COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

MODELING AND IDENTIFICATION OF THE DYNAMIC RESPONSE OF AN EXISTING THREE-STORY STEEL STAIRWAY

Triantafyllos K. Makarios[1], George D. Manolis[2], Ioanna Karetsou[3], Michael Papanikolaou[3] and Vasiliki Terzi[4]

[1] Assistant Professor
Institute of Static and Dynamic of Structures, Department of Civil Engineering, Aristotle University, Thessaloniki GR-54124, Greece
e-mail: makariostr@civil.auth.gr

[2] Professor
Institute of Static and Dynamic of Structures, Department of Civil Engineering, Aristotle University, Thessaloniki GR-54124, Greece
gdm@civil.auth.gr

[3] Dipl. Eng., MSc Civil Engineer
Institute of Static and Dynamic of Structures, Department of Civil Engineering, Aristotle University, Thessaloniki GR-54124, Greece
joykaretsou@hotmail.com , mipapani@civil.auth.gr

[4] PhD & Staff Member (E.D.I.P.)
Institute of Static and Dynamic of Structures, Department of Civil Engineering, Aristotle University, Thessaloniki GR-54124, Greece
terziv@civil.auth.gr

Keywords: Identification, mode shapes, modal response, acceleration time histories

Abstract. *In this work, we present a modal shape identification technique for an existing steel stairway located within the building complex comprising the Civil Engineering department of Aristotle University in Thessaloniki, Greece. The aforementioned flexible steel stairway was instrumented using a local multi-channel network of accelerometers in September 2014. We installed on the stairway two 12 bit-nominal resolution, digital uniaxial accelerometers of the type KUOWA-PCD-30A, connected by cables and with 'common time' and 'common start' characteristics. The dominant modes of vibration of the stairway were computed by the 'modal response acceleration time history' methodology. In parallel, a detailed finite element method model of the stairway was formed and calibrated according to the ambient vibration results. We note that the identification procedure used for the dynamic characteristics of spatial structures yields results that can be used to develop a family of numerical models for the stairway ranging from the simple single-degree-of-freedom system to highly detailed finite element method models, while some useful information on the theoretical procedure for the identification of modal shapes is included herein.*

1 INTRODUCTION

In the last few years, issues such as “monitoring” and “structural integrity” of structures is being developed rather vigorously by employing suitable, multi-channel networks for measuring acceleration response time-histories, followed by appropriate data processing techniques. If the structural response remains within the linear elastic range, then the recorded acceleration response time-histories contain the structure's modal response. It is well known that a clear deterministic relationship does not exist between ambient vibration loading or ground excitations as input and the ensuing structural response as output. Furthermore, the methodologies used to identify the dynamic characteristics (i.e., natural frequencies, mode shapes and modal damping ratios) of a structure must take into account the instrumentation configuration that is used on that structure. Thus, many methodologies are adapted to investigate dynamic structural response on a case-by-case basis (buildings, bridges, towers, dams, and also other categories of structures such as aircraft frames). Nevertheless, the main objective in all cases is to identify the dynamic characteristics of the structure through an analytic processing of the measured response. In the past, various deterministic and stochastic methods have been proposed (see Basseville *et al*, 2001; Brincker *et al*, 2001; Peeters & De Roeck, 2001; Wenzel & Pichler, 2005; Overschee & De Moor, 1996; Makarios, 2012 & 2013; Manolis *et al* 2014). However, the engineering community has yet to agree on a unique, well-documented procedure for identifying the dynamic characteristics of any structures or structural system through a detailed investigation of their measured response. The present paper identifies the dynamic characteristics of a specialized type of steel stairway with continuous mass distribution and stiffness. These stairways serve as escape routes from high-rise buildings, are placed externally to the building and can be classified as being very flexible structures. The analysis procedure presented in this work is adapted to the particular instrumentation configuration used on the steel stairway, supplemented by the method of “modal time-histories”. More specifically, the work is based on material drawn from the MSc diploma thesis of the authors, see Karetsou (2014) and Papanikolaou (2014), where ambient vibrations induced by ordinary, daily use of the stairway by pedestrians were used as the source of excitation to measure the stairway response. Within this framework, a parallel finite element modeling procedure (SAP 2000) was developed and the results obtained were calibrated against the experimental evidence. These finite element models were then reconfigured and used to reproduce details in the dynamic response of the stairway. This way, the numerical models help extend the results of the experimental effort and can be used to trace ageing and deterioration during the useful service life of the stairway.

2 THE STEEL STAIRWAY AND ITS MATHEMATICAL MODEL

The steel stairway under study is located in the courtyard behind the main building of the Civil Engineering Department at the Polytechnic School of Aristotle University in Thessaloniki, Greece, see Fig.1. The stairway was constructed in 1982 and consists of three spans which terminate respectively in three landing levels of a low rise annex building housing laboratories.

Fig.1: The steel stairway with side (left) and front (right) views

The second span is supported by two beams (IPN140 cross section), while the beams supporting the other spans as well as the landings are IPN100 cross sections. The columns supporting the beams at the second landing, as well as the main column of the third landing, are IPN140 cross section. The remaining columns of the structure are IPN100 cross sections. As illustrated in Fig.1, the IPN140 columns are connected by cross diagonal bars with RHS 60x40 cross section. The cross bars reinforce the later stiffness of the stairway and reduced the buckling length of the columns. The vertical stiffness of the landing is reinforced by the contribution of T50 beams on the first landing and IPN beams on the other two. Finally, the landings, the treads and the risers are made of sheet metal plates of 3mm thickness. The same material is used as protective sideway cover for the beams. Next, the instrumentation scheme used during a two week time period comprised three uniaxial accelerometers, as illustrated in Fig. (2).

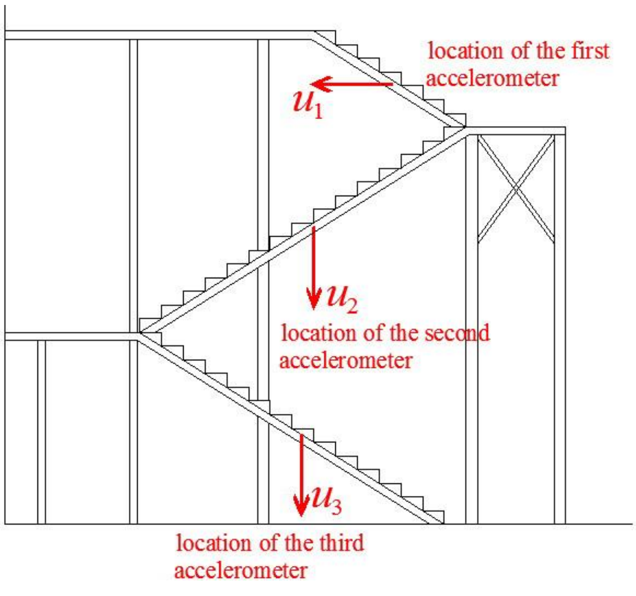

Fig.2: Location of accelerometers at important degrees of freedom of the structure

We note that the stairway under study cannot be modeled as a lumped mass system since there is no detectable concentration of mass at any level. Therefore, the structural system is strongly continuous with both continuous mass and stiffness distributions, and this causes difficulties in the identification of the eigen-modes. In accordance to the accelerometer placement scheme, the vector $\mathbf{u}$ corresponds to three basic degrees of freedom, and the generalized matrices for the mass $\mathbf{M}$, the stiffness $\mathbf{K}$ and the damping $\mathbf{C}$ of the steel stairway attain the following form:

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}, \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

Therefore, the equation of motion of the steel stairway subjected to environmentally-induced dynamic loads is the following:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{C}\,\dot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \mathbf{W}(t) \tag{1}$$

In the above, $\mathbf{W}(t)$ is the load vector encompassing the unknown ambient vibration loading on the specified degrees of freedom.
For this case, the solution of eq.(1) is of the following form:

$$\mathbf{u}(t) = \boldsymbol{\varphi}_1 q_1(t) + \boldsymbol{\varphi}_2 q_2(t) + \boldsymbol{\varphi}_3 q_3(t) = [\boldsymbol{\varphi}_1 \quad \boldsymbol{\varphi}_2 \quad \boldsymbol{\varphi}_3] \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix} = \boldsymbol{\Phi}\,\mathbf{q}(t) \tag{2}$$

where,

$$\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1 \quad \boldsymbol{\varphi}_2 \quad \boldsymbol{\varphi}_3] = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{bmatrix}, \qquad \mathbf{q}(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{bmatrix}$$

More specifically, $\boldsymbol{\varphi}_1$, $\boldsymbol{\varphi}_2$, $\boldsymbol{\varphi}_3$ are the corresponding three eigen-modes, $\boldsymbol{\Phi}$ is the modal matrix of the steel stairway and $q_i(t), i = 1, 2, 3$ are the corresponding time functions (i.e., generalized coordinates) of each eigen-mode
It can been proved that the system of eq. (1), which is particular to the placement scheme of the three accelerometers, is equivalent to a single degree of freedom vibrator response for each eigne-mode *i* and is described by eq. (3) below (see Makarios 2012 & 2013):

$$\ddot{q}_i(t) + 2\xi\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = \frac{\boldsymbol{\varphi}_i^T\,\mathbf{W}(t)}{\boldsymbol{\varphi}_i^T \mathbf{M}\,\boldsymbol{\varphi}_i} \tag{3}$$

Equation (3) shows that the exact solution for the calculation of the time varying modal functions $q_i(t)$ requires knowledge of the environmentally-induced dynamic loading vector $\mathbf{W}(t)$, which is of course impossible to estimate a priori. However, by observing eq. (3), it is clear that the exact solution is a superposition of the solution $q_{o,i}(t)$ to the homogenous equation and of the particular solution $q_{w,i}(t)$, which depends on the dynamic loading $\mathbf{W}(t)$:

$$q_i(t) = q_{o,i}(t) + q_{w,i}(t) \tag{4}$$

However, it is already known that in the free vibration case, the exact solution $q_{o,i}(t)$ for the homogenous equation of the structure coincides with the final, sought after solution. Consequently, suppose a theoretical situation where the structure vibrates, then the action of the dynamic loading $\mathbf{W}(t)$ ceases and its free vibration response is now being recorded. In this case, the condition that the solution $q_{o,i}(t)$ of the homogenous equation constitutes the final quested solution of the problem is fulfilled, namely:

$$q_i(t) = q_{o,i}(t) \tag{5}$$

More specifically, each term in the summation of eq.(2) represents the modal response movements $\mathbf{u}_i(t)$ for each i=1,2,3 eigen-mode:

$$\mathbf{u}_i(t) = \boldsymbol{\varphi}_i\, q_i(t) \quad \Rightarrow \quad \begin{bmatrix} u_{1i}(t) \\ u_{2i}(t) \\ u_{3i}(t) \end{bmatrix} = \begin{bmatrix} \varphi_{1i}\, q_i(t) \\ \varphi_{2i}\, q_i(t) \\ \varphi_{3i}\, q_i(t) \end{bmatrix} \tag{6}$$

From a combination of eq.(5) and eq. (6), the following conclusion is drawn: *If the dissociation of the modal responses* $\boldsymbol{\varphi}_i\, q_i(t)$ *for each degree of freedom of the system was feasible, then the direct estimation of the components of each one of the eigen-modes by the elimination of the time function* $q_i(t)$, *which is common for all the degrees of freedom of the same eigen-mode, would also be feasible*. Indeed, referring at the instant of the simultaneous extreme widths of vibration (*a, b, c*) for the of degrees of freedom (u_1 , u_2, u_3) as shown in Fig. 3, the components of the *i*-eigen-mode are directly described by eq.(7) below:

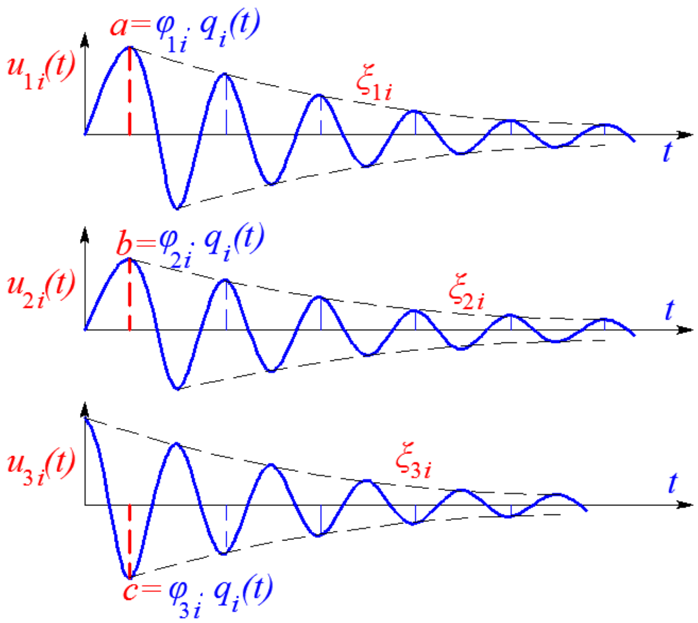

Fig.3: Modal time-histories for a three degree-of-freedom structural system

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \Rightarrow \begin{bmatrix} a/a \\ b/a \\ c/a \end{bmatrix} = \begin{bmatrix} \varphi_{1i} \\ \varphi_{2i} \\ \varphi_{3i} \end{bmatrix} = \begin{bmatrix} 1 \\ \varphi_{2i} \\ \varphi_{3i} \end{bmatrix} \tag{7}$$

The dissociation (or uncoupling) of the modal responses is achieved numerically by truncation of the harmonic having a frequency equal to the *i*-eigen-frequency of the structure. This truncation is achieved by the use of an appropriate digital filter applied to the initial recordings. This digital filter must permit only the transition of one specific frequency, which equals to the eigen-frequency of the structure, so long as the latter is already known. The eigen-frequency of the structure can be determined by the combination of the “peak-picking” technique applied to the Fast Fourier Transform diagrams of the recordings, followed by an examination of the phase difference between the extreme response values. This last point must take into account that a phase difference of 0 or π (rad) between different recordings corresponds to an eigen-mode. The aforementioned method is known as the "modal time-history method" and has been recently developed in Makarios (2012 & 2013). Thus, in the present work, the method is applied to the steel stairway, which is a system with infinite degrees of freedom having a continuous mass and stiffness distribution. This makes the identification of the important eigen-frequencies quite difficult.

3 INSTRUMENTATION PROCEDURE FOR THE STEEL STAIRWAY

The stairway was instrumented by a system of two 12 bit-nominal resolution, digital uniaxial accelerometers of the type KUOWA-PCD-30A. One sensor (ch.1) was used for the recording of the vertical accelerations, while the second sensor (ch. 2) was used for the recording of the horizontal accelerations, see Fig. 4. The fastening of the sensors on the lower surface of the treads was achieved by the use of silicon adhesives. The sensor recording the horizontal accelerations (reference accelerometer) was installed in the middle of the third span, while the sensor recording the vertical accelerations was alternately installed in the middle of the first and second spans, see Fig.2.

Fig.4: Installed accelerometers on the stairway underside during September 2014

Fig.5: Recorded unit of accelerometer network

The ambient vibration loading was realized during regular use, with people ascending and descending the steel stairway. A sufficient number of response recordings was made, and durations of two (2), five (5) and fifteen (15) minutes were considered, including the free vibration part of the response as well. The two accelerometers were connected to a logging unit for receiving data. The latter was connected to a laptop computer (see Fig.5). Using the appropriate software, visualization and calibration of the recordings took place. By selecting a range of 10000 μm/m and a calibration factor of 0.000829, the accelerometer in the vertical vibration direction was capable of recording the acceleration of gravity g=9.81m/s^2 under calm conditions. The software used offers the ability of exporting data in the .xls format. Therefore, the recordings were easily imported and processed in a spreadsheet (MS Excel program).

4 SIGNAL PROCESSING TECHNIQUE

During the processing of the recordings, multiple time windows were examined. The part of the forced vibration was truncated and only the part of the free vibration was taken into consideration. The noise was removed from the free vibration recordings by the use of appropriate filters (see Makarios 2012 & 2013). Finally, the Fast Fourier Transform of each degree of freedom was calculated. The main observation concerning the FFT diagrams refers to the lack of the usual “spikes” for certain eigen-frequencies due to the fact that the steel stairway does not respond as a discrete system, since it does not contain any sizeable lumped masses (see Figs.6 & 7).

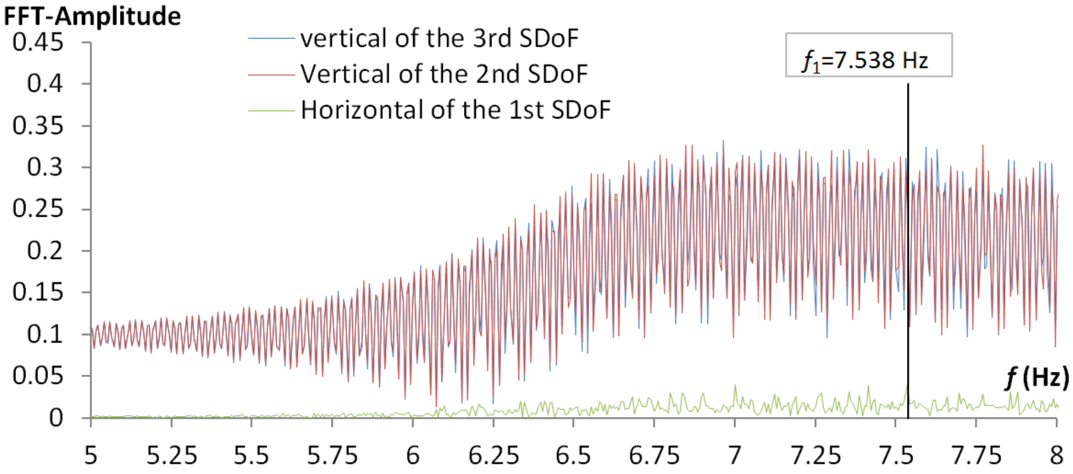

Fig.6: FFT of the three response components in the 5 - 8 Hz frequency range

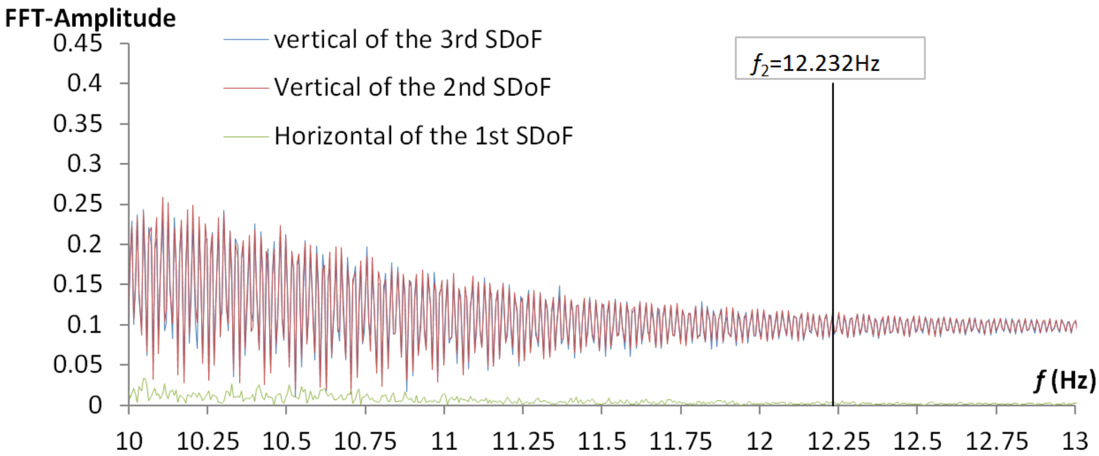

Fig.7: FFT of the three response components in the 10 - 13 Hz frequency range

On the contrary, the FFT displacement amplitudes presented a rather smooth uniformity, due to the fact that that the structure performs as a system with continuous mass and stiffness distribution. Combining the “peak-picking” technique with an estimation of the phase difference between the modal response histories, it is possible to detect whether or not the peaks observed refer to the modal response of the structure or simply stem from the external dynamic loading $\mathbf{W}(t)$. In this way, the peaks in the entire frequency range of FFT diagrams were sequentially examined. As a result, two frequencies, f_1=7.538Hz and f_2=12.232Hz, were estimated as being the eigen-frequencies of the structure. Indeed, the time histories corresponding to these two frequencies, present a phase difference of 0 and π rad respectively, as illustrated in Figs. 8 & 9. According to Fig.(3) and eq. (7), the modal components of the two eigenmodes of the stairway were computed by the SeismoSignal (2012) program and the following values were recovered:

$$\boldsymbol{\varphi}_1 = \begin{bmatrix} \varphi_{11} \\ \varphi_{21} \\ \varphi_{31} \end{bmatrix} = \begin{bmatrix} 1 \\ 41.405 \\ 47.50 \end{bmatrix}, \qquad \boldsymbol{\varphi}_2 = \begin{bmatrix} \varphi_{12} \\ \varphi_{22} \\ \varphi_{32} \end{bmatrix} = \begin{bmatrix} -1 \\ 51.16 \\ 44.00 \end{bmatrix}$$

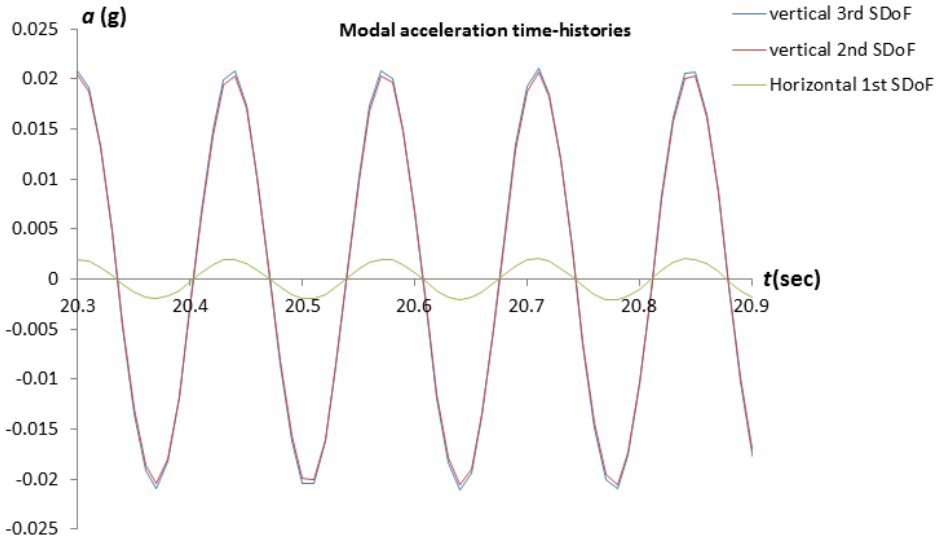

Fig.8: Modal acceleration time-histories for the first frequency f_1=7.538Hz, where the phase difference between the three components is zero

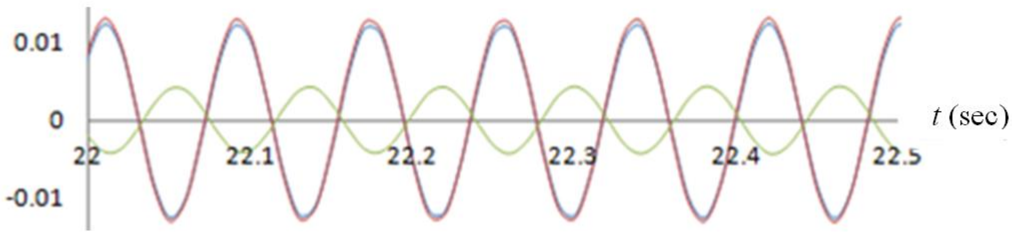

Fig.9: Modal acceleration time-histories for the second frequency f_2=12.232Hz, where the phase between the first component and the other two is equal to π.

Finally, the part of the free vibration of the modal time histories components was used in order to calculate the equivalent modal damping ratios, applying the equation of the logarithmic decrement in each one of the three modal components, see Table 1. The final average equivalent modal damping corresponds to the mean value of the aforementioned equivalent modal damping ratios.

First Eigen-frequency f_1=7.354 Hz	Equivalent viscous damping ratio of modal component	Mean equivalent viscous damping ratio ξ_1.
1st Degree of Freedom	0.0037	
2nd Degree of Freedom	0.0048	0.0045
3rd Degree of Freedom	0.0049	
Second Eigen-frequency f_2=12.232 Hz	Equivalent viscous damping ratio of modal component	Mean equivalent viscous damping ratio ξ_2.
1st Degree of Freedom	0.0073	
2nd Degree of Freedom	0.0065	0.0070
3rd Degree of Freedom	0.0071	

Table 1: Equivalent viscous damping ratios for the steel stairway

5 FINITE ELEMENT MODEL OF THE STEEL STAIRWAY

The finite element program SAP2000 was used for the numerical modeling of the steel stairway, see Fig. 10. Some of the most important features of this simulation are the following:

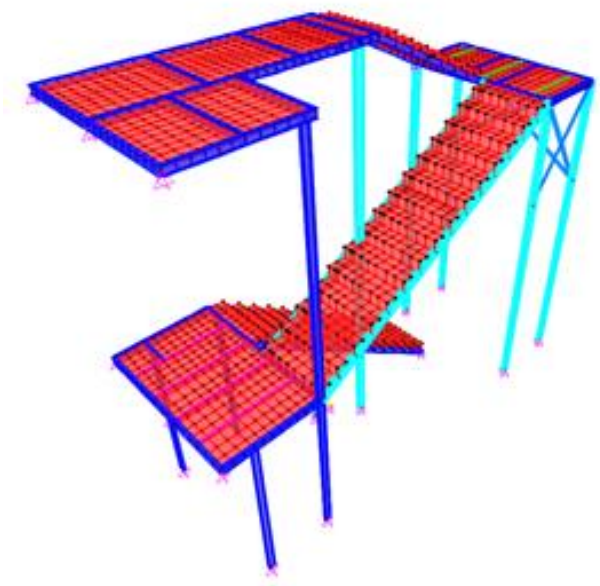

Fig.10: Model of the steel stairway using SAP2000

1. Beam elements were used to model the beams of the columns of the stairway.
2. Isotropic shell elements were used to model the surface elements of the stairway, such as the risers, the treads and the landings, which were made of sheet metal.
3. The total mass of the canopy was taken into account as lumped masses at the common nodes with the stairway beams.
4. The stiffness of the supporting beams was appropriately modified into order to take into account the stiffness of the railways. Furthermore, the mass and the weight of the railways were also taken into account.
5. The welded connections between all of the parts of the stairway were considered to be fully continuous connections.

In order to investigate the effect of the mass change at the landings due to live loads and the effect of the boundary conditions as well on the dynamic characteristics of the stairway, various parametric analyses were carried out. The values of the ten first eigen-frequencies of the optimized finite element model are presented in Table 2. The eigen-frequencies calculated by the use of the program SAP2000 yielded values close to 10Hz (or greater). The aforementioned results support the fact that the range of the real possible eigen-frequencies approaches or even includes the values of the eigen-frequencies estimated by the analytical-experimental results (i.e., frequency values f_1=7.538Hz and f_2=12.232Hz).

6 DISCUSSION AND CONCLUSIONS

In the present work, we present a method for the identification of the dynamic characteristics of a flexible metal structure which responds as a continuous (in terms of mass and stiffness distribution) dynamic system to environmentally-induced loads. This is a rather difficult problem in structural dynamics, since there is a spread of adjacently grouped eigen-frequencies with equivalent response amplitudes, and in the absence of any dominant frequency that would be detectable in terms of a bell or nail shaped bump in the FFT plots. In order to produce reliable results, the method of modal acceleration time histories was applied, which calculates the phase differences between the recorded degrees of freedoms in the total time history response. Moreover, it is noted that the set up of a finite element model for the steel stairway is equivalent to the development of a "discrete" model. Irrespective of how detailed this model is (in terms of use of large numbers degrees-of-freedom), it will always produce results that deviate from the real response. This means that for flexible and asymmetric types of structures, the dynamic characteristics according to the finite element modeling will not always correspond to reality. Nevertheless, the numerical model is capable of determining an acceptable range of eigen-frequencies, but without the possibility of convergence in terms of better accuracy. The on-site simultaneous recordings from the multichannel system of accelerometers, under the appropriate analytic processing, can produce a very good estimation of the real values of eigen-frequencies and eigen-modes, even in the case of continuous systems. It is of course acknowledged that the use of finite element models contributes to the identification of the eigen-frequency range, narrowing the search for the true eigen-frequencies during the processing of the experimental recordings. However, the development of a numerical modal is not absolutely necessary for the identification of the dynamic characteristics of the flexible structure, since the method of the modal time history accelerations method is autonomous. The true value of finite element models in our case lies elsewhere: Once calibrated, they can be used in conjunction with non-periodic dynamic response recordings to establish the structural decay over time as the stairway remains under continuous use.

Output Case	StepType	Step Num	Period	Frequency	Circ. Freq	Eigenvalue
Text	Text		(sec)	(cyc/sec)	(rad/sec)	$(rad/sec)^2$
MODAL	Mode	1	0.098988	10.102	63.474	4029
MODAL	Mode	2	0.084927	11.775	73.983	5473.5
MODAL	Mode	3	0.078137	12.798	80.413	6466.2
MODAL	Mode	4	0.074687	13.389	84.126	7077.2
MODAL	Mode	5	0.072954	13.707	86.126	7417.6
MODAL	Mode	6	0.067339	14.85	93.306	8706
MODAL	Mode	7	0.057517	17.386	109.24	11934
MODAL	Mode	8	0.049105	20.365	127.95	16372
MODAL	Mode	9	0.045839	21.815	137.07	18788
MODAL	Mode	10	0.044003	22.726	142.79	20389

Table 2: Modal Periods and frequencies by FEM program SAP2000

REFERENCES

[1] M. Basseville, A. Benveniste, M. Goursat, L. Hermans, L. Mevel & H. Auweraer. Output-only subspace based structural identification: from theory to industrial testing practice. *Journal of Dynamic Systems, Measurement and Control, Transactions of the ASME: Special Issue on Identification of Mechanical Systems*; vol. 123, no. 4 pp. 68–76, 2001.

[2] R. Brincker, L. Zhang & P. Andersen. Modal identification of output-only systems using frequency domain decomposition. *Smart Materials and Structures Journal;* vol. 10: pp. 441–45, 2001.

[3] B. Peeters & G. De Roeck. Stochastic system identification for operational modal analysis: a review. *Journal of Dynamic Systems, Measurement and Control, Transactions of*

the ASME: Special Issue on Identification of Mechanical Systems; vol. 123, no. 4, pp. 659–67, 2001.

[4] H. Wenzel & D. Pichler. *Ambient Vibration Monitoring*. John Wiley & Sons, Chichester, England, 2005.

[5] P. Overschee & B. De Moor. *Subspace Identification for Linear Systems*. Kluwer Academic Publishers, Dordrecht, 1996.

[6] T. Makarios. Identification of the mode shapes of spatial tall multi-storey buildings due to earthquakes. The new modal time histories method. *Journal of the Structural Design of Tall and Special Buildings*. vol. 21, pp. 621–641, 2012.

[7] T. Makarios. Identification of building dynamic characteristics by using the modal response acceleration time-histories in the seismic excitation and the wind dynamic loading cases. Chapter 4 in *Accelerometers: Principles, Structure and Applications*, edited by P.S. de Brito Andre & H. Varum, Nova Science Publishers, New York, pp. 7-113, 2013.

[8] G.D. Manolis, A. Athanatopoulou-Kyriakou, K.D. Dragos, A. Arabatzis, A. Lavdas & C.Z. Karakostas. Identification of pedestrian bridge dynamic response trough field measurements and numerical modeling: Case Studies. *Journal of Theoretical and Applied Mechanics*, vol. 44, no. 2, pp. 3-24, 2014.

[9] I. Karetsou, *Instrumentation of steel stairway and identification of its dynamic characteristics: Model of analysis*. MSc Diploma Thesis, Postgraduate Program on Earthquake Engineering and Seismic Design of Structures, Civil Engineering Department, Aristotle University, Thessaloniki, Greece, November 2014 (in Greek).

[10] M. Papanikolaou, *Identification of dynamic characteristics of steel stairway using recordings and development of suitable model*, MSc Diploma Thesis, Postgraduate Program on Earthquake Engineering and Seismic Design of Structures, Civil Engineering Department, Aristotle University, Thessaloniki, Greece, November 2014 (in Greek).

[11] *SeismoSignal, Version 5.1.0*. Seismosoft Ltd, Pavia, Italy, 2012.

[12] *SAP 2000 Nonlinear Version 11.03, User's Reference Manual*, Computer and Structures Inc., Berkley, California, 2007.