

VIBRATIONS OF SPORTS STADIUM GRANDSTAND STRUCTURE DUE TO CROWD-JUMPING LOADS

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Abstract. *In recent years the destiny of stadiums has much changed. Formerly the stadiums were used occasionally. The present stadiums are much more versatile; take place in them may concerts and a variety of sporting events, while inside grandstands are located commercial units. As a result, the requirements for the design of these structures are more rigorous, in particular for dynamic loads. The design of grandstands has also evolved. This is due to the requirements of spectators who wish to have a perfect, direct and unobstructed view of anything, and be as close as possible playing action. In parallel, economic considerations limit grandstands supporting structure while maximizing sitting places. This combination leads to longer and more flexible brackets which in combination of spontaneous human behavior raises problems of vibration of the structure. A crowd of people can generate significant dynamic loads especially when it involves rhythmic jumping. Such situations occur during sporting events or concerts. The rhythm of music leads to synchronization behavior of people gathered in the grandstands. Such harmonic force may raise the issue of a resonant vibration, resulting in exceeding the limit states. The paper refers to computational examination of theoretical model of one grandstand in stadium in Krakow. The dynamic interaction function is applied as the dynamic loading reflects to actual behavior of the fans in the grandstands. The calculation of the dynamic response is carried out in the Abaqus 6.12 code. To best replicate reality, a numerical model was built based on a construction project taking into account the actual properties of the structural materials. The study analyzed the convergence of solutions determining the first three natural frequencies of the structure. The first calculated natural frequency ($f = 3.24\text{Hz}$) is close to frequency ($f = 2.8\text{Hz}$) which can generate fans crowd. On the other hand the second natural frequency of the structure ($f=4.5\text{Hz}$) is almost twice of frequency of "Labado" dance. The applied dynamic force cause increasing of stresses in structural elements about 40 - 60%. The level of calculated stresses as well as displacements of grandstand is safe for the structure.*

1 INTRODUCTION

In recent years the destiny of stadiums has much changed. Formerly the stadiums were used occasionally, once every two weeks, when the local team played their matches. The present stadiums are much more versatile; take place in them may concerts and a variety of sporting events, while inside grandstands are located commercial units. As a result, the requirements for the design of these structures are more rigorous, in particular for dynamic loads.

The design of grandstands has also evolved. This is due to the requirements of spectators who wish to have a perfect, direct and unobstructed view of anything, and be as close as possible playing action. In parallel, economic considerations limit grandstands supporting structure while maximizing sitting places. This combination leads to longer and more flexible brackets which in combination of spontaneous human behavior raises problems of vibration of the structure. A crowd of people can generate significant dynamic loads especially when it involves rhythmic jumping. Such situations occur during sporting events or concerts. The rhythm of music leads to synchronization behavior of people gathered in the stands. Such harmonic force may raise the issue of a resonant vibration, resulting in exceeding the limit states.

There are known examples of sports stadiums stands disasters that took place virtually anywhere in the world. These disasters have caused the death and injury of many spectators. There were various reasons for those disasters. Analysis of the causes of grandstands stadium disasters have also contributed to their design changes and reducing capacity of stadium. As example the findings of the report from the Hillsborough disaster occurred on 15 April 1989 at the Hillsborough Stadium in Sheffield, England resulted in the elimination of standing terraces at all major football stadiums in England, Wales and Scotland. Collapse of wooden terrace of Ibrox Stadium (Glasgow – 1902) resulting in the death of 26 people led to the prohibition of wooden scaffold type terraces in favour of terraces with a base of earth banking. The collapse of upper part of the makeshift scaffold-type stand in Bastia Stadium in 1992 were caused by lack of time, failing safety procedures, and fraudulent actions what resulted in critical construction errors. It led to the death of 18 fans and injuring over 2,000 more. In 26 November 2007 eight people were killed and about 150 were injured as a result of the collapse of the stands on one of the sectors of the stadium in the city of Salvador in the Brazilian state of Bahia.

The paper refers to computational examination of theoretical model of one grandstand in stadium "Vistula" in Krakow. The dynamic interaction function is applied as the dynamic loading. The loading reflects the actual behavior of the fans in the grandstands. This forcing corresponds to a dance called "Labado". The calculation of the dynamic response is carried out in the Abaqus 6.12 code. To best replicate reality, a numerical model was built based on a construction project taking into account the actual properties of the structural materials. The first calculated natural frequency ($f = 3.24\text{Hz}$) is close to frequency ($f = 2.8\text{Hz}$) which can generate fans crowd. On the other hand the second natural frequency of the structure ($f = 4.5\text{Hz}$) is almost twice of frequency of "Labado" dance. The applied dynamic force cause increasing of stresses in structural elements about 40 - 60%. The level of calculated stresses as well as displacements of grandstand is safe for the structure.

2 CHARACTERISTICS OF DYNAMIC LOAD GENERATED BY THE CROWD

The main issue the analysis is to define the dynamic structural loads. In humans, it is highly dynamic nature. The study loads and impacts caused by man engaged in medical science, in particular, orthopedics and biomechanics. As a result of studies of people received load model

delivered by the man on the ground. As the first results of the vertical component of the pressure that causes pedestrian, Skorecki published in 1966 [1]. Since then began to construct test platform for testing load that causes a man on the move. The main reason for conducting this study was progression in the sport. Data from measurements performed on the most popular Kistler platforms enable to assume the dynamic load induced by man in motion – comp. Fig.1.

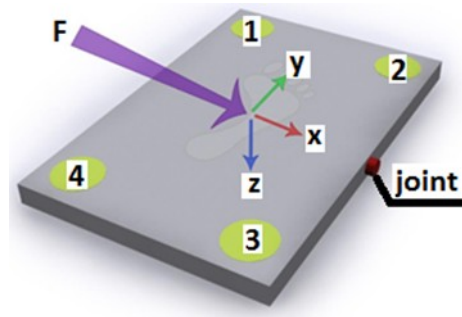


Figure 1: Distribution of sensors and forces on the Kistler's platform [2].

The basic element of the platform is a rigid, rectangular plate supported at four corners of the piezoelectric gauges providing information about the vertical and horizontal load. These professional made measuring devices are now standard in research of forces transmitted by the man on the ground during normal operation and sport activities. Knowledge of the impact on the ground in time and space used in sports to evaluate the forces resulting in human osteo-muscular system, sport techniques, sports footwear type assessment, and many other biomechanical factors [1]. Biomechanics research results made it possible to determine the load that transfers to the structure by a man on the move. The vertical component of the load has a decisive influence on the induced deformations and vibrations of structures. Basic human physical activity is cyclical, and on this basis, a classification of human physical activity of the corresponding frequency range is introduced in Table 1 [1].

Type of activity	Full range Hz	Slow, Hz	Average, Hz	Fast, Hz
Marching	1,4-2,4	1,4-1,7	1,7-2,2	2,2-2,4
Run	1,9-3,3	1,9-2,2	2,2-2,7	2,7-3,3
Jumps	1,3-3,4	1,3-1,9	1,9-3,0	3,0-3,4

Table 1: Classification of human physical activity [1].

2.1 Human impact – jumps

Jumps (jumping in place) are physical activity, which is characterized by four phases: preparation (lowering), reflection (climb), flight and landing – cf. Fig. 2.

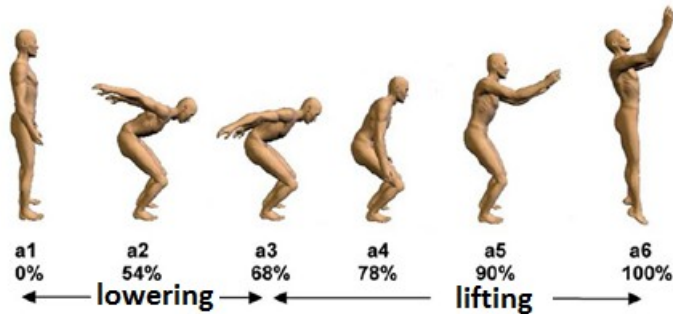


Figure 2: Phase of human movement during jump [1].

The main parameters characterizing the rhythmic leaps and jumps are frequency as a function of time and time of the dynamic interaction. Depending on the type of jumps, the frequency is typically in the range of 1.8 to 3.4 Hz. Vertical dynamic effects are mainly generated while jumping. The time of this interaction depends primarily on: frequency, bounce height, weight of person, gender, type of footwear, the conditions of the substrate surface (soft surface) and its movements – cf. Fig. 3. Jumping's in place simultaneously on both feet at the same time is the most dangerous for the constructions [3].

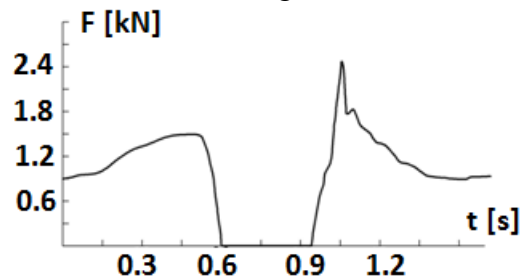


Figure 3: The dependence of the impact F of vertical jump vs time [1].

Jumping is physical activity that, unlike the gait is practically not sensitive to vibrations of the substrate (in the sense of the generated load). This is due to the fact that the vertical movement of the body can achieve the displacement amplitude times greater than the displacement of the substrate. Displacement of basic components of human body due to typical jump amounts to approx. 0.5 m.

2.2 Dynamic loads for dancing and jumping - standard recommendations

In dynamic analysis it is often convenient to express the applied loading as a Fourier series representing the variation of load with time as a series of sine functions. Any periodic loading can be decomposed in to a combination of a constant load and several harmonics. Synchronized dynamic loading [4] caused by activities such as jumping and dancing are periodic and mainly depend upon:

- a) the static weight of the jumper(s)/dancer(s) (G);
- b) the period of the jumping/dancing load(s) (T_p);

c) the contact ratio (α), i.e. the ratio of the duration within each cycle when the load is in contact with the floor and the period of the jumping/dancing.

Mathematically the load at any instant (t) may be expressed as:

$$F(t) = G \left[1 + \sum_{n=1}^{\infty} r_n \sin \left(\frac{2\pi n}{T_p} t + \varphi_n \right) \right] \quad (1)$$

where:

n - the number of the harmonic being considered 1, 2, 3, ...;

r_n - the dynamic load factor for the n^{th} harmonic;

φ_n - the phase lag of n^{th} harmonic;

T_p - jump period.

The values of r_n and φ_n are functions of the value of the contact ratio α . In practice for the evaluation of displacement and stresses, only the first few harmonics need be considered as the structural response at higher values is generally not significant. It is generally sufficient to consider the first three harmonics for vertical loads and the first harmonic for horizontal loads. For the calculation of acceleration, additional harmonics will need consideration. Table 2 gives typical values for four various activities [5, 6].

Activity	Contact ratio α
Pedestrian movement	$\frac{2}{3}$
Low impact aerobics	
Rhythmic exercises	$\frac{1}{2}$
High impact aerobics	
Normal jumping	$\frac{1}{3}$
High jumping	$\frac{1}{4}$

Table 2: Typical values of the contact ratio α for various activities [5].

The resultant values of r_n and f_n for a given period of dancing T_p or a jumping frequency ($1/T_p$) may be obtained from literature (e.g. [4]). For individual loads the frequency range that should be considered is 1.5 Hz to 3.5 Hz and for larger groups 1.5 Hz to 2.8 Hz as coordinated movement at the higher frequencies is impractical. For a large group the load $F(t)$ calculated from equation (1) may be multiplied by 0.67 to allow for lack of perfect synchronization. Vertical jumping also generates a horizontal load which may be critical for some structures, e.g. temporary grandstands. A horizontal load of 10 % of the vertical load should be considered.

Equation (1) represents the vertical action of one person performing any rhythmic movement. At the time of its cessation, the load is reduced to a static load. The values of coefficients r_n and φ_n of the formula (1) can be determined using functions (2) and (3) developed by Ji, Ellis and Wang [5, 7]. These functions, however, only apply to the division of activities described in Table 2.

$$r_n = \begin{cases} \frac{\pi}{2} & \text{if } 2n\alpha = 1 \\ \left| \frac{2\cos(n\pi\alpha)}{1-(2n\alpha)^2} \right| & \text{if } 2n\alpha \neq 1 \end{cases} \quad (2)$$

or

$$\varphi_n = 0 \text{ if } 2n\alpha = 1$$

$$\varphi_n = \left\{ \begin{array}{l} \tan^{-1} \left(\frac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)} \right) - \pi, \frac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} < 0 \\ -\frac{\pi}{2}, \sin(2n\pi\alpha) = 0 \\ \tan^{-1} \left(\frac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)} \right), \frac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} > 0 \end{array} \right\} \text{ if } 2n\alpha \neq 1 \quad (3)$$

Values of the first six Fourier coefficients series, which can be regarded as sufficient for its implementation are contained in Table 3.

		n=1	n=2	n=3	n=4	n=5	n=6
$\alpha=2/3$	r_n	9/7	9/55	2/15	9/247	9/391	2/63
	φ_n	$-\pi/6$	$-5\pi/6$	$-\pi/82$	$-\pi/6$	$-5\pi/6$	$-\pi/2$
$\alpha=1/2$	r_n	$\pi/2$	2/3	0	2/15	0	2/35
	φ_n	0	$\pi/2$	0	$-\pi/2$	0	$-\pi/2$
$\alpha=1/3$	r_n	9/5	9/7	2/3	9/55	9/91	2/15
	φ_n	$\pi/6$	$-\pi/6$	$-\pi/2$	$-5\pi/6$	$-\pi/6$	$-\pi/2$

Table 3: Fourier coefficients and phase lags for the different contact ratios [8].

Figure 4 shows the course of the function (3.4) at time applying 3 and 12 coefficients of a Fourier series. The diagram illustrates two cycles of "normal jumping" with frequency equals 2 Hz and the contact ratio $\alpha = 1/3$. Contact ratio $\alpha = 1/3$ is equivalent to that two-thirds of each cycle of the body has no contact with the ground. For the calculations it was assumed normalized load of 1.0 corresponding to the static weight of the jumper ($G_s = 1$). It is easy to note that the higher coefficients of the Fourier series ($n > 3$), are not critical to this maximum. However, it can affect the response of construction, in particular at the critical value of the frequency ratio to the natural frequency of the structure.

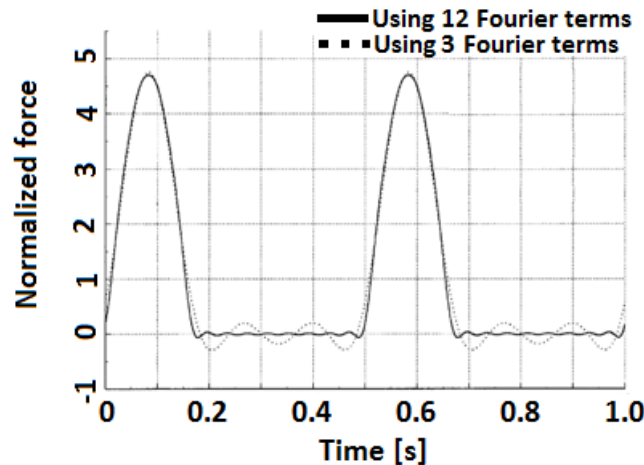


Figure 4: Load-time history for two jumps with frequency equals 2 Hz [8].

Equation (1) describes the dynamic load derived from one person. This can be extended to the case of the populated grandstand. Multiplying the static effects in this case will be lower than in the case of one person. This is due to imperfect coordination in jumping group of spectators in relation to the one person. Therefore, the load equation for a crowd jumping will be as follows:

$$F(f, x, y, t) = G(x, y) \left(1 + C_e \sum_{n=1}^{\infty} r_n \sin \left(\frac{2n\pi}{T_p} t + \varphi_n \right) \right) \quad (4)$$

where:

$F(f, x, y, t)$ – distributed force which varies with time at a comfortable frequency,

$G(x, y)$ – density and distribution of static loads from spectators,

C_e – crowd dynamic coefficient, $C_e \in (0, 1)$

The C_e coefficient depends on the coordination of people in the group, the type of jumps (dance) and the frequency of the rhythm of the music. Numerical simulations in which the phase lag of each person treated as a random variable normally distributed subordinate, shows a reduction of C_e coefficient of one-third in the case of large groups [9]. On the basis of the results shown in [9], the C_e coefficient can be described as function of number of people (v) in form:

$$C_e(v) = -0.0799 \ln(v) + 0.9797 \quad (5)$$

Another way to cover a greater number of people is correction of the Fourier series. These values were determined experimentally by Ji and Ellis [5], but only for the first three terms of the series - cf. Equation (6).

$$F(f, x, y, t) = G(x, y) \left(1 + \sum_{n=1}^{\infty} r_{n,v} \sin \left(\frac{2n\pi}{T_p} t + \varphi_n \right) \right) \quad (6)$$

where:

$$r_{1,v} = 1.61v^{-0.082}$$

$$r_{2,v} = 0.94v^{-0.24}$$

$$r_{3,v} = 0.44v^{-0.31}$$

3 ANALYZED STRUCTURE AND ITS NUMERICAL MODEL

3.1 Characteristics of the structure

Analyzed the northern grandstand is single level and is part of the municipal stadium in Krakow - cf. Fig. 5. Grandstand, consists of five sectors and allows the presence of 5806 spectators. The main structural element of the grandstand is RC frame. The frame consists of six columns with cross section 1.4 x 2.2 m, connected in two rows of girders, the lower section 1.6 x 0.96 and the upper 2.0 x 2.2 m - cf. Figure 4.8. The frame is the base for steel sloping beams (IKS 1000-3) supporting prefabricated concrete slabs with a thickness of 9cm. In the mid-span beams are based on the columns (HEB 550), which, by means of girders (HKS 550x5) are connected with a RC frame.

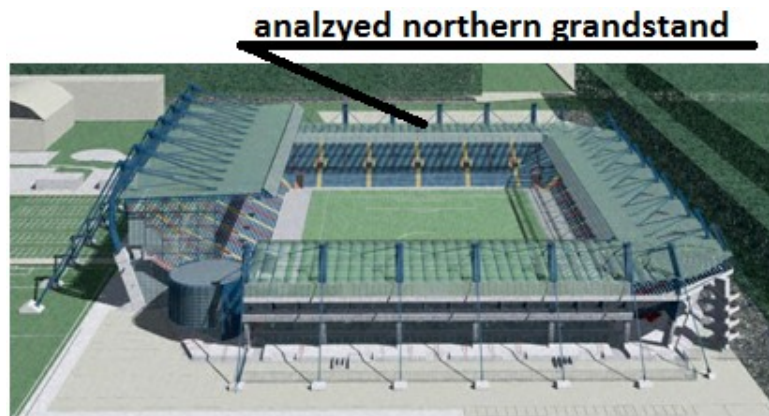


Figure 5: Visualization of the municipal stadium in Krakow and analyzed grandstand [10].

Cores of the main roof superstructure are designed with a tubular cross-sections of diameter $D = 1040$ mm and a thickness of 80 mm. Pylons support the truss girders at the distances of 9.9 m. Truss girders are braced by means of trusses at a spacing of 3.3 m. They support structure under purlins – comp. Figs. 6, 7. Roof's covering is made of polycarbonate cellular structure.



Figure 6: North grandstand view of the stadium [11].

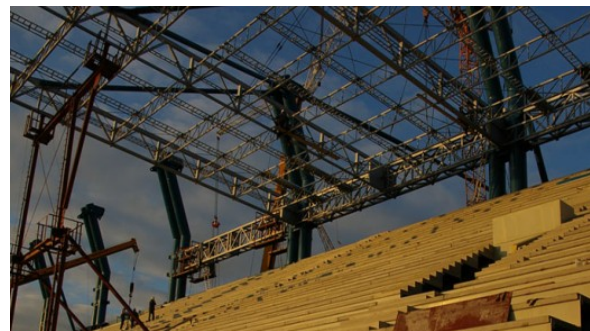


Figure 7: Construction of the roof of the north grandstand [11].

Tables 4 and 5 present the basic geometrical and structural data of the analyzed grandstand.

Building area	2937.66 m ²	
The total area of closed spaces	3588.88 m ²	
Cubature of closed spacer	17233.85 m ³	
The dimensions of the stands with peripheral structural elements	length	103.00 m
	width	50.86 m
	height	28.83 m
Number of seats	5806	

Table 4: Basic technical data of the analyzed grandstand.

Concrete	B30 (C25/30)
Reinforcing steel	A-III 34GS
Structural steel	St3S, 18G2A

Table 5: The used materials.

3.2 Numerical model of the grandstand

3D numerical model of the grandstands is made in the Abaqus 6.12 code - cf. Fig. 8. The beam - shell model is adopted considering the geometry of the object. The model assumes simplification. At the support points full restraint were used ($u_x, u_y, u_z, r_x, r_y, r_z = 0$), what means that modeling of piles and flexibility of the substrate were neglected. Stairs of the grandstand are modeled using shell elements. Double span panels are connected to the main structure by means of joints. Joints also used in the connections between columns and sloping girders as well as in nodes connecting roof and pylons. Other connections are defined as rigid as they are welded or monolithic (RC). Due to the low stiffness roof cover with polycarbonate its modeling was neglected. The model uses linear elements of "B31" and the shell "S4R". Selection of an appropriate finite element mesh density was adopted on the basis of the convergence of the natural frequency (see Section 4). The material characteristics of the structure are listed in table 6.

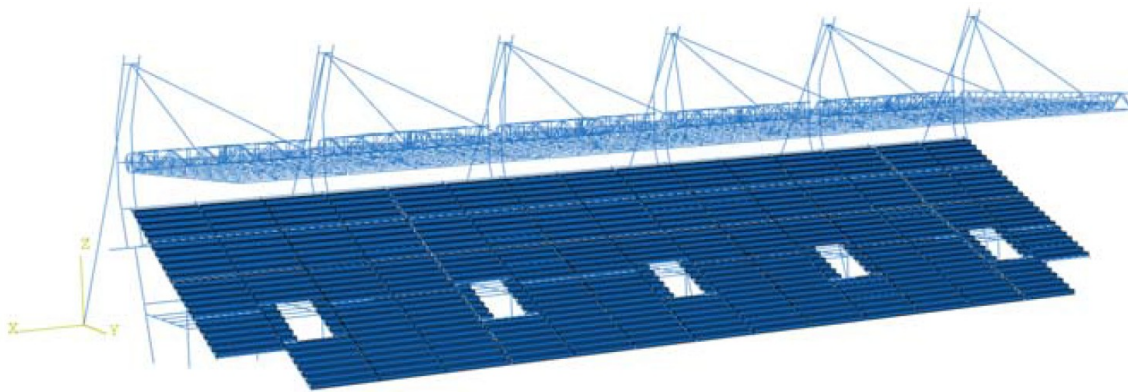


Figure 8: Numerical model of the analyzed grandstand.

Concrete	density	2600 kg/m ³
	Young's modulus	37 GPa
	Poisson ratio	0.15
Steel	density	7850 kg/m ³
	Young's modulus	210 GPa
	Poisson ratio	0.33

Table 6: Material characteristics of the structure.

4 RESULTS OF THE NUMERICAL ANALYSIS

4.1 Sensivity analysis

Several different grid mesh were analyzed and their influence on the convergence of the value of the first three natural frequencies of the model. Three finite element meshes for preliminary analysis were adopted. Data concerning number of elements and values of natural frequencies are summarized in Table 7. Several different grid mesh were analyzed and their influence on the convergence of the value of the first three natural frequencies of the model. Three finite element meshes for preliminary analysis were adopted. Data concerning number of elements and values of natural frequencies are summarized in Table 7.

Number of elements	f_1 [Hz]	f_2 [Hz]	f_3 [Hz]
14910	3,18	4,40	5,33
47854	3,24	4,50	5,40
160936	3,24	4,49	5,41

Table 7: The sensitivity of frequency on the number of elements.

4.2 Calculated natural frequencies and assumed dumping

The first stage of dynamic analysis concerned the calculation of the natural frequency of the structure to the value of 8.4 Hz according to [6]. The calculated values are summarized in the table 8.

No. of frequency	Value of the natural frequency [Hz]
1	3.24
2	4.50
3	5.40
4	6.70
5	7.50
6	>8.40

Table 8: The calculated natural frequencies of the analyzed grandstand.

In this paper Rayleigh damping was adopted (proportional to mass and stiffness) and the coefficients of proportionality were determined from equation (7):

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \quad (7)$$

using the following data:

- $f_1 = 3.24$ Hz, $f_2 = 4.50$ Hz,
- damping coefficients ξ_1 and ξ_2 corresponding to f_1 and f_2 are equal 1.3% and 1.5% respectively.

For the above data, the proportionality coefficient values are equal to $\alpha = 0.186$ and $\beta = 0.00083$.

4.3 Applied dynamic loads

The weight of one person was assumed equal to 0.78kN, which, with a capacity of 5806 people on the grandstand gives the average load equal to 3 kN/m². Two functions of dynamic load (jumping) were used in further analysis. The first one refers to dance "Labado" which describe the actual behavior of the spectators in the grandstand. Jump function during one period T_p describes equation (8):

$$F_s(t) = \begin{cases} K_p G_s \sin \frac{\pi}{t_p}, & \text{if } 0 < t < t_p \\ 0, & \text{if } t_p < t < T_p \end{cases} \quad (8)$$

where:

G_s – static action,

t_p – contact time,

K_p - coefficient determining the growth of the dynamic interaction calculated from eq. (9):

$$K_p = \frac{\pi}{2\alpha} \quad (9)$$

This function, excluding the impact of static influence is shown in fig. 9.

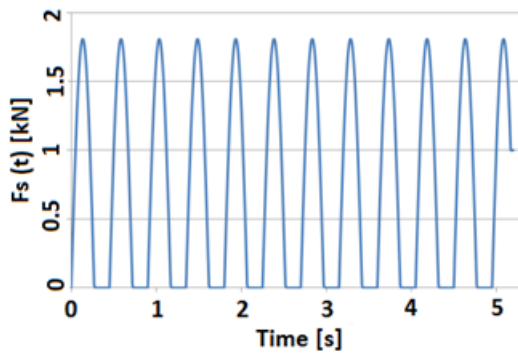


Figure 9: A function of load (dance "Labado") vs. time, at frequency of 2.2 Hz.

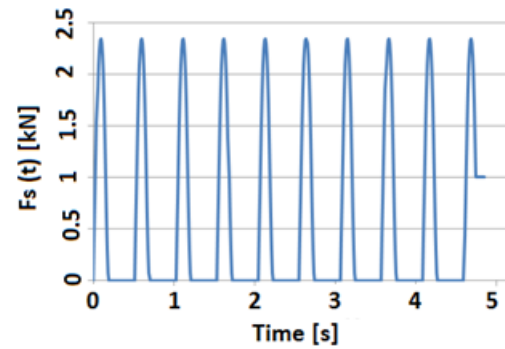


Figure 10: A function of load vs. time, at a frequency of 1.9 Hz.

The second function was obtained from equation (1) for jumps frequency equal 1,9Hz using data from table 3 and assuming value of C_e equal 0.5 - cf. Fig. 10.

4.4 Selected elements and values to be analyzed

Dynamic response analysis was performed for the 3D adopted model of grandstand and two assumed loads. This chapter presents the results of analyzes for a few selected elements of the grandstand - cf. Fig. 11. Highlighted items are:

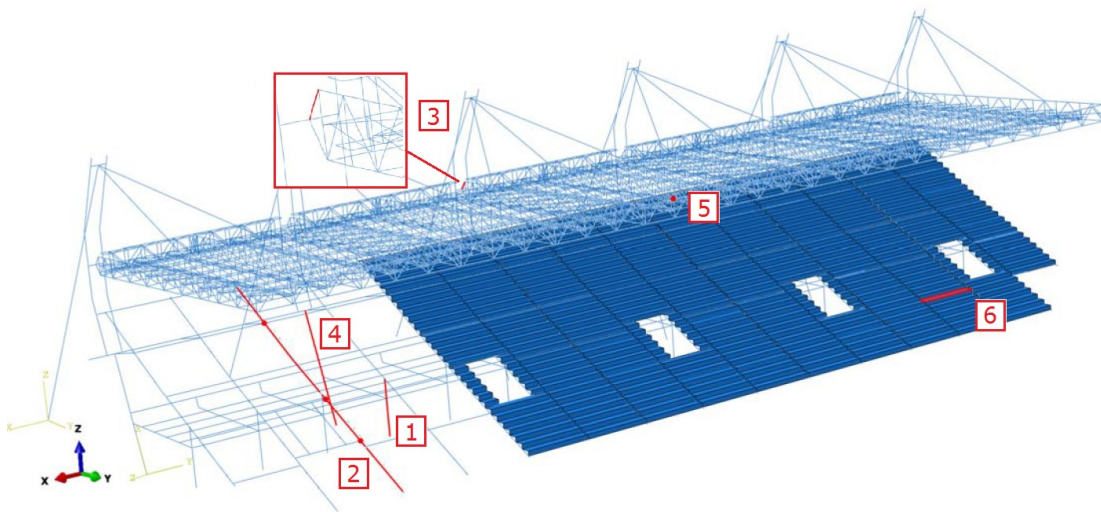


Figure 11: Analyzed elements.

- 1 – HEB 550 steel column; HMM stress,
- 2 – IKS 1000-3 sloping girder; HMM stress, displacement u_z ,
- 3 – RK 200x6.3 cross brace; HMM stress,
- 4 – RC column 1.4 x 2 m; principal stress,
- 5 – free end of the roof girder; displacement u_z ;
- 6 – plate of audience; displacement u_z .

In order to obtain the maximum output value (from the weight and dynamic loads) it has been examined several locations of people hopping on grandstand. These situations illustrate the figures 12 - 16.

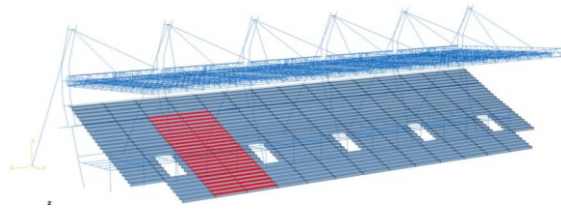


Figure 12: Location of fans no. 1.

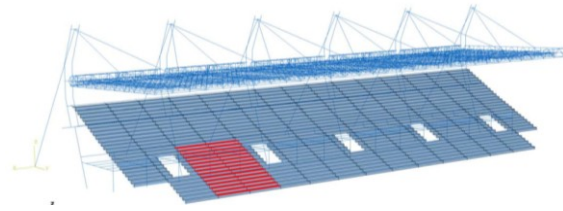


Figure 13: Location of fans no. 2.

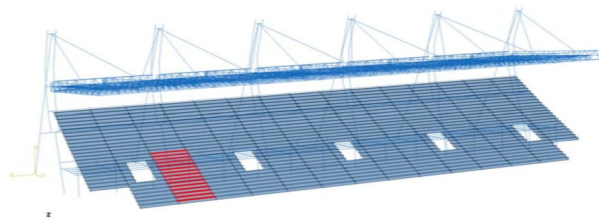


Figure 14: Location of fans no. 3.

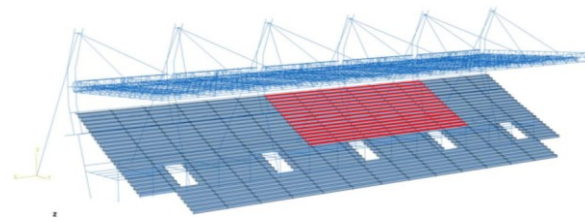


Figure 15: Location of fans no. 4.

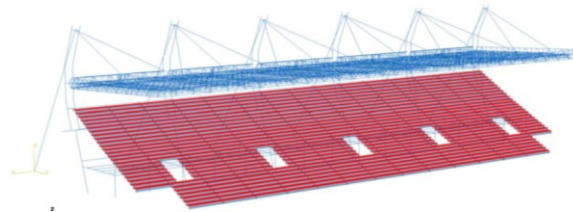


Figure 16: Location of fans no. 5.

4.5 The dynamic response of the grandstand

The response of the grandstand due to applied loads was calculated by integrating the equations of motion using the module Abaqus / Implicit. The resulting maximum stress values (HMH - for steel parts, the principal - for RC elements) and vertical displacements were compared with static and the codes' values. Table 9 summarizes the results of stress HMH for steel elements. Changes of the values of these stresses over time are shown in Fig. 17 - 19. The response of the grandstand due to applied loads was calculated by integrating the equations of motion using the module Abaqus / Implicit. The resulting maximum stress values (HMH - for steel parts, the principal - for RC elements) and vertical displacements were compared with static and the codes' values. Table 9 summarizes the results of stress HMH for steel elements. Changes of the values of these stresses over time are shown in Fig. 17 - 19.

Element number	Static value	Dynamic value for excitation with 1.9 Hz	Dynamic value for excitation with 2.2 Hz
1	25.5 MPa	44.2 MPa	39.0 MPa
2	104.2 MPa	172.9 MPa	169.6 MPa
3	46.0 MPa	66.6 MPa	55.1 MPa

Table 9: Values of HMH stress in the steel elements.

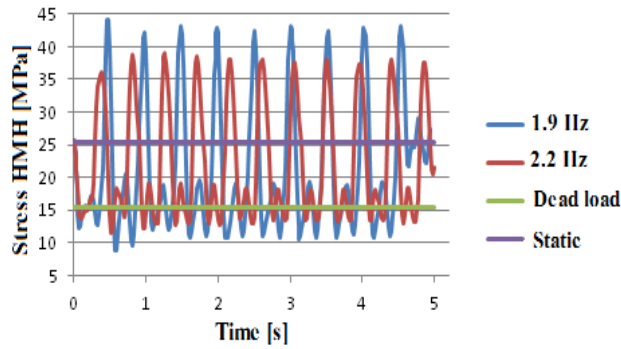


Figure 17: Stress HMM in element no. 1.

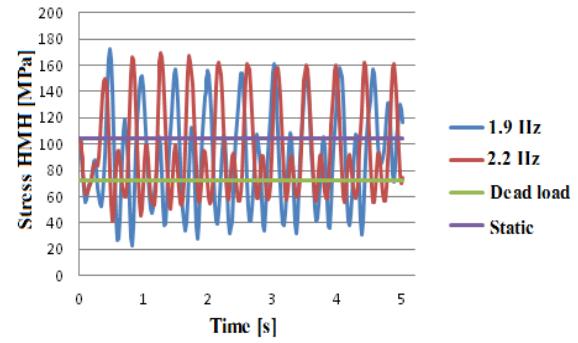


Figure 18: Stress HMM in element no. 2.

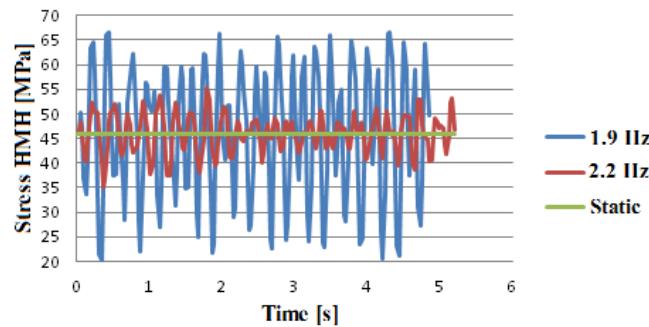
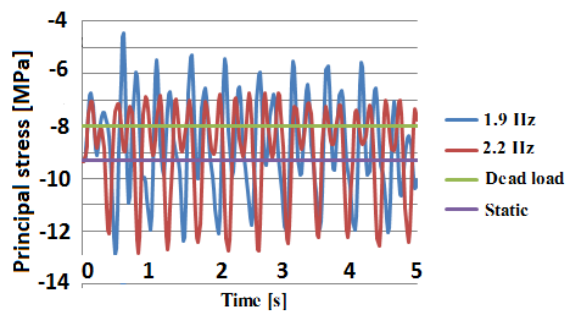
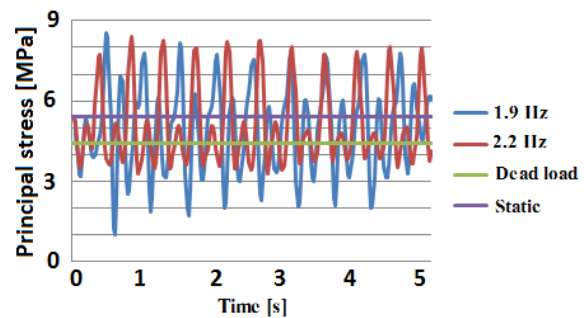


Figure 19: Stress HMM in element no. 3.

Table 10 presents the results of principal stresses for one of RC column in support cross-section - element no. 4. The graphs in Figures 20 and 21 illustrate the change of these values over time.

Part of the column	Static value	Dynamic value for excitation with 1.9 Hz	Dynamic value for excitation with 2.2 Hz
Tension	5.4 MPa	8.5 MPa	8.4 MPa
Compression	-9.3 MPa	-12.9 MPa	-12.8 MPa

Table 10: Values of principal stresses in element no. 4.

Figure 20: Stress σ_x in element no. 4 - compression.Figure 21: Stress σ_x in element no. 4 - tension.

The limit values according to the EN 1992-1 and EN 1993-1 are equal:

- for concrete C25/30:

$$\frac{f_{ctd}}{f_{ctk}} = \frac{f_{ctd}}{f_{ctk}} \quad (10)$$

where:

- characteristic value of compressive strength of concrete,
- partial factor of concrete,
- design value of compressive strength of concrete,
- the average value of the axial tensile strength of concrete.

- for steel S235:

$$\frac{f_{yk}}{f_{yk}} = \frac{f_{yk}}{f_{yk}} \quad (11)$$

where:

- yield strength - the characteristic value,
- material factor,
- yield strength - the design value.

Table 11 shows the vertical component of displacements for elements no. 2, 5 and 6. Changes of vertical displacements for elements no. 2 and 5 vs. time are shown in Figure 22 and 23.

Element number	Static value	Dynamic value for excitation with 1.9 Hz	Dynamic value for excitation with 2.2 Hz
2	1.6 cm	3.89 cm	4.12 cm
5	3.80 cm	5.46 cm	4.40 cm
6	0.02 cm	0.25cm	0.52cm

Table 11: Displacement values for the elements no. 2, 5, and 6.

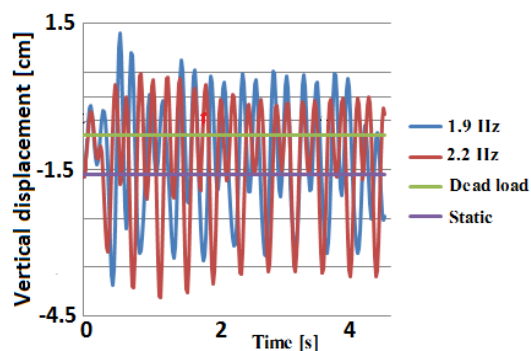


Figure 22: Vertical displacement of the element no. 2 in the middle of the span vs. time.

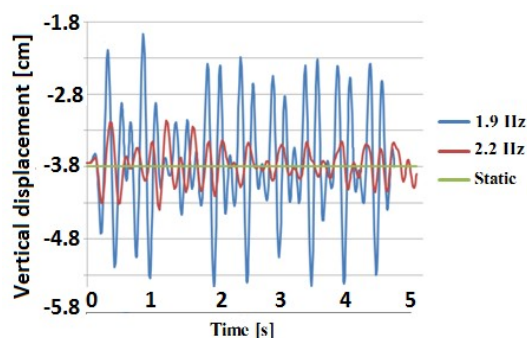


Figure 23: Vertical displacement of the end of the roof girder vs. time (element no. 5).

Span of the roof girder between supports is 1418cm. So the allowable deflection according to polish standard is equal 1/200 of the span, ie. 7.1cm. Maximal vertical displacement of the free end of the roof beam is 3.48 cm - cf. Fig. 23.

5 CONCLUSIONS

The study deals with the problem of vibration of the grandstand the municipal stadium in Krakow caused by spontaneous behavior of spectators. Calculations were carried out in the ABAQUS 6.12. Dynamic load was applied with characteristics that led one of the speedway stadiums in Poland to the state of emergency. Analysis of the results of calculation adopted structural model allowed us to formulate the following conclusions:

- I. The first natural frequency of the structure ($f_1 = 3.24$ Hz) is close to maximal frequency of dynamic load which can be generated by crowd ($f = 2.8$ Hz).
- II. The second natural frequency of the structure ($f_2 = 4.5$ Hz) is almost twice greater than frequency of excitation by dance called „Labado” ($f = 2.2$ Hz). It results increasing level of vibrations of construction elements.
- III. Dynamic excitation causes an increase in stress in the elements of a 40% - 66%. Excitation also has an impact on the rooftop structures and increases the effort of some elements.
- IV. Neglect of dynamic effects in the design of the grandstand entertainment facilities can be fatal. In order to prevent potential excessive vibrations of the structure should ensure a sufficiently high first natural frequency.
- V. In the absence of resonance assuming dynamic load with a lower frequency it results in an increased dynamic effects for the system. The higher the frequency the lower the spikes and hence a greater dynamic load.
- VI. The level of calculated stresses and displacements does not endanger the safety of use of the grandstand.

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