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SENSOR PLACEMENT OPTIMIZATION USING ENSEMBLE KALMAN FILTER AND GENETIC ALGORITHM

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Abstract. *A robust methodology is proposed in this study for determining the optimal locations of sensors in a structure to extract the most informative measurement data for the purpose of parameter estimations. First, the Ensemble Kalman Filter (EnKF) is used as a history matching method to predict and update the state and model parameters of the system. Then, a Genetic Algorithm (GA) approach is applied to determine the best locations of the sensors in the system through a minimization procedure, where the objective function to be minimized is represented by the mismatch between the predicted values and the actual measurements. The robustness and efficiency of the proposed method are demonstrated by developing the optimal sensor configuration for a shear building subjected to El Centro earthquake excitation at its base and using synthetic measurements of displacements and velocities of different floors.*

1 INTRODUCTION

With the development of sensors and other monitoring devices, detecting the damage in structures became an easier task. Using sensor based structural health monitoring (SHM) made data abundantly available and continuously obtained in real time. As such, one of the main challenges that remain lies in post processing this data. Therefore to save money and get only the necessary data, this study deals with optimizing the sensor locations to efficiently use the generated data for system identification and damage detection.

Many methods were proposed in the literature to determine the optimal sensor placement for damage detection and parameter estimation. These methods can be classified into two main categories: Optimization-based and Selection-based procedures [1]. The most common optimization-based methods used in the literature include the Genetic Algorithm (GA), the Simulated Annealing (SA), the Tabu Search (TS) and their different variations. The GA is a search technique used to solve optimization problems using random operations based on natural evolution operations (selection, crossover, and mutation). The algorithm starts by generating a possible set of solutions, called “population of chromosomes”, each having a certain fitness function. After performing a series of natural operations on the initial population, a new better population is created. The algorithm is repeated, each time starting with the new created population, until some predefined end condition is satisfied. L. Yao et al. used the GA for sensor placement optimization for a space structure and a photo-voltaic array, and suggested to use the determinant of the Fisher information matrix as the fitness function in the algorithm [2]. H. Y. Guo et al. proposed to use an improved version of the GA (improved crossover and mutation) to determine the optimal locations of sensors in a SHM system consisting of a 2D truss structure [3]. The SA is an optimization method, named after the physical process of annealing in thermodynamics and used for finding a global minimum or maximum for problems having multiple local minima and maxima [4, 5]. P. L. Chiu et al. proposed to use the SA algorithm to solve the problem of sensor placement and tested it on small and large sensor fields [6]. The TS is a “meta-heuristic” optimization procedure used to avoid being trapped at a local optimum in neighborhood search problems [7, 8, 9][7]. The algorithm creates and updates a “tabu list” of previously visited solutions to prohibit (Tabu means something prohibited) considering them again. The second class of methods, selection-based procedures, consisting mainly of selecting the best sensor locations, is facilitated using information theory measures (i.e. entropy or mutual information) which quantify the uncertainty associated with random variables. Therefore locations where sensors have low information content are eliminated and locations where sensors have good information content are considered optimum. In [10][10], K Yuen et al. used the Bayesian framework to compute the uncertainty in model parameters, then used the information entropy to determine the optimal sensor locations. They applied their method on two numerical examples, an 8 DOF chain mass-spring model and a 40 DOF truss model, having uncertain excitations.

Combined approaches, based on both optimization (more specifically GA) and selection methods, are extensively used in the literature for determining the optimal sensor locations. C. Papadimitriou et al. proposed a methodology where the Bayesian approach is used to compute the uncertainty in the parameters, then the GA is applied to minimize the information entropy over the set of possible sensor configurations [11]. The methodology proposed was applied on two numerical examples, a 9-story building and a 29-DOF truss structure. C. Papadimitriou also used the information entropy to determine the optimal sensor configuration in [12]. In this work, he compared the GA with the sequential sensor placement (SSP) approach based on the computational burden needed to determine the optimal sensor locations and applied his methodology on two numerical examples, a 10 DOF chain-like spring-mass model and a 240

DOF 3D truss structure. E. B. Flynn et al. also combined the Bayesian framework with the GA approach to determine the best locations of sensors and applied their approach to active sensing field [13]. H. M. Chow et al. used a combination of the Bayesian statistical framework, the information entropy and the GA to determine the optimal sensor configuration of a 3D finite element model of a transmission tower modeled in ANSYS [14, 15].

In this paper, a combination between the GA and the Ensemble Kalman Filter (EnKF) is presented to tackle the problem of optimal sensor placement. The EnKF is a sequential data assimilation technique, used in the literature for structural health monitoring (SHM) purposes. R. Ghanem et al. showed the ability of the EnKF to be a good estimator of the system state in [16]. It is combined with a non-parametric modeling technique and applied to a four story shear building subject to El Centro earthquake excitation at its base. G. Evensen provided an overview on the EnKF method and its use for state and parameter estimation problems, then illustrated the advantages of using this filter when dealing with high-dimensional or non-linear systems [17]. To the authors' knowledge, this paper is considered as a first attempt to combine the GA and the EnKF for the purpose of optimal sensor placement, but it should be noted here that these two techniques were used together for other purposes in the past few years. H. Li et al. combined the GA with the EnKF for optimal permeability distribution of a 3D model of landfills [18]. The GA was first used to generate initial ensembles of the permeability distribution of the landfill model. The EnKF was then used to estimate this distribution and update it based on synthetic real-time measurements. The algorithm was repeated till it converged to the optimal distribution. J. Lyons et al. used the EnKF for history matching and updating a reservoir model, then combined it with the GA for well placement optimization [19].

The paper is divided into five sections. Section 2 describes the methodology used, combination between GA and EnKF, and the mathematical formulation of each of the two techniques separately. The third section presents the numerical example consisting of a four story building subjected to seismic excitation at its base. Section four exposes and discusses the results. Finally, a general conclusion is drawn in section five.

2 GA-ENKF METHODOLOGY FOR OPTIMAL SENSNOR PLACEMENT

The methodology proposed in this paper starts by randomly selecting an initial population of sensor locations as a first step for applying the GA. The next step is to determine the objective function of the problem, which is taken to be the difference between the actual sensor measurements and the predicted values. The role of the EnKF starts at this point, it is used to estimate and update the system state and get the needed predicted values to calculate the objective function. The fitness of each individual is next evaluated and preference is given to the ones with the least mismatch between the measured and predicted data. These individuals will be selected as the parents from which GA will give birth to better offsprings (new sensor locations) after performing the crossover and mutation operations. Selection of the best individuals from the new solutions will then be based on their fitness values. The algorithm will be repeated until the convergence condition is satisfied and the best locations of the sensors are determined.

The next sub-section presents a summary about the definition of the GA and its different operations, followed by a sub-section describing a brief mathematical formulation of the EnKF method.

2.1 The Genetic Algorithm (GA)

The GA is a search technique that uses the theory of natural evolution to solve optimization problems. It was first introduced by John Holland in 1975 [20]. The algorithm starts by randomly creating a population of individuals, from which two fittest parents are selected. These initial parents undergo a series of natural evolution operations to create new better offsprings, from which the best two new parents are selected based on their fitness. This is repeated until a certain termination criterion is satisfied.

The GA can be summarized in the following outline [14, 15, 21]:

1. Initialization: The algorithm starts by randomly generating an initial set of individuals or solutions (Population), where each individual is represented by a chromosome. There are many encoding methods in the literature to represent the individuals of a population, but the most used way is the binary string (1 or 0) where each chromosome is represented by one binary string. For the case of optimal sensor location problems, if for example two sensors are available to be placed on a four-story building and the individual is encoded as 1010, this means that one sensor is placed on the first floor and the second sensor is placed on the third floor.
2. Fitness evaluation: The next step is to determine the fitness or objective function to be optimized, that is problem dependent, and evaluate the fitness of each individual of the initial population.
3. Creation of new solutions: The following steps must be repeated until convergence:
    i. Selection: In this step, two parents (sub-population) are selected from the population based on their fitness values to produce new offsprings (children). The fitter the chromosome, the better it has chance to be selected to propagate its genetic information.
    ii. Crossover (Recombination): This operation is used to create new offsprings by recombining genes from the selected parent chromosomes. A crossover point or site should be randomly selected, the part of the strings before this point of the first parent is combined with the part of the strings after this point of the second parent to create the first offspring, whereas the second offspring is created by combining the remaining two parts of the strings from the two parents.

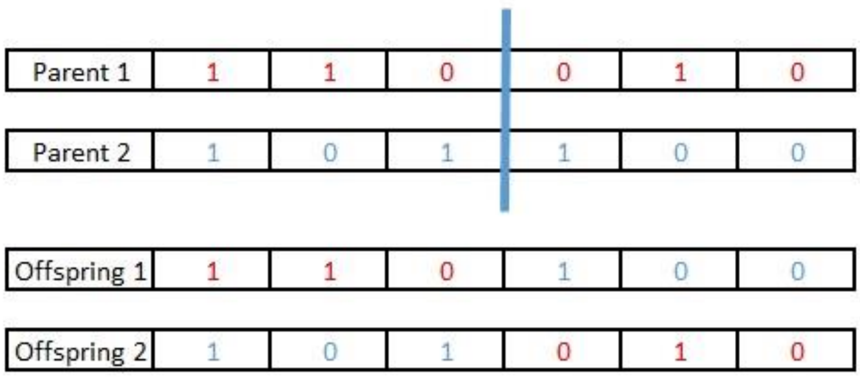

Figure 1: Crossover Example.

    iii. Mutation: This operation randomly changes some of the portions of the new individuals to avoid being trapped into a local optimum. For binary encoding, mutation randomly flips some bits from 0 to 1 or from 1 to 0.

Original Offspring 1	1	1	0	0	1	0
Original Offspring 2	1	0	1	1	0	0
Mutated Offspring 1	1	0	1	1	0	0
Mutated Offspring 2	1	0	0	0	1	1

Figure 2: Mutation Example.

iv. The new, hopefully better, solutions are used as parents now. The fitness of the new offsprings should be evaluated and the loop should be repeated until convergence to the best individuals.

4. Termination conditions: For each problem, many termination conditions can be predefined, those may include: number of generations, time of the run, plateau (no more improvement of the best solution), and many others.

2.2 The Ensemble Kalman Filter (EnKF)

The standard Kalman Filter (KF) [22] is an optimal estimator used for the state approximation of linear dynamic systems perturbed by Gaussian white noise using observations subjected to Gaussian error statistics. For the case of large and nonlinear systems, the Ensemble Kalman Filter (EnKF) is widely used in the literature to overcome most of the drawbacks of the standard KF. The EnKF was first introduced by Evensen in 1994 [23] as a Monte Carlo approximation of the ordinary KF. It consists of a forecast step where an ensemble of realizations is propagated forward in time and then corrected in the analysis step, whenever observation data are recorded.

The first step in the EnKF is to evaluate the ensemble matrix A, holding the ensemble members x_i,

$$A = \left(x_1, x_2, ..., x_N\right) \quad A \in R^{n \times N}, x_i \in R^n \tag{1}$$

where, n is the size of the model state vector and N is the number of ensemble members.

The next step is to evaluate the ensemble mean and the ensemble perturbation matrices as follows,

$$A = A1_N \, \overline{A} \in R^{n \times N} \tag{2}$$

$$A' = A - \overline{A} = A\left(I - 1_N\right) \quad A' \in R^{n \times N} \tag{3}$$

where, $1_N \in R^{N \times N}$ is a matrix having all its elements equal to 1/N.

Consequently, the ensemble covariance matrix is defined as,

$$P = \frac{1}{N-1} A' A'^T \quad P \in R^{n \times n} \tag{4}$$

Let D be an ensemble of observation matrix holding the measurement vectors $d \in R^m$

$$D = \left(d_1, d_2, ..., d_N\right) \quad D \in R^{m \times N} \tag{5}$$

where m is the number of measurements at each occurrence, and

$$d_j = d + \in_j \quad j = 1, ...N \tag{6}$$

where, ϵ is the measurement noise vector.

The measurement error covariance matrix is defined as,

$$R = \frac{1}{N-1} \gamma \gamma^T \quad R \in R^{m \times m} \tag{7}$$

where, γ is the ensemble of perturbations expressed as

$$\gamma = \left(\in_1, \in_2, ..., \in_N\right) \quad \gamma \in R^{m \times N} \tag{8}$$

The assimilation equation in matrix form is expressed as,

$$A^a = A^f + KG\left(D - HA^f\right) \tag{9}$$

where, the superscripts f and a represent the forecast and analysis states respectively, H is the observation matrix connecting the true state to the observations and KG is the Kalman Gain defined as follows,

$$KG = P^f H^T \left(HP^f H^T + R\right)^{-1} \tag{10}$$

3 NUMERICAL EXAMPLE

The numerical problem in this paper is similar to the one found in [16, 24, 25] with some slight modifications. It consists of a four-degree-of-freedom shear building, as shown in figure 1 below, and subjected to El-Centro earthquake excitation at its base. The stiffness is assumed to be constant and equal to $K = 7.5 \times 10^6 N/m$ on all the floors, the damping ratio is also assumed to be constant with a value of 5% for all modes, the mass for each floor is taken to be 5000 Kg.

A predefined change in the hysteretic behavior of the first floor of the system is imposed after the earthquake excitation hits the structure.

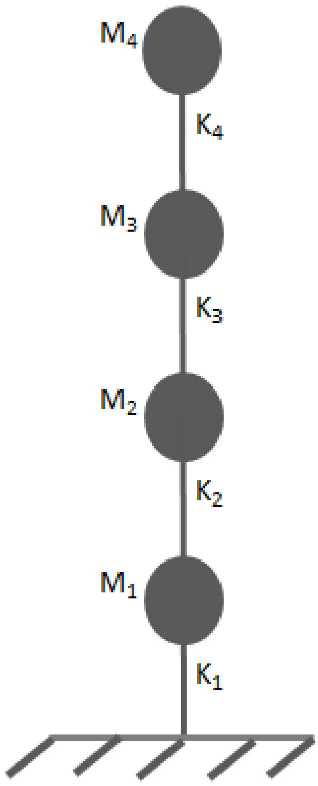

Figure 3: Four-story shear building.

It is assumed that only two sensors are available, therefore the purpose of this numerical problem is to determine the optimal locations of these two monitoring devices.

The first and second floors are randomly selected as an initial population of sensor locations. The next step consists of determining the fitness function, which is the difference between the measured displacements and velocities of each floor and their respective predicted values.

For this problem, the measurements data are synthetically generated by the Bouc-Wen model. Therefore the equation of motion is expressed as

$$M\ddot{u}(t) + C\dot{u}(t) + \alpha K_{el} u(t) + (1-\alpha) K_{in} z(x,t) = -M\ddot{u}_g(t) \tag{11}$$

where M is the mass matrix, C is the damping matrix, K_{el} and K_{in} are respectively the elastic and inelastic stiffness matrices, α is ratio of the post yielding stiffness to the elastic stiffness, τ is an influence vector, u is the displacement vector, x is the inter-story drift vector, and z is the evolutionary hysteretic vector of dimension n and whose ith component is expressed by the Bouc-Wen model by

$$\dot{z}_i = A_i \dot{x}_i - \beta_i |\dot{x}_i| |z_i|^{n_i - 1} z_i - \gamma_i \dot{x}_i |z_i|^{n_i} \quad i = 1,\ldots n \tag{12}$$

where A, β, γ are the Bouc-Wen model parameters, whose values are shown in the table below

Model Parameter	Pre-Change (before 5 sec.)	Post-Change (after 5 sec.)
α	0.15	0.15
β	0.1	1000
γ	0.1	1000
A	1	1
n	1	1

Table 1: Bouc-Wen model parameters.

On the other side, the predicted values of the displacements and velocities are calculated using an inverse model that is used to detect the behavior of the system under consideration. The inverse model is expressed by the following equation

$$M\ddot{u}(t) + F(u,\dot{u}) = -M\ddot{u}_g(t) \tag{13}$$

where F is the non-parametric representation of the non-linearity (non-linear restoring force), whose ith component for the case of the Ensemble Kalman Filter is given by

$$\begin{aligned} F_i(u,\dot{u}) &= a_i(u_i - u_{i-1}) + a_{i+1}(u_i - u_{i+1}) + b_i(u_i - u_{i-1})^3 + b_{i+1}(u_i - u_{i+1})^3 + c_i(\dot{u}_i - \dot{u}_{i-1}) \\ &+ c_{i+1}(\dot{u}_i - \dot{u}_{i+1}) + d_i(u_i - u_{i-1})(\dot{u}_i - \dot{u}_{i-1}) + d_{i+1}(u_i - u_{i+1})(\dot{u}_i - \dot{u}_{i+1}) \end{aligned} \tag{14}$$

where $\{a_i\}, \{b_i\}, \{c_i\}, and \{d_i\}$ are the chaos coefficients of the unknown parameters to be identified.

Based on the fitness value of each individual, the ones having the lowest mismatch between the predicted and actual data have a better chance to be selected, hoping they will produce better offsprings after undergoing the crossover and mutation processes.

4 RESULTS

The problem is solved using the Matlab's genetic algorithm function (*ga*) [26],

$$X = ga(@\,FitnessFunction, N\mathrm{var}\,s, A, b, A_{eq}, b_{eq}, LB, UB, options) \tag{15}$$

This function is part of the global optimization toolbox in Matlab, it is used for minimization purposes. In this case, *ga* function is used to give the optimal sensor locations as an output (vector *X*). The inputs of this function are the following:

1. The fitness function: for this problem, the fitness function is the mismatch between the measured displacements and velocities synthetically generated by the Bouc-Wen model, and their predicted values generated by applying the EnKF method.
2. The number of variables present in the problem are $N\mathrm{var}\,s = 2$.
3. Linear constraints represented by the linear inequalities $A \times X \le b$ and the linear equalities $A_{eq} \times X = b_{eq}$. There are no constraints for this special problem.
4. Variables' bounds: a lower bound $LB = 1$ (story 1) and an upper bound $UB = 4$ (story 4) are used.

5. Options: the Matlab function *gaoptimset* is used to impose customized options to the GA. The following table lists the optimization parameters used as inputs for *gaoptimset* function.

Parameter	Value
Number of Population	3
Number of Elite individuals	2
Number of Generations	20
Stall Generations	10
Function Tolerance	10^{-10}
Initial Population	[1 2]

Table 2: Genetic Algorithm parameters.

where, the elite individuals represent the individuals having the best fitness function and consequently having the highest chance to be selected.

The algorithm stops if the number of generations is exceeded or if the average change in the fitness function value is less than the function tolerance over the stall generations. Figure 5 shows the stopping criteria of this numerical problem. It seems that the algorithm has stopped before the number of generations is reached, which means that the change in the fitness function over 10 successive generations is less than 10^{-10}. For this problem, the time and the Stall time limit (Stall (T)), which is the time over which the fitness function exhibits no improvement, are not included as stopping criteria.

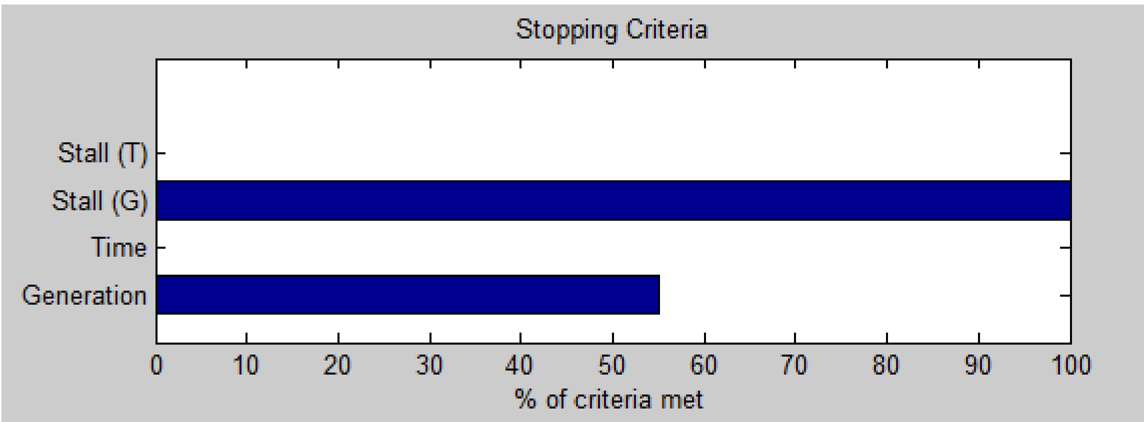

Figure 5: Stopping Criteria.

The default mutation and crossover operations, embedded within Matlab, are applied on this example [26]. The default mutation function is a Gaussian mutation that adds a random Gaussian number with zero mean to each entry of the population. The default crossover function creates a random binary vector (for example, for the four-story example, the random binary vector can be [1001]), and combines the genes of parent 1 at the locations of the ones and the genes of parent 2 at the locations of the zeros (for example, if for the first parent of chromosomes the sensors are placed at floors 1 and 3, parent 1 = [1010], and if the sensors for the second parent are placed at floors 2 and 4 , parent 2 = [0101], the offspring becomes [1100], so the sensors will be placed at floors 1 and 2).

The best fitness values (black dots) and the mean fitness values (blue dots) are represented at each generation in figure 6. It can be seen that the best and the mean values of the fitness function coincide at generation 4. As mentioned before, the algorithm converged before attaining the maximum number of generations because there was no improvement in the value of the fitness function for 10 successive generations.

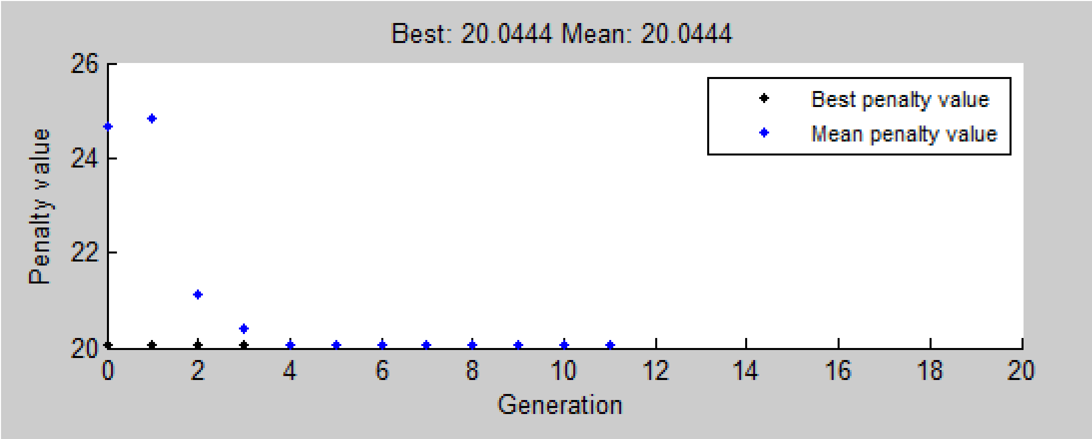

Figure 6: Best fitness value at each generation.

At the end of each generation, the best two locations of the sensors are determined based on their penalty value, the mismatch between the predicted and the measured displacements and velocities is minimal. This penalty value is calculated using the EnKF, which is a sequential data assimilation method that estimates the previously mentioned parameters *a*, *b*, *c* and *d* and the system state at each loop of the proposed algorithm. Figures 7 and 8 show the displacement and velocity estimates of floor 3 at the first generation and last generation, respectively. The comparison between these two figures shows the clear improvement in the mismatch between the EnKF predictions and the Bouc-Wen values for both the displacement and the velocity when sensors are placed at their optimal locations. The best locations of the two sensors are found to be on floors 2 and 3.

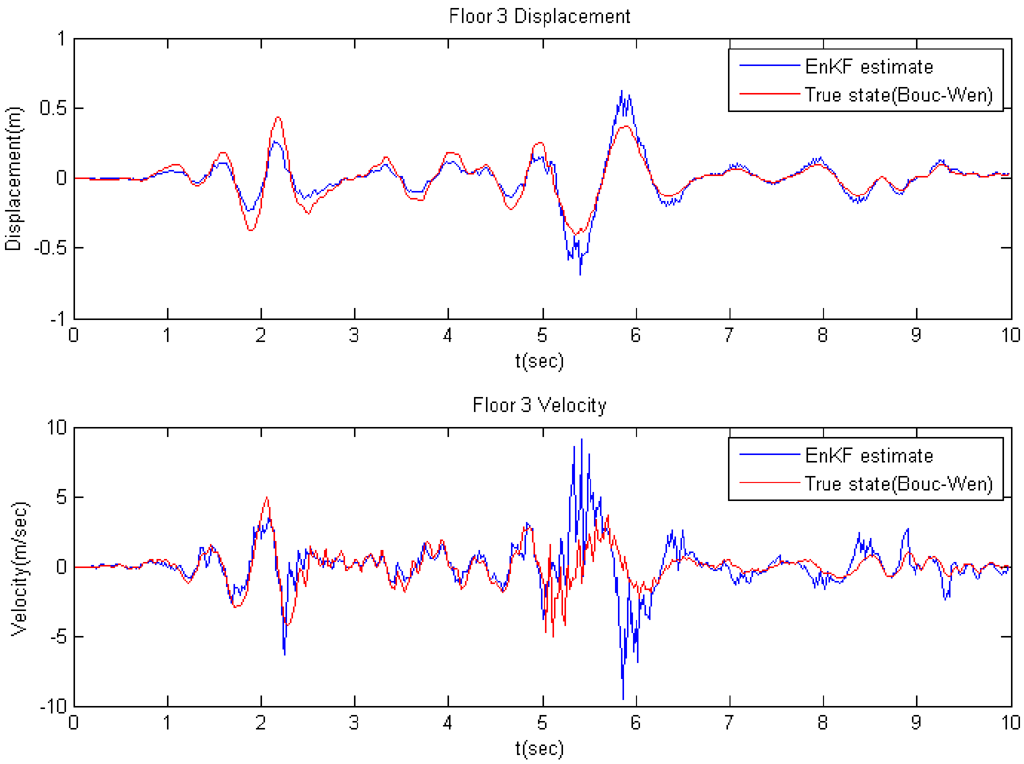

Figure 7: Estimates of the floor 3 displacement and velocity (at first generation).

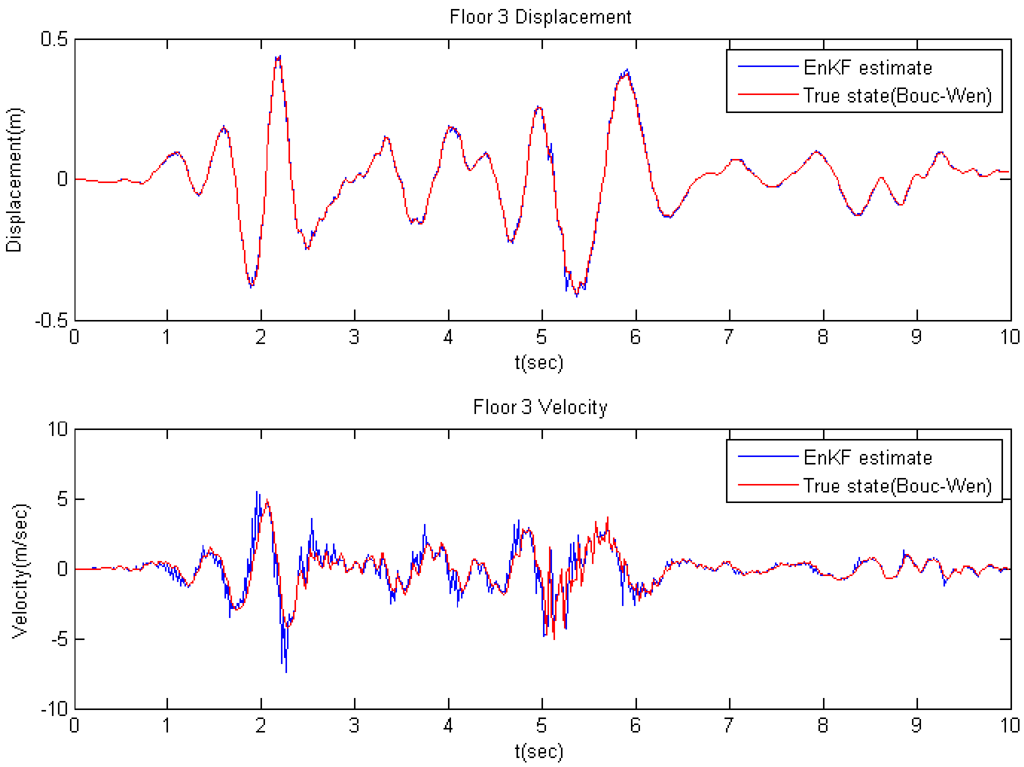

Figure 8: Estimates of the floor 3 displacement and velocity (at final generation).

5 CONCLUSIONS

This paper presented a new methodology for determining the optimal sensor locations to get the most necessary data for the purpose of system identification. This methodology is a combination between the genetic algorithm and the ensemble Kalman filter methods. The efficiency of the method was tested on a numerical example, consisting of determining the best locations of two available sensors on a four-story shear building. First the GA was used to generate an initial population of sensor locations from which the fittest individuals were selected as the best parents. The fitness function in this paper was assumed to be the difference between the synthetically generated measurements data and their respective predicted data from the EnKF. New better solutions resulted from applying some natural evolution operators on the parents chromosomes. The algorithm was repeated till convergence and the best locations of the two sensors were presented.

Future work will include a more complicated high-dimensional structure, to test the efficiency and the robustness of the proposed methodology in optimizing the sensors locations as well as their number for system identification and state estimation purposes.

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