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CALIBRATION OF CONSTITUTIVE MACRO MODEL FOR A LEAD-CORE BEARING DEVICE: IDENTIFICATION BASED ON RESULTS OBTAINED BY FINITE ELEMENT ANALYSIS

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Abstract. *An identification procedure based on genetic algorithm is discussed in this paper. The procedure is aimed to calibrate a single-degree-of freedom (SDOF) model capable to simulate the mechanical response of lead-core bearing device for passive seismic isolation.*
The identification is based on data obtained by finite element simulation. An accurate finite element model is built. All components of the bearing device: lead-core, rubber layers and steel shims are discretely modeled. The lead-core bearing device is seen as a multiple component system. Constitutive laws are defined on the meso-scale for all materials: lead (-core), rubber, steel. Furthermore a typical identification test is simulated to obtain the shear stress-shear strain relationship which characterizes the response of the lead-core bearing device.
In the SDOF model describing the response of the bearing device on the macro-scale, a damage variable is introduced to account for mechanical damage accumulated in the lead-core.

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1 INTRODUCTION

Single-degree-of- freedom (SDOF) models capable of simulating the mechanical response of lead-core bearing device for passive seismic isolation are discussed in this contribution. SDOF models have evolved to differential equations models which involve variables design to account for degradation effects.

Model parameters are in general identified through comparison with reference data. Reference data is commonly obtained by experimental testing- e.g. identification test.

Alternatively reference set of data can be obtained by finite element analysis.

Details of the identification procedure are discussed.

2 SDOF MODELS

The widely used Bouc-Wen model ([1], [2], [3], [4], [5]) is described by the following set of equations:

$$Q = \alpha \frac{Q_y}{d_y} u + (1-\alpha) \frac{Q_y}{d_y} Z \tag{1}$$

$$\frac{dZ}{dt} = -\gamma \left| \frac{du}{dt} \right| Z |Z|^{(\eta-1)} - \beta \frac{du}{dt} |Z|^{\eta} + A \frac{du}{dt} \tag{2}$$

The first equation defines the constitutive relation $Q=Q(u)$ which characterizes the bearing device. Q stands for the shear force generated in the bearing device and u is the lateral displacement (see **Грешка! Източникът на препратката не е намерен.**). Q_y denotes the shear force in the lead-core bearing device at the moment of yielding of the lead-core; d_y denotes displacement at yielding.

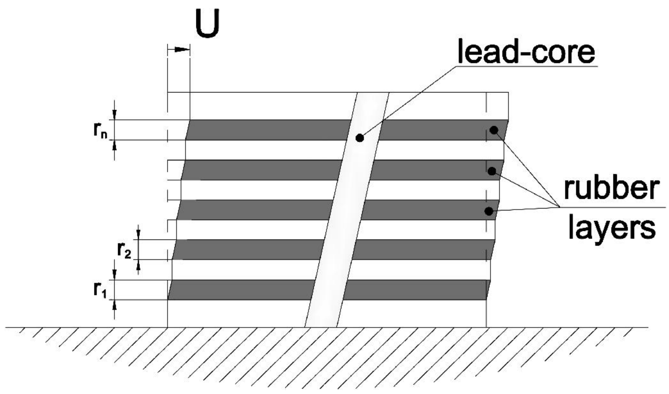

Figure 1: lead-core bearing; neutral position is depicted with dashed line

In equation (1) α is the ratio between the post- yield to pre-yield stiffness and Z is a hysteresis parameter. The evolution of Z is defined by the differential equation (2). In this equation γ, η, β and A are model parameters which are to be calibrated.

It should be noted that constitutive relation describing a lead-core bearing device is typically defined by postulating the relation $Q=Q[\gamma(t)]$ where γ is referred to as shear strain e.g. the ratio between the shear displacement u and the total high of the rubber layers ($\sum_{i=1}^{n} r_i$) in the lead-core bearing device (Figure 1):

$$\gamma = \frac{u}{\sum_{i=1}^{n} r_i} \tag{3}$$

As an alternative, a model in which a variable associated with material degradation is considered. This model, as already stated postulates a relation between shear stress and shear displacement:

$$\tau = f(\gamma) \tag{4}$$

Shear stress is conventionally split into three components:

$$\tau = \tau_e + \tau_1 + \tau_2 \tag{5}$$

The first term- τ_e denotes an elastic contribution, and the two others- overstresses relaxing with time. A variable accounting for damage accumulation is incorporated in the second term- τ_1. The evolution of the damage variable q_e is defined as a function of the shear strain and of a model parameter p_e [6]:

$$\frac{dq_e}{dt} = \begin{cases} \zeta_e \left| \frac{d\gamma}{dt} \right| \left(0{,}5|\gamma| - q_e\right) & if \quad q_e \le 0.5|\gamma| \\ 0 & if \quad 0.5|\gamma| \le q_e \le 1 \end{cases} \tag{6}$$

As it can be seen, in the above formulation it is presumed that material degradation depends on the shear strain and on the shear strain rate.

There exist also constitutive relations in which damage variable is introduced in the stress-strain relationship of the material [7]:

$$\sigma = E(D)\varepsilon \tag{7}$$

The above equation describes

All material parameters should be identified through curve fitting.

3 REFERENCE DATA

Material parameters occurring in the constitutive laws are commonly identified through comparison with experimental data. Authors support the idea that identification of the material constants in the SDOF models can be done on the basis of data obtained by finite element analysis ([8], [9]).

An explicit and accurate finite element model of the bearing device is set up. Geometry is reproduced in high level of detail.

Material models are defined for each component of the bearing device: lead-core, rubber, steel shims.

SDOF constitutive models are commonly calibrated on the basis of experimental data obtained by identification test (see for example [10]). Therefore an identification test is simulated in an implicit transient dynamic analysis.

4 CURVE FITTING

In this section stochastic algorithms which can be implemented in a numerical procedure for identification of model constants are discussed. The focus is on the genetic algorithms [11]. The input of the procedure is formed by a lot of 'chromosomes': sets (or vectors) of model parameters.

A test function (*TF*) is defined to assess the proximity of a Stress-strain (or Shear force- shear displacement relationship) to a chosen reference curve.
TF supplies a criterion whether a set of points $\{P_i^{(t)}=(u_i^{(t)},Q_i^{(t)})\}$ in the criterion space is close to the reference set $\{P_i^{(r)}=(u_i^{(r)},Q_i^{(r)})\}$.
For each iteration an optimal set is chosen by using TF. In the context of the 'genetic' algorithm other sets are manipulated, e.g. subjected to cross-over and mutations.
Pairs of arbitrary sets of model constants are compared. Pairs should be randomly chosen. This can be achieved by using one of the following schemes: a roulette wheel, a stochastic remainder, ranking tournament selection etc. ([13], [15]. Another strategy for generating random numbers, specifically a quasi-random sequence is discussed in the next section.
Thus for two compared sets S_1 and S_2 the set which is less convergent to the reference curve

$$TF_i^{(out)} > TF_j^{(out)} \tag{8}$$

is modified as follows:

$$S_j^* = fS_i + (1-f)S_j \tag{9}$$

where f is a random number ($f \in [0,1]$).

Test function is defined on the basis of the error estimation (following [12]):

$$e_i = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Q_i^{(m)} - Q_i^{(FEM)}}{\Delta Q}\right)^2 \tag{10}$$

In equation (10) superscripts *m* and *FEM* denote data points of the curve obtained by using the SDOF model and the curve obtained by finite element analysis respectively; *n* is the number of data points in the compared curves and ΔQ is the difference between maximum and minimum values of the shear force generated in the lead-core bearing:

$$\Delta Q = Q_{\max} - Q_{\min} \tag{11}$$

Test functions corresponding to each set of model parameters is evaluated by using the following set of equations:

$$PPTF_i = 1 - \frac{e_i}{\max(e_j)} \qquad j = 1..n \tag{12}$$

$$PTF_i = PPTF_i - \min\{PPTF_j\} \qquad j = 1..n \tag{13}$$

$$TF_i = \frac{PTF_i}{\max\{PTF_j\}} \qquad j = 1..n \tag{14}$$

In the following iteration the set S_i is kept as it stands whereas the set S_j is replaced by the set S_j* as defined above.

5 QUASI RANDOM SEQUENCE

A quasi random sequence can be constructed in the following way [15]:

$$q_{i,j} = e_1 V_j^{(1)} * e_2 V_j^{(2)} * \ldots * e_m V_j^{(m)} \quad i = 1 \ldots n \tag{15}$$

In equation (15) q_{ij} are coordinates of a point Q_i. The reference number of a given point $Q_i=(q_{i,1}, q_{i,2} \ldots q_{i,n})$ must be presented in binary format:

$$i = e_m \ldots e_2 e_1 \tag{16}$$

The quasi random sequence can be constructed by using the following arithmetic algorithm [15]:

$$m = 1 + \left[\frac{\ln(i)}{\ln(2)} \right] \tag{17}$$

$$q_{i,j} = \sum_{k=1}^{m} 2^{1-k} \left\{ \frac{1}{2} \cdot \sum_{l=k}^{m} \left[2.\left\{ i 2^{-l} \right\} \right] \left[2.\left\{ r_j^{(i)} 2^{k-1-l} \right\} \right] \right\} \quad j = 1 \ldots n \tag{18}$$

In equation (17) i stands for the number of a given point of the quasi random sequence. In equation (18) $[q]$ denotes the integer part of a given quantity q whereas $\{q\}$- the fractional part of q.

6 CURVE FITTING

Reference “shear displacement- shear force” curve obtained by finite element analysis as outlined in paragraph 3 is depicted in Figure 2.

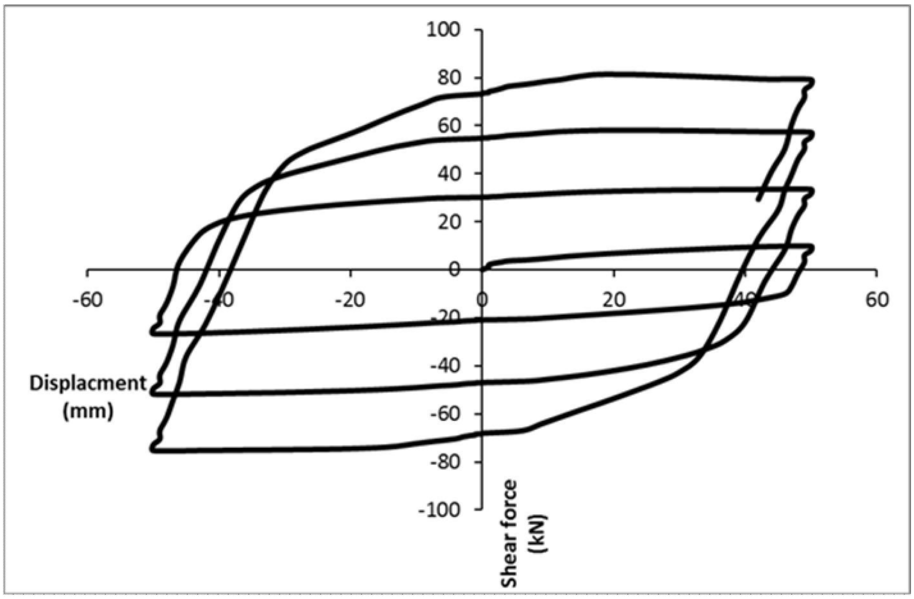

Figure 2: Reference Load- shear displacement curve obtained by finite element analysis

After defining initial data sets containing model parameters

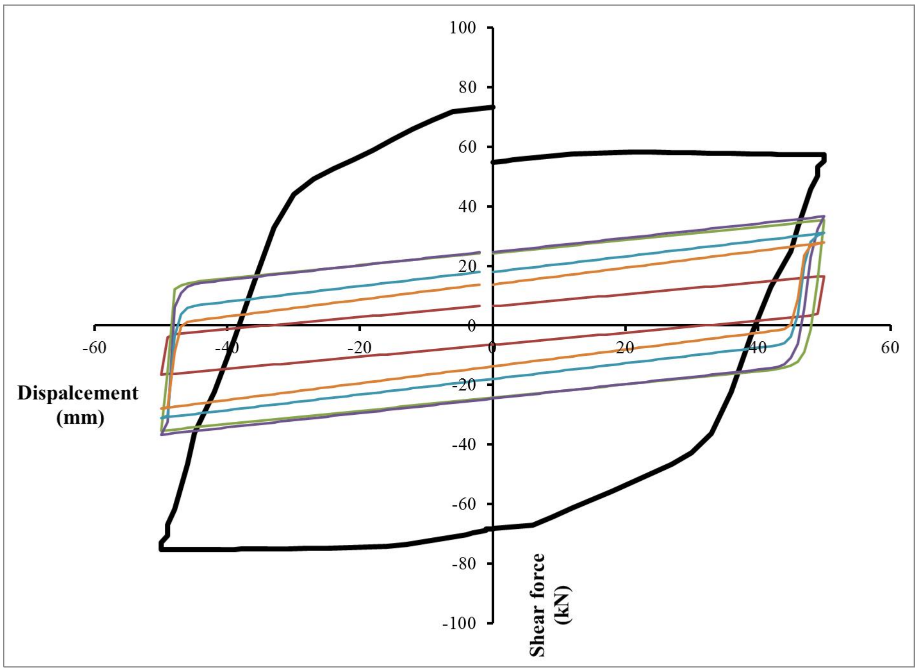

Figure 3 : Reference hysteresis curve (in black) obtained by finite element analysis and SDOF produced curves corresponding to different initial data sets: first iteration

the proximity of each set of model parameters (Figure 3) to the reference data is evaluated by using a test function as outlined in paragraph 4.

In further iterations sets of model constants which are estimated as less convergent are manipulated to search a better fit with the reference hysteresis curve (Figure 4)

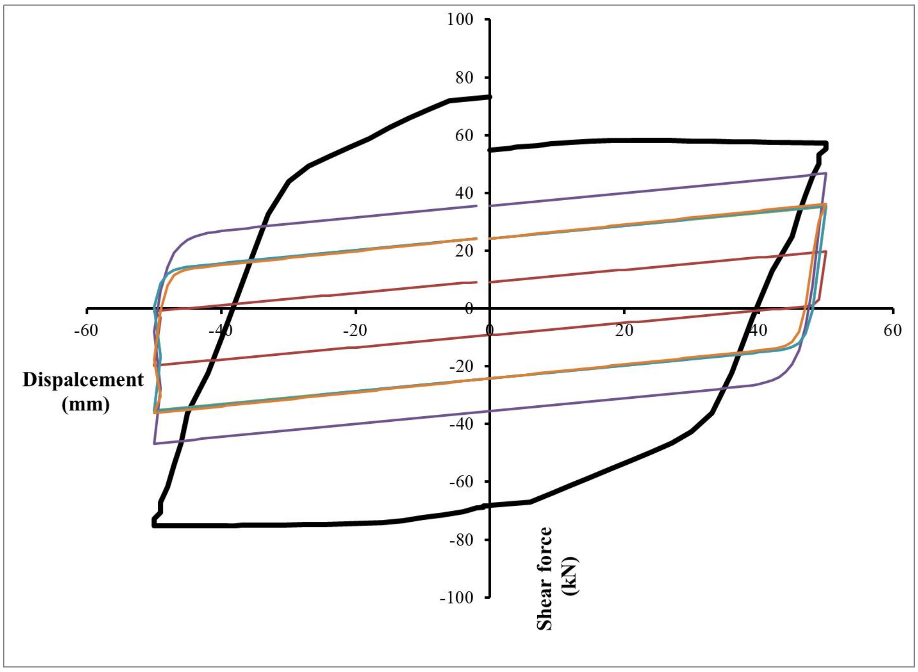

Figure 4: Reference hysteresis curve (in black) obtained by finite element analysis and SDOF produced curves corresponding to different initial data sets: further iterations

and to choose eventually a set giving maximum possible approximation of the reference shear force- displacement curve (Figure 5).

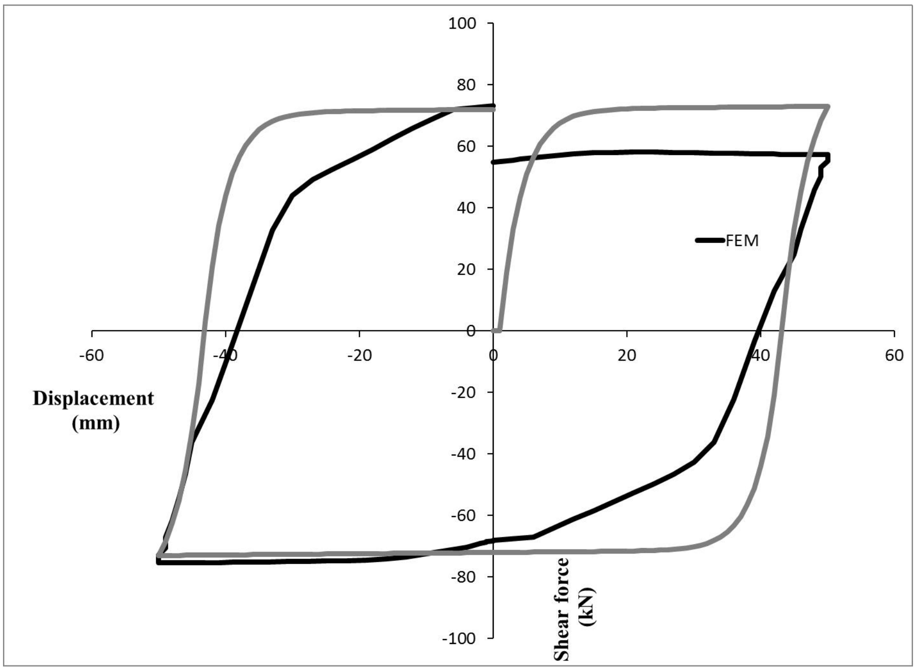

Figure 5: Shear force- displacement relationships obtained by finite element modeling (I black) and by Single-Degree-of-Freedom (Bouc-Wen) model (in grey).

7 CONCLUDING REMARKS

In this contribution steps of a numerical procedure designed to calibrate the parameters of single-degree of freedom models aimed to simulate the mechanical response of lead-core bearing devices for passive seismic isolation have been discussed.

It should be noted that authors support the idea for parameter identification partially or gully based on results obtained by finite element analysis instead of using experimentally obtained reference data.

The identification procedure is based on a genetic algorithm. It starts by defining a number of initial set of model parameters. The output from the single-degree-of-freedom model produced by each is compared to the reference hysteresis curve obtained by finite element analysis. A test function based on error estimation is used to compare output curves produced by different data sets. Before the next iteration of the identification process datasets are manipulated in order to assure a better fit with reference hysteresis curve in subsequent iterations. The manipulation rule accounts for test functions associated to each data set.

Further work is related to the "automation" of the identification algorithm as well as to the identification of mechanical parameters of a single-degree-of-freedom model accounting for material degradation.

This contribution appears in the framework of collaboration between the first author and the Earthquake Engineering Research Center of the University of Iceland (affiliation of the other authors) started in 2014.

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