

## VALIDITY DOMAIN OF AN ENERGETIC METHOD FOR THE ESTIMATION OF MODAL DAMPING IN THE CASE OF NON LINEAR LOCALIZED DISSIPATION

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**Abstract.** *Modal damping estimation errors can be introduced when using assumptions such as proportional damping or a diagonal generalized matrix of localized dissipation at interfaces [1]. Previous work has studied the estimation of modal damping in bolted joints using many methods [2, 3]. One such method, used in this work, is the iterative energetic method (EM), developed in [4]. The present study investigates the EM for calculating the modal damping associated with localized nonlinear dissipative interfaces of a global linear structure. This method has limitations in two cases: localized linear damping [5] and localized nonlinear damping. The validity domain of the EM in the linear case was studied previously in [5]. The main conclusions were that EM is not valid in the following conditions : when eigenfrequencies are closely spaced, when localized dissipation exceeds a specific damping level, and if the response is not projected on a single mode of vibration. To the best of our knowledge, the non-linear limitations of the EM are still unknown and have yet to be identified in the literature. The main goal of this paper is to investigate the iterative EM and define its validity domain for harmonic excitations as a function of the level of external structural loads and the level of nonlinear damping. We propose to investigate these parameters and their influence for the prediction of equivalent modal damping in a simple beam structure with a bolted nonlinear interface. Through use of an academic example with exact solutions, we will be able to quantify the error of our predictions, and thus provide accurate validity domains for the method.*

## 1 INTRODUCTION

The problem of damping remains an important challenge when computing the vibration levels of assembled structures. Virtual tools, Computer Aided Design and Finite Element Simulations are used to predict both mass and stiffness matrices with good accuracy, but damping is often badly estimated. This leads to an erroneous estimation of vibration levels. Generally the localized dissipation in the interfaces presents nonlinear effects. These nonlinear effects must be taken into account when modeling damping locally. It is assumed here that the structure has a globally linear behavior. The main interest of this study is to investigate the iterative EM and define its validity domain for harmonic excitation as a function of the level of external loads and the level of nonlinear localized damping. An exact method is used as reference to investigate the influence of these two parameters and compare the resulting error in the modal damping estimation.

## 2 Estimation of modal damping

To estimate modal damping in assembled structures with localized nonlinearities two computational methods will be considered herein. The first method uses a time integration solver of a globally nonlinear system and will be considered as the reference method. The second method is the *Iterative Energetic Method* (IEM), which is based on the estimation of the modal damping by the ratio between the dissipated energy and the maximal potential energy, over a cycle of stationary vibration. Localized damping can be induced in several manners, for example, by viscoelastic materials, pressure loss in fluids and solid friction. In this work, in order to model bolted joints a nonlinear viscoelastic damping model is used to represent a bolted interface.

### 2.1 Reference Method

In the reference method, the system will be considered as a globally nonlinear system :

$$M\ddot{u} + Ku + C\dot{u} + f_{nl}(t, \dot{u}) = f_{ext}(t) \quad (1)$$

where

- $M, K$  et  $C$  are respectively mass, stiffness and damping matrices
- $f_{ext}(t) = F \cos(\omega t)$  Harmonic excitation
- $u(t) = \{u_1, u_2, u_3, \dots, u_n\}^T$  Displacement vector
- $\dot{u}(t) = \{\dot{u}_1, \dot{u}_2, \dot{u}_3, \dots, \dot{u}_n\}^T$  Velocity vector
- $f_{nl}(t, \dot{u})$  Nonlinear dissipation force

The nonlinear dissipated force describes the friction in the bolted interfaces. The  $i^{th}$  interface is expressed by the following expression :

$$f_{nl}^i(t, \dot{u}) = (c_0^i + c_1^i |\Delta \dot{u}_i|) \Delta \dot{u}_i \quad (2)$$

where  $c_0^i [N/(m/s)]$  and  $c_1^i [N/(m/s)^2]$  are two coefficients of the nonlinear viscous damping,  $\Delta \dot{u}_i = u_j - u_k$  is the relative velocity, and  $j, k$  are the nodes of the  $i^{th}$  interface.

The calculation of the modal damping is performed into two steps : The first step is to evaluate the response of the system using a time integration solver, for example the Runge-Kutta method [6]. This step requires the transformation of the second-order equation (1)

$$\ddot{u} = M^{-1}(f_{ext}(t) - f_{nl}(t, \dot{u}) - Ku - C\dot{u}) \quad (3)$$

and by applying a change of variable  $y_1(t) = u(t)$  and  $y_2(t) = \dot{u}(t)$  on the equation (3) we obtain the following first-order system

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = M^{-1}(f_{ext}(t) - f_{nl}(t, y_2) - Ky_1 - Cy_2) \end{cases} \quad (4)$$

The second step consists in the calculation of the modal damping with the following expression

$$\xi_\nu = \frac{1}{4\pi} \frac{E_\nu^{diss}}{E_\nu^{pot}} \quad (5)$$

Dissipated energy is calculated by the following expression :

$$E_\nu^{diss} = \int_0^T \dot{u}(t)^T f_c(t) dt + \sum_{i=1}^{n\_inter} \int_0^T \Delta \dot{u}_i(t)^T f_{nl}^i(t) dt \quad (6)$$

where

- $T = \frac{2\pi}{\omega_\nu}$  Cycle of periodic vibration of mode  $\nu$
- $f_c(t) = C\dot{u}(t)$  Dissipated force due to the material
- $n\_inter$  Number of interfaces in the system

Potential energy is calculated as follow :

$$E_\nu^{pot} = \frac{1}{2} u(\omega_\nu)^T K u(\omega_\nu) \quad (7)$$

where

- $u(\omega_\nu) = \|u_{max}\| e^{j\varphi}$  Frequency response deduced from the temporal response
- $\|u_{max}\|$  Maximum amplitude of the time response in a vibration cycle of the established regime when the structure is excited at the resonance frequency  $\omega_\nu$
- $\varphi$  Dephasing angle between the Dofs and the external load

The above method was implemented in Matlab and will be considered as the reference method when comparing damping results with the iterative energetic method IEM.

## 2.2 Iterative Energetic Method IEM

The Iterative Energetic Method consists in calculating modal damping using a multiscale approach which is divided in two main steps : step (1) local nonlinear contact analysis to estimate energy dissipation and step (2) estimate modal damping for the globally linear elastodynamic structure.

The present method is based on the ideas presented in earlier work [4, 5]. The main advantage of the IEM method is to avoid the high computational burden associated with to reference approach. It is based on calculating the modal damping by the ratio between dissipated energy and maximal potential energy, over a cycle of periodic vibration, as described in the equation (5). Meanwhile the estimation of both dissipated and potential energies requires an accurate characterization of the response levels of the system and the latter remains an approximation since they depend a priori on a knowledge of the different dissipation mechanisms. An iterative approach, as described by figure(1), is thus needed to converge when estimating the modal damping for the global system.

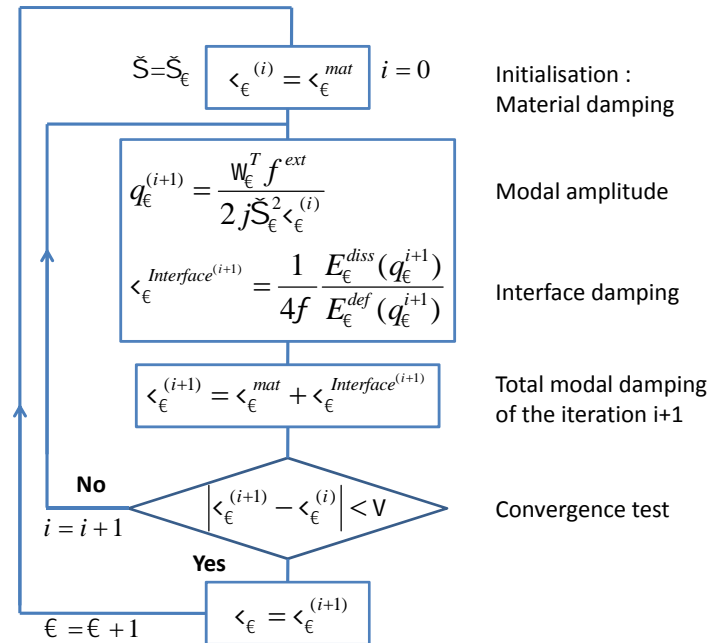


Figure 1: Flowchart of iterative process

32

The Flowchart of figure (1) presents the iterative process of the IEM. To calculate the  $\nu^{th}$  total modal damping  $\xi_\nu$ , we assume damping is equal to material damping ( $\xi_\nu^0 = \xi_\nu^{mat}$ ) in the iteration 0. The modal amplitude  $q_\nu$  is then calculated using this incorrect material damping. Next the interface damping  $\xi_\nu^{Interface}$  is calculated. The sum of these two sources of modal dissipation due to material and interface effects provides the estimation of the total modal damping in the iteration 1. If the difference between the successive damping is more than the convergence coefficient  $\varepsilon$ , the process continues to the next iteration to correct consecutively the modal amplitude  $q_\nu^{(i+1)}$ , the energies and the next modal damping estimation. These steps are repeated iteratively until convergence. Once the convergence obtained the eigenmode is actualized ( $\omega_\nu = \omega_{\nu+1}$ ) to calculate the next modal damping of the structure.

### 3 Simulations

In this section the system and design parameters will be presented first followed by an investigation of the validity domain of the IEM.

#### 3.1 System description

The system studied is composed of a beam clamped on one end and connected to a localized nonlinear dissipative interface on the other end. The system is discretized into ten 2D beam finite elements (2 DOFs per node). The full model has 20 DOFs. The excitation force is applied to node number 1 of the beam.

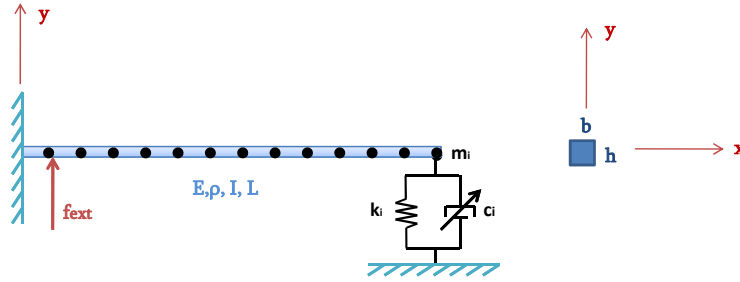


Figure 2: Simple beam with localized nonlinear dissipative interface

The dissipation force is expressed in terms of the velocity by the equation :

$$f_c(t, \dot{u}) = c_i \times \dot{u}_i \quad (8)$$

and the nonlinear viscous model is expressed by the following expression

$$c_i = c_0 + c_1 |\dot{u}_i| \quad (9)$$

where  $c_0$  and  $c_1$  are two damper coefficients of the nonlinear viscous damping. The used parameters are performed in table (1).

Beam properties			Interface properties		
$E$	Steel Young modulus (GPa)	210	$k_i$	Linear stiffness (N/m)	$10^6$
$\rho_s$	Steel density ( $kg/m^3$ )	7800	$c_0$	Viscous damper (N/(m/s))	20
$h$	Beam thickness (m)	$2 \cdot 10^{-2}$	$c_1$	Viscous damper ( $N/(m/s)^2$ )	[0,...,400]
$b$	Beam width (m)	$2 \cdot 10^{-2}$	$m_i$	Mass (Kg)	$2 \cdot 10^{-4}$
$L$	Beam length (m)	0.6			

Table 1: Physical and geometric properties of the system.

The first three modes of the associated conservative system are  $f_1 = 214.31Hz$ ,  $f_2 = 505.50Hz$  and  $f_3 = 1205.85Hz$ .

### 3.2 Validity domain of IEM

The proposed IEM has been implemented on an academic example in MATLAB. Two numerical simulations have been performed in order to highlight its domain of validity. The goal of this section consists in studying the influence of the following parameters on the accuracy of this method: external structural loads level and nonlinear localized damping level.

#### 3.2.1 Level of external structural loads

To investigate the effect of high levels of external structural loads a series of simulations have been performed by changing the load amplitudes while keeping the damping parameters constant. In this section, the studied parameters are performed in table (2).

Case	Force (N)	Case	Force (N)
1	F = 1 N	6	F = 80 N
2	F = 5 N	7	F = 100 N
3	F = 10 N	8	F = 120 N
4	F = 25 N	9	F = 150 N
5	F = 50 N		

Table 2: Load amplitude simulation parameters.

Figures (3) and (4) show the comparison of modal damping between the IEM and the reference method respectively for the cases  $F = 10$  N and  $F = 150$  N. The error of estimating damping is greater for the case of high loads ( $F = 150$  N) than the case of low loads ( $F = 10$  N).

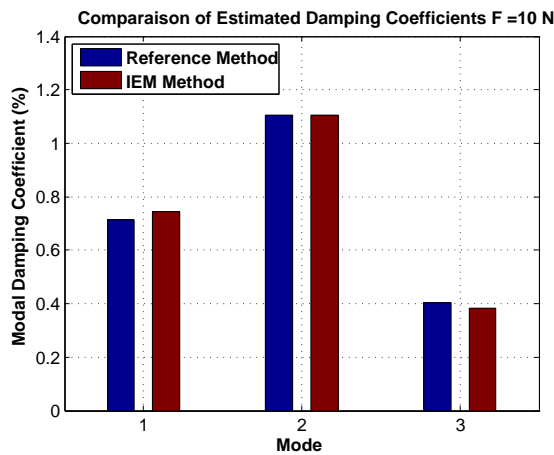


Figure 3: IEM vs. Reference  $F = 10$  N

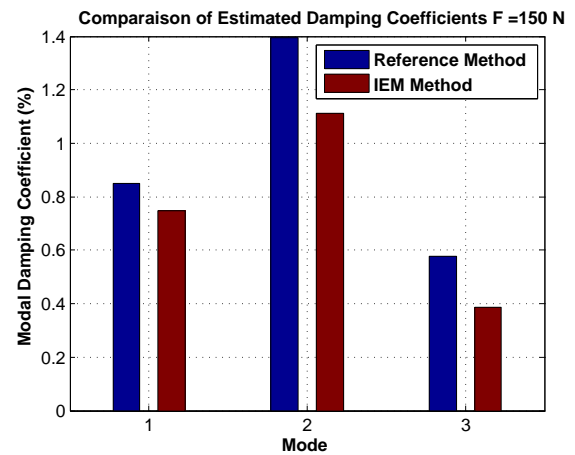


Figure 4: IEM vs. Reference  $F = 150$  N

The evolution of the reference modal damping estimation with the increase of the external load is given in Figure (5). Here we are interested in the first three modes because in this case others modes give a few modal damping coefficients ( $\xi_v \ll 1\%$ ). Figure (6) presents the error of damping estimation committed by the IEM. The estimated error increases with increasing external load level as described in the curves of the figure (6). Figure (6) shows that the error is minimal for the first three modes when  $F = 10$  N. In the following we will take this value to excite the system when studying the influence of nonlinear localized damping.

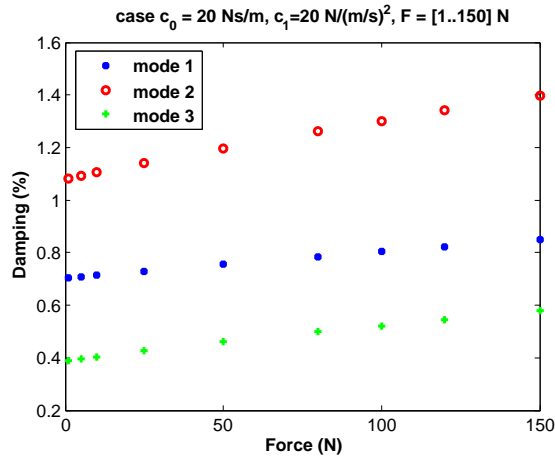


Figure 5: Modal damping estimated by Ref method (%)

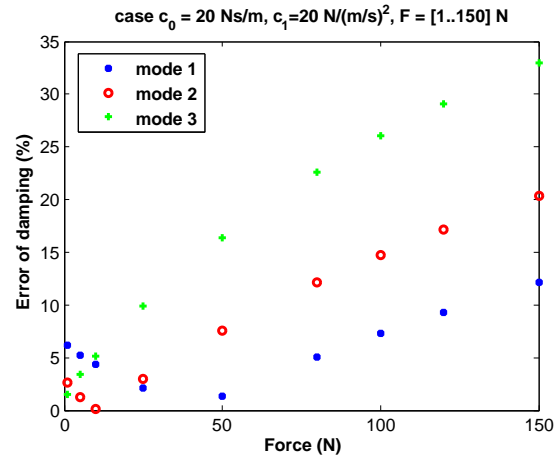


Figure 6: Error of damping by IEM method (%)

### 3.2.2 Level of nonlinear localized damping

To investigate the effect of high level of nonlinear localized damping a series of simulations have been performed by changing the damping amplitudes while keeping the loads amplitude parameter constant. In this section, the parameter values used are indicated in table (3).

Case	Damping $c_1$	Case	Damping $c_1$
1	0	5	150
2	20	6	200
3	50	7	300
4	100	8	400

Table 3: Nonlinear localized damping simulation parameters

To have an idea of the level of the localized nonlinear damping we compare the restoring force versus velocity between two cases : Figure (7) shows a nonlinear localised damper level with a nearly linear behavior and Figure (8) shows a high nonlinear localised damper level.

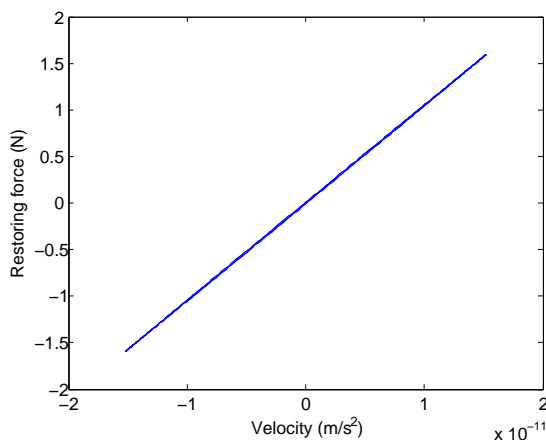


Figure 7: Few nonlinear localised damper level

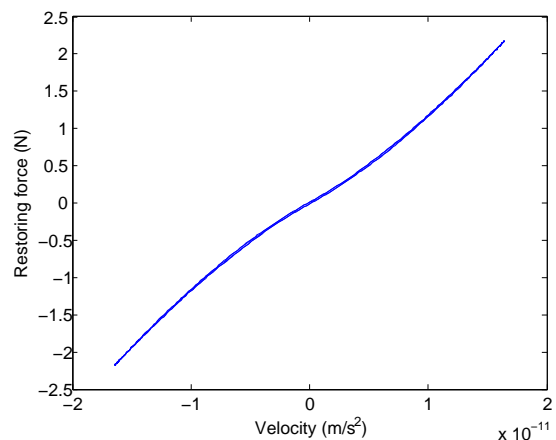


Figure 8: High nonlinear localised damper level

The evolution of the reference modal damping estimation with the increase of the localized nonlinear damper level is given in Figure (9). Figure (10) presents the evolution of the estimation error for the three first modes of the system. When the nonlinear damper level increases the error of damping estimation by IEM increases too.

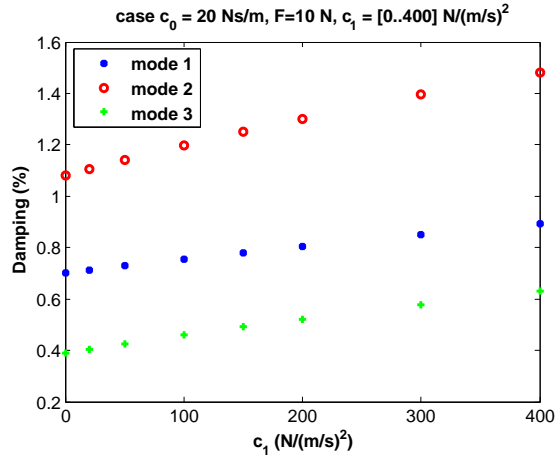


Figure 9: Modal damping estimated by Ref method (%)

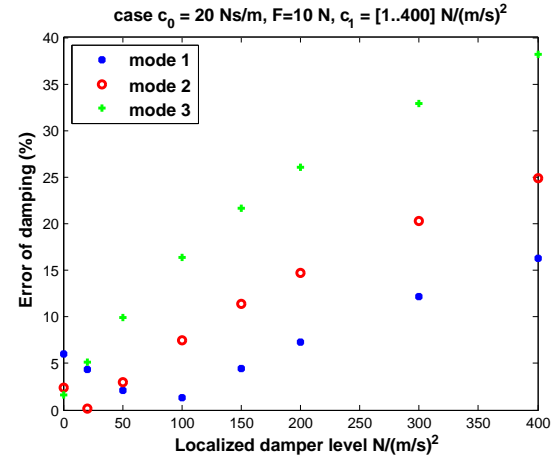


Figure 10: Error of damping by IEM method (%)



## 4 CONCLUSIONS

In this paper, the iterative energetic method is presented and investigated for specific conditions in nonlinear interface joints in order to study its limitations. The method is based on the ratio between the dissipated energy in the interfaces and potential energy of the global structure. Firstly, the authors compared the modal damping calculated by two methods with the aim of demonstrating the effects of the external structural loads. Secondly, the effects of the localized damping level when nonlinear localized viscous damping increases was investigated. Finally, when dealing with nonlinear systems, the IEM method is computationally faster than the reference method but it is less accurate for estimating precise results of modal damping in these both cases of high level of external structural loads and high nonlinear localized damper level.

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