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RESONANCE EVALUATION WITH GEOMETRIC STIFFNESS AND CREEP OF SLENDER BEAM OF REINFORCED CONCRETE

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Abstract. *This article assesses the occurrence of resonance with the effects of geometric stiffness and creep in the vibration of a prestressed reinforced concrete beam. A mathematical model based on the Rayleigh method is used, designed to represent a simply supported beam, intended to function as the base of an engine. The gross cross-section of the beam is set to an arrangement of passive reinforcement that is able to resist the effort provided in the simulation, treated by the method of homogenized section. Creep is taken into account through a three-parameter rheological model that conduces to a temporal modulus of elasticity; a normal force of compression reproduces the post tensioning force, which changes the stiffness, and consequently, the natural frequency of vibration of the structure with time. The results from the numerical simulation indicate resonant and non-resonant schemes between the natural frequency of the beam and the frequency of the engine predicted in the mathematical considerations.*

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1 INTRODUCTION

The dynamic characteristics of a structure depend, basically, on its stiffness and mass. With these two elements, the natural frequencies and modes of vibration of the system are determined. However, the initial stiffness of a structure can be affected by the so-called geometric stiffness, a function of the acting normal force. In the case of compression force, the stiffness of the structure decreases, also reducing the natural frequencies of vibration. A class of structures of socio-economic-strategic importance for the national industry are machine bases, which are subject to vibrations induced by the supported equipment. These vibrations can affect the safety of the structure itself and generate detrimental effects on the equipment and the quality of the manufactured product. They can also make the working ambience unsuitable for operators. All industrial sectors are subject to these problems, including oil exploration, production and refining, mining, wind energy, atomic energy, as well as bridges and viaducts for road and rail use.

Although equipment support structures are, as a general rule, over-dimensioned, and therefore not subject to the effects of geometric stiffness, the tendency of modern structural engineering is towards increasingly slender elements, made possible by materials that are more efficient and lightweight, and having more and more powerful structural analysis capabilities. One of these features is prestressed concrete, represented by the presence of a steel bar or cable inside the structure that compresses it, the purpose of which is to reduce the effects of tension on flexion. In the case of beams subjected to periodic excitation, it is assumed that the original design has taken care to distance the natural frequencies of the system from those of the excitation, considering that, by hypothesis, the prestressing force decreases the stiffness of the element and, consequently, its natural frequencies, which may lead to unexpected, potentially dangerous resonance regimes. In the opposite direction, the presence of the prestressing can provide a form of control of this same vibration, where a resource is available to remove the structure of the resonant regime, if perceived in the preliminary stages of design. In one way or another, a satisfactory analysis solution to most engineering problems comes from a consideration that is easily implemented in analytical and numerical-computational formulations: the geometric stiffness. The influence of geometric stiffness has been studied in several contexts, both in laboratory tests and in comparison with the finite element method (MEF) by Wahrhaftig et al. [1]–[4].

The problem is aggravated when the material itself changes its elastic properties, such as in the case of creep, which represents the gradual increase of deformation with time. This is a typical phenomenon of concrete structures because it is a viscoelastic material. It must be considered when verifying the stability of slender pieces compressed under the Ultimate Limit State (ULS), since these have their stiffness modified in function of the rheology of the material itself. An approximate and satisfactory solution can be found by considering creep through flexural bending over time. To evaluate these aspects, a numerical simulation has been performed, assuming an idealized section of a beam as an engine base. A rheological model of the three parameters has been used to obtain the variable modulus of elasticity. A model, including geometric stiffness, distributed and concentrated masses, is derived based on the Rayleigh method and solved for a range of axial compression load values. The results made it allowed us to verify the resonant and non-resonant response of the system.

2 BASIC CONSIDERATIONS

2.1 Prestressing in reinforced concrete

A piece can be considered as prestressed reinforced concrete when it is subjected to the action of the so-called prestressing forces and of permanent and variable loads, so that the concrete is not subjected to tension or it occurs below the limit of its resistance. As an example, take the normal stresses beam diagrams of the prestressed beam of Figure 1, where *P* is the prestressing force, M_P the bending moment due to eccentricity of the load P, M_p is the bending moment due to uniformly distributed load *p*, and *R* is the resultant, each one of these with their corresponding normal stresses. Under the conditions presented, the lower fibres of the beam, under positive bending moment, will have the tension stresses overturned by the superposition of those produced by the normal stress of the applied stress eccentrically.

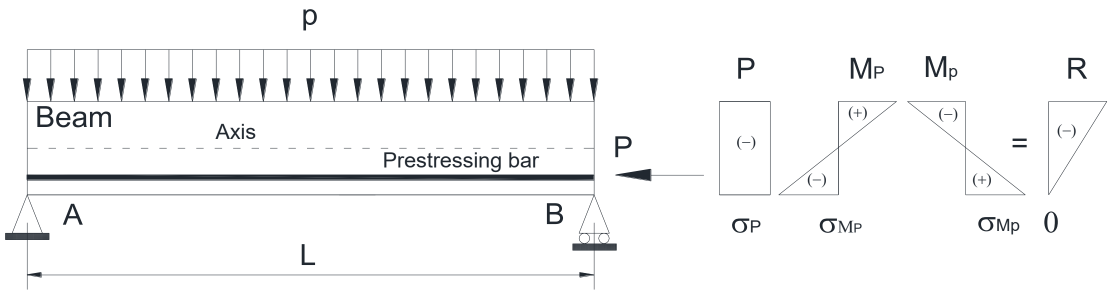


Figure 1: Normal stresses in a prestressed beam.

Prestressed concrete was developed scientifically from the beginning of the last century. Prestressing can be defined as the artifice of introducing, in a structure or a part, a previous state of stresses, in order to improve its resistance or its behaviour in service, under the action of several effects. Due to the characteristics of the concrete as a structural material, the use of prestressing can bring a great advantage from the economic point of view. When comparing the cost of a prestressed structure with a similar one of conventional reinforced concrete, there is a reduction in the final cost of the structure due to the reduction of steel reinforcement, affirms Oliveira [5]. In addition, the prestressing allows the part to overcome large spans, improves the control and reduction of deformations and fissures. It can also be used for structural recovery and reinforcement, as well as for slender systems and prefabricated or precast parts. There are three types of prestressing systems: a) prestressing with initial adherence; b) prestressing with posterior adherence; and c) prestressing without adherence. The latter type is composed of a post-tensioning system characterized by the slipping freedom of the steel reinforcement in relation to the concrete, along the whole extension of the cable, except for the anchorages.

In a non-adherent prestressing, the cables or chutes are wrapped in two or three layers of resistant paper. The wires and paper are painted with bituminous paint in order to tension them after the concrete has hardened. The bitumen avoids the penetration of the cement cream inside the cable and, in this way, it eliminates adhesion between the concrete and the reinforcement, comments Calduro [6]. The prestressed concrete is a composite material of the aggregate mixture and a cement paste associated with prestressing cables and/or passive reinforcing bars. Because of the combination of several materials, these structures develop a highly complex behaviour, presenting a non-linear response, which is due, among other factors, to time-dependent effects, such as the creep of the concrete, affirms Jost [7].

2.2 Mathematical model for the nonlinear vibration problem

Consider a rotary machine mounted on a beam subjected to a pre-tensioning force, without adhesion. It is known that such forces affect the geometric stiffness and, consequently, the values of the undamped free-vibration frequencies. If the structure is designed, as is usually the case, to have frequencies farther from the machine's service speed rotation, the changes in the frequency due to geometric stiffness may lead to the appearance of potentially dangerous resonance conditions.

Take a beam model of Bernoulli-Euler applied to a simply supported beam *AB* of length *L* and inertia *I*, intended to function as the base of an engine E_g, composed of viscoelastic material, represented by the temporal modulus of elasticity *E(t)* as shown in Figure 2. A normal force of compression *P* reproduces the post-tensioning force, which changes the stiffness, and consequently, the natural frequency of vibration of the structure with time. The eccentricity between the engine axis and the part is initially ignored. The vertical displacement of the central joint is the generalized coordinate of the system.

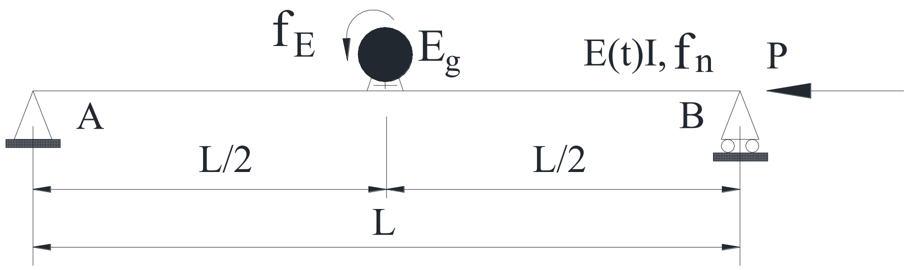


Figure 2: Beam model.

By using the Rayleigh method, the undamped vibration frequency in its first mode is obtained.

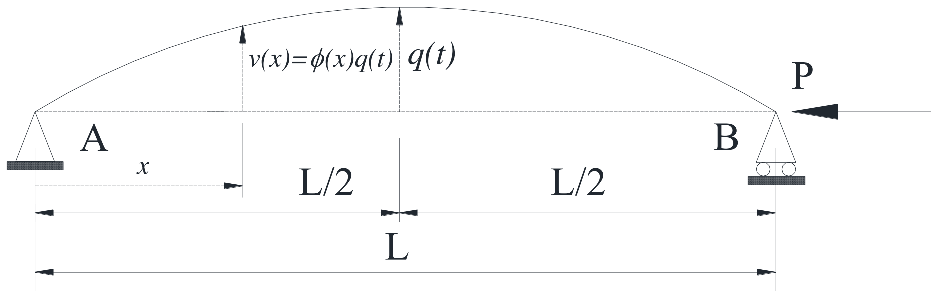


Figure 3: Rayleigh method.

Consider that the vertical displacement of a generic section of the beam in Figure 3 is given by:

$$v(x,t) = \phi(x)\, q(t), \tag{1}$$

in which $\phi(x)$ is a shape function that attempts to define the boundary conditions in the supports and value 1 in the central section of the beam, whose displacement with time is *q(t)*. In this case, one adopts the shape function $\phi(x) = sin\ (\pi x/L)$, which is the exact solution of the problem without the *P* load. A prime mark will denote a derivative of the function in relation to *x* (Lagrange's notation).

Applying the Rayleigh method, one has the conventional bending stiffness, K_0, as a function of the material elasticity and the geometry of the cross, which is equivalent to:

$$K_0 = \int_0^L E(t)I \;\; \phi'' \;^2 \; dx = \frac{\pi^4 E(t)I}{2L^3} \tag{2}$$

where *E(t)I* is the known flexural bending with viscoelasticity, represented by multiplication of the temporal material modulus of elasticity with the inertia of the section in relation to the considered movement, the vertical vibration mode (1st mode). In turn, the geometric stiffness, K_G, as a function of the normal force of compression (or even tension), is equivalent to:

$$K_G = P\int_0^L \;\; \phi' \;^2 \; dx = \frac{P\pi^2}{2L} \tag{3}$$

The total generalized mass of the system is found by calculating $M = M_C + M_V$ where M_C is the concentrated mass at the middle span and M_V is the mass coming from the beam self-weight given by:

$$M_v = \int_0^L m_V \; \phi(x)^2 dx = \frac{m_V L}{2} \tag{4}$$

in which m_V represents the total mass per length unit. Finally, the frequency of undamped free vibration (in rad/s) is found by way of Eq. (5):

$$\omega = \sqrt{\frac{K}{M}} \tag{5}$$

Considering the total beam stiffness as $K = K_0 - K_G$, the free undamped frequency of vibration of the 1st mode is found, in Hertz, admitting the compressive force as positive, by:

$$f = \frac{\omega}{2\pi} = \frac{1}{2}\left[\frac{\pi^2 E(t)I - PL^2}{L^3 \;\; L\, m_V + 2Mc}\right]^{\frac{1}{2}} \tag{6}$$

For a better understanding of the Rayleigh method and the importance of the geometric stiffness to the structural analysis, the work of Leissa [8] and Levy [9] should be consulted.

2.3 Mathematical solution for representing the creep

It is conceptually convenient to consider classic viscoelastic models in which only two types of parameters, relating to elasticity and viscosity, appear, report Armijo et al. [10]. Classic viscoelastic models are obtained by arranging springs and dampers, or dashpots, in different configurations. Springs are characterized by elastic moduli and dashpots by viscosity coefficients. The best known of these mechanical models are the Maxwell model, containing a spring in series with a dashpot, and the Kelvin–Voigt model, containing a spring and dashpot in parallel.

One model used to represent creep is the three-parameter model, in which the elastic parameter E_0 is connected to the viscoelastic Kelvin–Voigt model with parameters E_1 and η_1, which is a simplification of the Group I Burgers model. The three-parameter model sufficiently describes the viscoelastic nature of many solids and is often used to study the phenomenon in various scientific fields. The total deformations of the Kelvin–Voigt model are given by $\varepsilon = \varepsilon^e + \varepsilon^v$, where ε^e is the deformation of the elastic model and ε^v is the deformation of the Kelvin–Voigt model. When differentiated with respect to time, the total deformation is obtained as

$$\varepsilon = \dot{\varepsilon}^e + \dot{\varepsilon}^v \tag{7}$$

which is the constitutive equations of the elastic and Kelvin–Voigt models, respectively. Considering $E_1 = E_0$ as the modulus of elasticity for both parts of the rheological model,

$$\sigma = E_0 \varepsilon^e \text{ and } \dot{\sigma} + \frac{E_0 + E_0}{\eta_1}\sigma = E_0 \dot{\varepsilon} + \frac{E_0 E_0}{\eta_1}\varepsilon \tag{8}$$

are found. From the previous equations, one derives the following differential equation:

$$\sigma = E_0 \varepsilon^v + \eta_1 \dot{\varepsilon}^v \tag{9}$$

where $\sigma = 0$ for $t < 0$ and $\sigma = \sigma_0$ for $t > 0$, with t representing the time and $t = 0$ the instant of loading application. As the stress remains constant, the stress derivate with respect to time is zero. Applying the previous stress condition, the following ordinary differential equation is found:

$$E_0 \dot{\varepsilon} + \frac{E_0 E_0}{\eta_1}\varepsilon = \sigma_0 \tag{10}$$

for which the general solution for $t > 0$, taking the initial condition $\varepsilon(0) = \sigma_0/E_0$, is

$$\varepsilon(t) = \sigma_0 \left[\frac{1}{E_0} + \frac{1}{E_0}\left(1 - e^{-\frac{E_0}{\eta_1}t} \right) \right] \tag{11}$$

Obviously, if the stress level remains constant, the modulus of elasticity should decrease concurrently with increasing strain:

$$E(t) = \frac{1}{\frac{1}{E_0} + \frac{1}{E_0}\left(1 - e^{-\frac{E_0}{\eta_1}t} \right)} \tag{12}$$

The previous solution for consideration of the viscoelastic behaviour of materials was used by Wahrhaftig et al. [11] to evaluate the stability of a slender wooden column. However, it is of interest, at this moment, to make clear that the present work is a numerical speculation, which takes into account the creep of the concrete by assuming a viscoelastic rheological model of three parameters.

3 NUMERICAL SIMULATION

The beam gross cross-section was estimated with a passive reinforcement arrangement capable of resisting the predicted load in the simulation, being treated by the homogenized section method, with geometry as indicated in Figure 4. The modulus of elasticity of the concrete was calculated according to NBR 6118/2014 [12] recommendations, following Eq. (13), for a concrete characteristic compressive strength, f_{ck}, equal to 30 MPa.

$$E_0 = \alpha_i \cdot 5600\sqrt{f_{ck}} = 26838.405\ MPa;\ \alpha_i = 0.8 + 0.2 \cdot \frac{f_{ck}}{80MPa} = 0.875 \tag{13}$$

The reinforced concrete specific weight γ_c was obtained for a material density ρ of 2,500 kg/m^3 and a gravitational acceleration g of 9.8061 m/s^2, therefore γ_c = 24.52 kN/m^3.

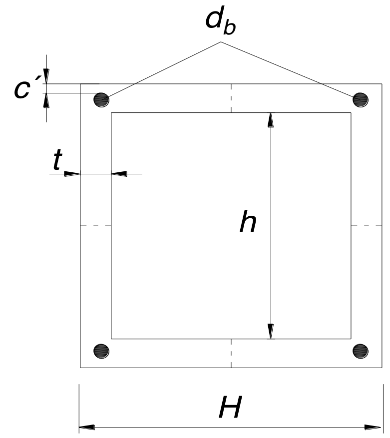


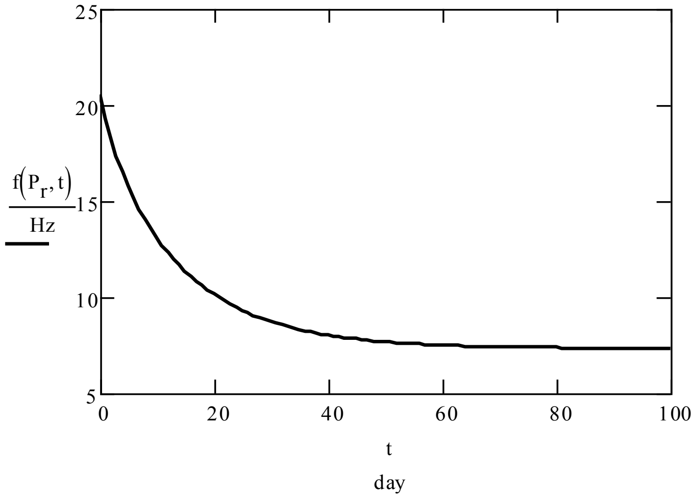
Figure 4: Beam section characteristics.

Section data:

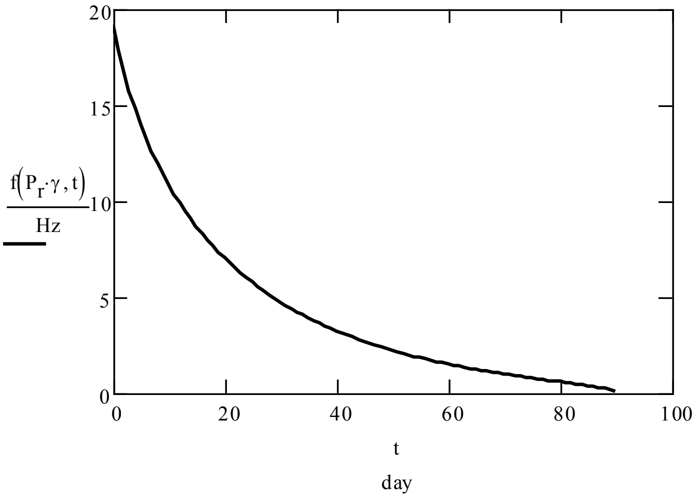
- External height: $H = 16$ cm
- Internal height: $h = H - 2 \cdot t = 6$ cm
- Wall thickness: $t = 5$ cm
- Concrete cover: $c' = 2.5$ cm
- Reference beam span: $L = 3$ m
- Reinforced bar diameters: $d_b = 8$ mm
- Number of reinforcement bars: $nb = 4$
- Total inertia: $I = \frac{H^4 - h^4}{12} = 5353$ cm^4
- Gross section area: $A = H^2 - h^2 = 220$ cm^2

The concrete section was homogenized by the transformation of the steel bars of the reinforcement, which led to a homogenization factor of 1.0616 to be considered in the material and geometric properties of the beam section. For the simulation, all the elements that constitute physical parts to be added to the system, such as the bar used in prestressing systems and an electric induction motor that represents periodic excitation, were considered as lumped or distributed masses.

By fixing the force on the section-resistant capacity, one can observe the variation of the natural frequency of the beam with time when considering creep, as shown in Figure 5(a). A safety factor of 1.17042570 to be applied to the loading can be found, which defines the beam collapse at the 90th day, as can be seen in Figure 5(b). By varying force *P*, which represents a non-adherent post-tension force, from zero to the resistant capacity of the section, the variation of the natural frequency of the beam is obtained, given in the graph in Figure 6. There, it is possible to see that, with the increase of the axial compression force, the beam frequency decreases, since the geometric portion (K_G) stiffness is changed, consequently decreasing the total stiffness (*K*) of the beam.



(a) Frequency with time for capacity of the section



(a) Safety factor γ – Collapse at 90th day

Figure 5: Frequency of the beam with time with viscoelasticity.

Since the motor rotation is set at 1,200 rpm (20 Hz), represented by the dotted horizontal line, there is no resonance without consideration of creep, but the resonance appears when the natural frequency of the beam is calculated with the introduction of the creep. For ten days after application of load, for example, the resonant regime can be observed by the intersection of two curves, dotted (horizontal) and dashed (sloped), defining exactly for which prestressing force the phenomenon occurs at that instant (210 kN).

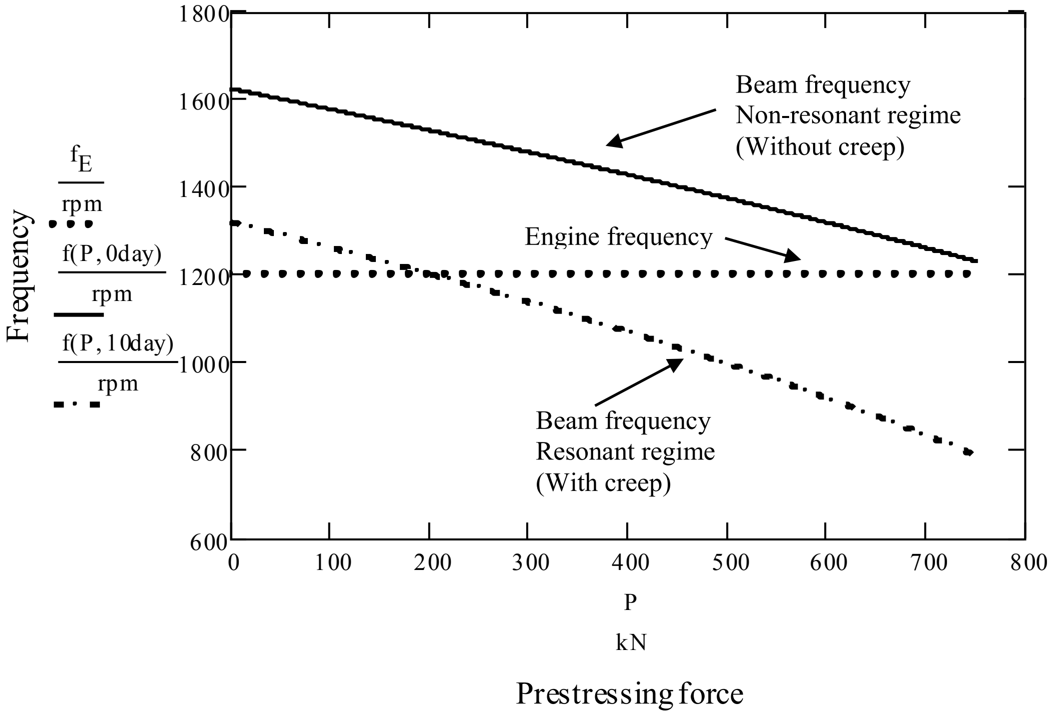


Figure 6: Resonant and non-resonant frequencies as a function of axial compression force *P*.

4 CONCLUSIONS

- In this article, a numerical simulation of a reinforced-prestressed concrete beam as a support for rotating machines was performed.
- For the vibration analysis, the creep, which is an intrinsic material property, was introduced due to the slenderness of the beam, revealing a resonant regime not foreseen in the linear analysis (without creep).
- The effect of geometric stiffness produced by the horizontal loading and the corresponding possibility of introducing resonant regimes in the structural support system were demonstrated by calculating their frequencies.
- It can be concluded, therefore, that due to the increase of the axial compressive force, resonance conditions can occur, as represented by the intersection of the curves in Figure 6. In the present study, resonance occurs when the axial compression force reaches 210 kN, at 10 days. Other instants might also be considered.
- Since the force of post-tension decreases the stiffness of the beam, this can lead to the resonant regime if it has not been previously evaluated in the structural analysis.
- The technique studied in this article offers an efficient tool to provide the removal of the support structure of that unwanted regime, avoiding the production of harmful effects on the equipment, fabricated products and work environment of the operators.
- In further work, it is necessary to introduce normative criteria, perform experimental activity and evaluate the influence of the prestressing bar stiffness on the structural response.

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