qualitative assessment
a)	20 % average error with a standard deviation inferior to 10 %.	Very good estimation of the localization of the pic value and of the structure oscillation natural frequency.
b)	Average error inferior to 10 % with a standard deviation inferior to 20 %.	
c)	No satisfactory prediction for the foundation residual horizontal displacement and rotation. After rectifying the foundation settlement according to the error observed during the plastic parameters calibration: average error inferior to 10 % with a standard deviation of 5 %.	The modeling of energy dissipation due to the soil plastic deformation is satisfactory. The macroelement performs well at identifying which earthquakes have the most critical impact on the foundation settlement and residual rotation. This property is less valid concerning the foundation residual horizontal displacement.

Table 8: Summary of the results for the macroelement validation procedure

Figures 14 to 18 illustrate the elements of dynamic response required to assess the macroelement qualitative behavior and precision in the case of Lucerne earthquake (PGA = 0,6 g).

Table 9 and Figure 19 detail the validation results for each acceleration time history.



a) Bedrock level

b) Ground level

Figure 12: Lucerne acceleration time history

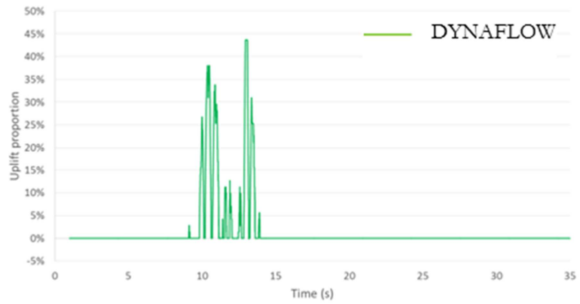
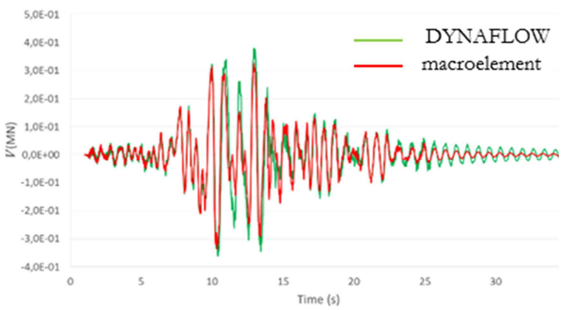
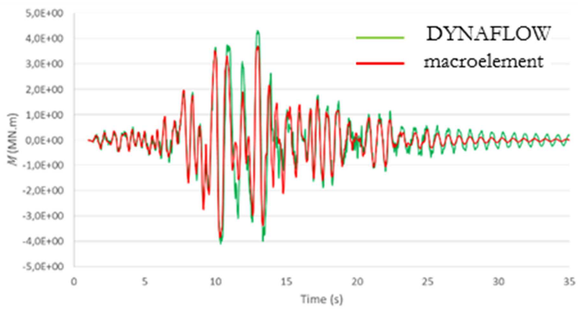


Figure 13: Foundation uplift kinematics



a) Shear force



b) Bending moment

Figure 14: Pier base reactions

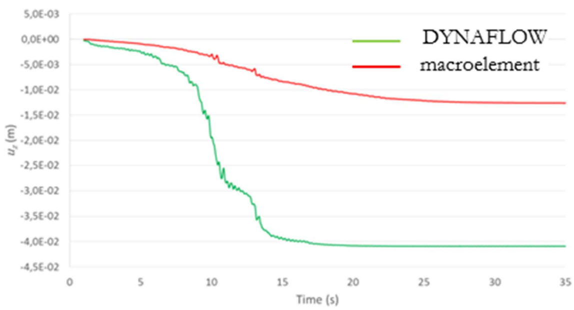
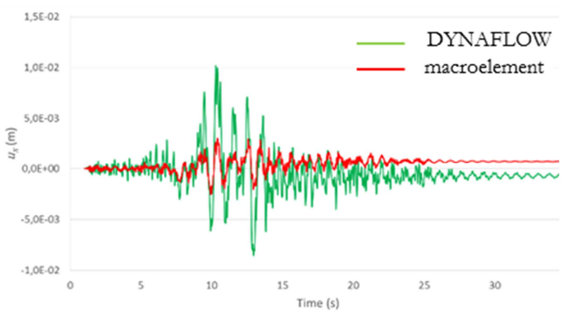
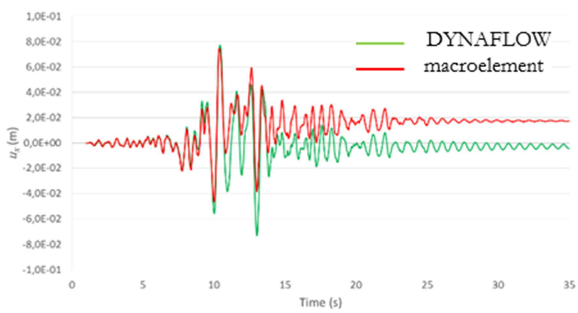


Figure 15: Foundation vertical displacement

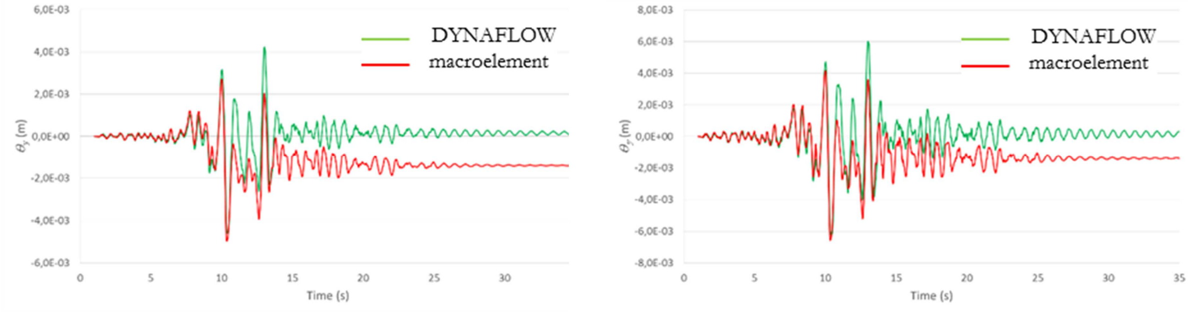


a) Foundation



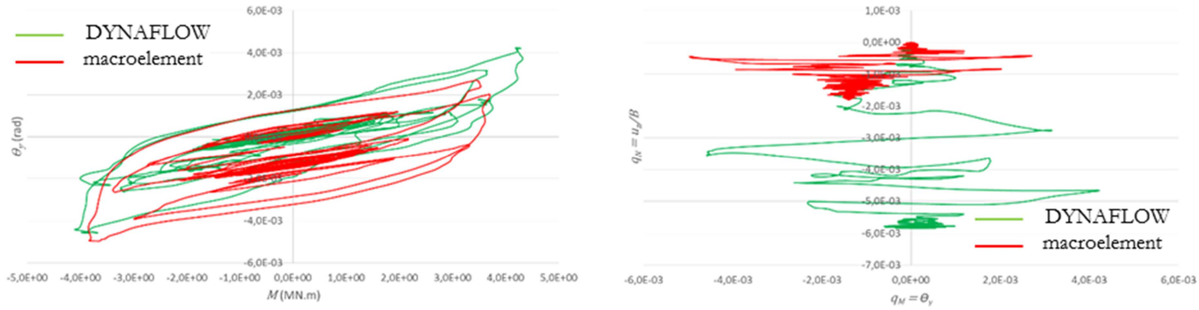
b) Deck

Figure 16: Horizontal displacements



a) Foundation b) Deck

Figure 17: Rotational displacements



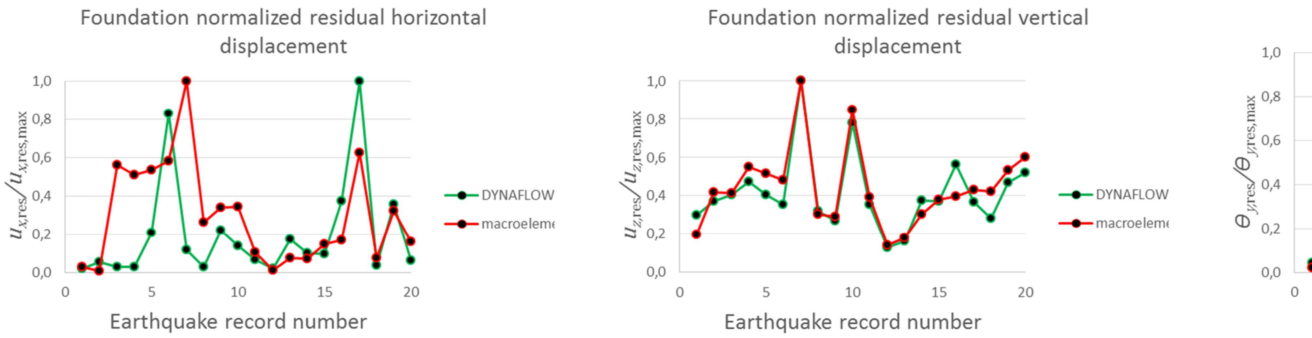
a) Foundation M - θ_y behavior b) Foundation q_N - q_M behavior

Figure 18: Macroelement qualitative assessment

Earthquake record n°	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$V_{\max}$	-11	-13	-20	-18	-25	-24	-22	-16	-12	-22	-11	-6	-2	-17	-19	-31
$M_{\max}$	-12	-14	-19	-19	-23	-24	-21	-13	-14	-20	-11	-8	2	-20	-17	-19
$u_{x,\max}^{\text{deck}}$	-2	-22	-3	-7	6	-4	11	34	-9	-5	-3	-7	-3	-32	-3	-7
$\theta_{y,\max}^{\text{deck}}$	7	-17	1	-2	17	2	18	38	-2	-2	5	-8	12	-27	5	5
$u_{x,\text{res}}$	-10	-93	>10²	>10²	64	-55	>10²	>10²	-2	56	1	-65	-73	-54	-3	-71
$u_{z,\text{res}}$	-82	-68	-71	-68	-64	-62	-72	-74	-70	-69	-69	-69	-69	-77	-71	-80
$\theta_{y,\text{res}}$	56	73	>10²	>10²	>10²	>10²	>10²	>10²	>10²	>10²	>10²	-19	>10²	>10²	>10²	>10²

Table 9: Assessment of macroelement error (in %)

$\overline{x}$ the average, x_m the median and σ the standard deviation values. The assessment of macroelement erro
$\frac{V_{\text{max,test}}-V_{\text{max,ref}}}{V_{\text{max,ref}}}$, with the tested value corresponding to macroelement calcu



a) Horizontal direction   b) Vertical direction   c) Rota

Figure 19: Foundation normalized residual displacements and rotation

5 CONCLUSION

The analysis of the macroelement current state of development and application domains encouraged further investigations in its calibration and validation procedures.

The following results are obtained concerning the calibration of the macroelement parameters:

- The establishment of calibration tables for the elastic parameters describing the foundation uplift in the case of the rectangular footing.
- The proposal of a standardized methodology to calibrate plastic parameters, characterized by a cyclic overturning moment loading of the foundation.

The macroelement validation procedure is performed using the model of a typical bridge pier structure relying on a strip foundation. Accuracy is evaluated by considering for each criterion two indicators: the average relative error and the standard deviation.

The macroelement ability to predict the maximum displacements/rotation of the superstructure and reaction forces/overturning moment developed at the foundation is demonstrated. Macroelement calculation seems to provide sufficient accuracy for contributing to final justifications of a structure when its design is governed by these criteria.

The macroelement is not that good in predicting the foundation residual displacements and rotation, with the exception of its settlement when its calculation is rectified by the error observed during the calibration of plastic parameters. Nonetheless, the accuracy of the qualitative information provided is sufficient for parametric studies carried out during preliminary design.

Finally, the macroelement calibration and validation procedures highlight the fact that development efforts should be focused on improving the macroelement hypoplastic model, compromising the needs for model simplicity and accuracy in modeling all aspects of dynamic response.

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