Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2017) 3506-3518

ECCOMAS
Proceedia

COMPDYN 2017
6th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15–17 June 2017

SEISMIC DESIGN OF ABOVEGROUND STORAGE TANKS CONTAINING LIQUID

Martin Sivy[1], **Milos Musil**[1]

[1] Slovak University of Technology, Faculty of Mechanical Engineering
Namestie slobody 17, 812 31 Bratislava, Slovakia
e-mail: {martin.sivy, milos.musil}@stuba.sk

Keywords: Liquid Storage Tanks, Earthquake, Seismic Vulnerability, Seismic Characteristics, Simplified Models, Eurocode 8.

Abstract. *Liquid storage tanks are important components of liquid transmission and distribution systems and should be properly designed to withstand dynamic loadings which can take several forms. One of them is the investigation of tanks subjected to a seismic excitation. During seismic activity, a specific interaction between the tank and the liquid occurs. It is expressed as a vibration of the tank, its walls and contained liquid. The impulsive (lower) portion of the liquid moves in unison with the structure while the convective (upper) portion represents the free surface moving against a wall which results in a sloshing effect. The procedure for the design of tank-liquid systems for earthquake resistance is covered in various international and national standards. The paper deals with the seismic analysis of the aboveground vertical liquid storage tanks of circular cross section with different slenderness parameters (from broad to tall tanks) intended to compute the dynamic properties (natural frequencies and respective modes of oscillation) and the response of the flexible tank – liquid system to seismic loading using the response spectrum method. The paper also focuses on the comparison of dynamic properties of the tank-liquid system when it is resting on a rigid or flexible foundation. For tanks supported on softer soils, the dynamic response may be significantly different from those supported on rigid foundations. In addition to the seismic analysis, the paper is dedicated to comparing the acquired results from analyses computed by FE method (in ANSYS Multiphysics) with the results from analytical calculations of a tank containing liquid introduced in Eurocode 8.*

Peer-review under responsibility of the organizing committee of COMPDYN 2017.
doi: 10.7712/120117.5661.17061

1 INTRODUCTION

In contemporary global industrial development and improvement of production efficiency, emphasis must be given to the improvement of safety in facilities using new and advanced technical resources that will ensure effective prevention of large industrial accidents. In a broader context, the word "safety" can be expressed by the term loss prevention so as to prevent the loss of human life and in doing so, also ensuring the integrity of property, production and the environment [1].

There are some quantitative and qualitative methods in hazard identification (HAZOP, DOW index, fault tree analysis, selection method, etc.) to prevent negative consequences of transmission and processing systems (e.g., heat exchangers, evaporators, drying devices, cooling system, etc.) in ordinary operation (more introduced in [2]).

On the other hand, the systems must withstand external loadings of different nature (static, dynamic) as well. Requirements for the suppression of possible loading in future operation must be reflected in the design process of the device which are mostly included in various international, national or company standards and/or guidelines. One of the most critical external events affecting safety and operation efficiency of systems is seismic excitation (e.g., seismic testing of structures is described in [3]).

Large capacity tanks and vessels are one such device which is assessed in terms of vulnerability against seismic effects. Tank-liquid systems are commonly used in the storage of various liquids in various sectors of industry (e.g., nuclear, chemical, food, etc.). Seismic analysis of liquid storage tanks requires special considerations which take into account time-dependent hydrodynamic forces and pressure exerted by the liquid on the tank wall and bottom. Knowledge of these hydrodynamic effects is essential in the seismic design of tanks. Inadequately designed tanks in the past exposed to strong ground motions led to damage, ruptures and failures of tank accessories. Furthermore, when tanks store flammable or toxic liquids, disastrous effects, such as uncontrolled fire, explosion or toxic dispersion arose [2]. Therefore, tanks must be designed to maintain their integrity before, during and after a seismic event to prevent negative future effects.

Based on experimental research and analytical results of G. W. Housner [4, 5, 6], A. S. Veletsos [7, 8], P. K Malhotra [9] and others, different provisions for seismic resistance of different tank-liquid systems (e.g., aboveground, buried, elevated, etc.) have been developed. Some of them were adopted in international codes and guidelines dedicated to a seismic resistance of these systems, e.g., AWWA, ACI, API, Eurocode 8 and NZSEE.

2 BASIC CONCEPT

Evaluation of hydrodynamic forces due to lateral base excitation requires modeling and dynamic analyses of the tank-liquid systems. The complexity of the investigated liquid storage tanks may be simplified by mechanical models. The most used equivalent mechanical model is the one proposed by G. W. Housner [4] which converts investigated tank-liquid system into a spring-mass system. Housner's model or eventually its modification is introduced in the codes aimed at the structural design for earthquake resistance (e.g., in Eurocode 8 [10]).

This mechanical analogy assumes that the contained liquid can be divided into two regions. The first, impulsive portion of the liquid moves in unison with the tank as a rigid body and induces impulsive hydrodynamic pressure on the tank wall and its base. In the equivalent model, it is replaced by an impulsive mass m_i rigidly attached to the tank walls at a height h_i (h_i'). A second or convective portion of the liquid represents the free surface which undergoes sloshing motion and exerts convective hydrodynamic pressure on the tank wall and base. In the simplified model, this zone is substituted by an infinite number of convective masses m_{cn}

attached to the tank wall at height h_{cn} (h_{cn}') by a spring of stiffness k_{cn}. Each convective mass represents the effective liquid mass that oscillates in each particular slosh mode [11].

Parameters introduced in Fig. 1 like liquid masses and heights are used in basic seismic characteristics such as base shear, overturning moment and hydrodynamic pressure. Procedure for parameters calculation for cylindrical storage tanks can be found e.g. in [10, 12].

Mechanical models were first developed for tanks with rigid walls which were modified for short and slender tanks. Subsequently, Haroun and Housner [6] and Veletsos [7] developed models for flexible tanks. Malhotra [9] further simplified models proposed by Veletsos [7]. Observing and comparing the parameters of the rigid and flexible tank-liquid systems it was concluded there is no significant difference in the results obtained from these models [13].

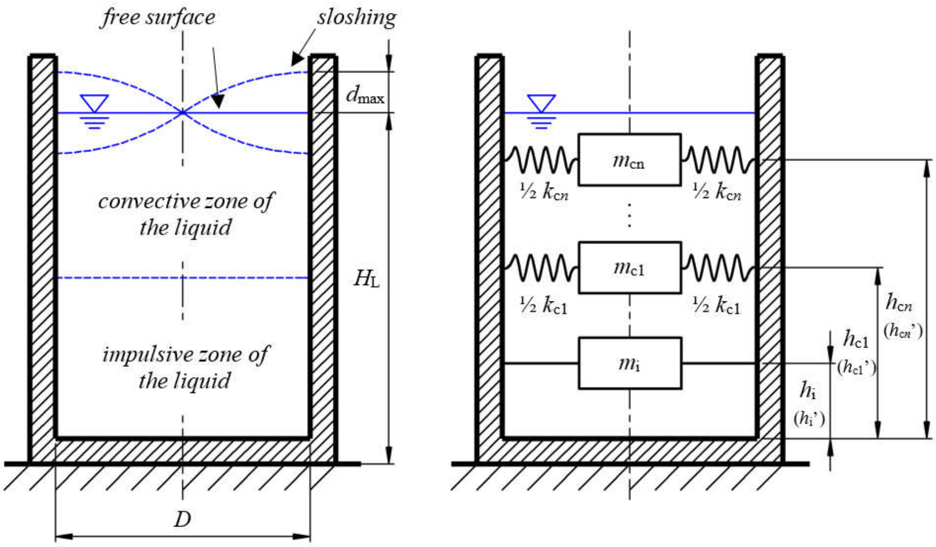

Figure 1: Equivalent spring-mass mechanical model

In addition to spring-mass models, other equivalent models were developed in which the convective liquid can be substituted by a system of simple pendulums, each of mass m_{cn}, and length l_{cn}. For observing nonlinear phenomena (e.g. at sloshing frequencies), the aforementioned linear models do not provide an adequate representation of the liquid free-surface. Therefore, models capable of describing relatively large amplitudes (spherical pendulum model) or strongly nonlinear motion (pendulum describing impacts with the tank walls) must be employed [12].

Analytical methods are suitable only for the simplest systems. For complex structures, analytical approach may be insufficient and inaccurate. Therefore, numerical approaches have been developed. The most widely used is the method based on finite elements (FEM). This interaction can be investigated by applying elements based on different formulation, such as the added mass concept, Lagrangian or Eulerian methods. Using fluid elements, a definition of fluid-structure interaction (FSI) at interface between structure and fluid is required to couple their displacements.

3 SEISMIC RESPONSE OF ABOVEGROUND LIQUID STORAGE TANKS

The paper is focused on the seismic analysis of aboveground vertically oriented liquid storage tanks with circular cross section and differing slenderness parameters γ (liquid height to tank radius – 0.5, 1 and 2). The investigated models are of radius R (5 m), wall thickness t (5e-3 m) and heights H (4, 6 and 11 m). The tanks are filled with liquid (water) with free surface heights H_L (2.5, 5 and 10 m). The aim is to calculate dynamic properties such as impulsive and convective frequencies and respective modes of oscillation. Subsequently, a seismic response to a given earthquake loading is calculated using a response spectrum method.

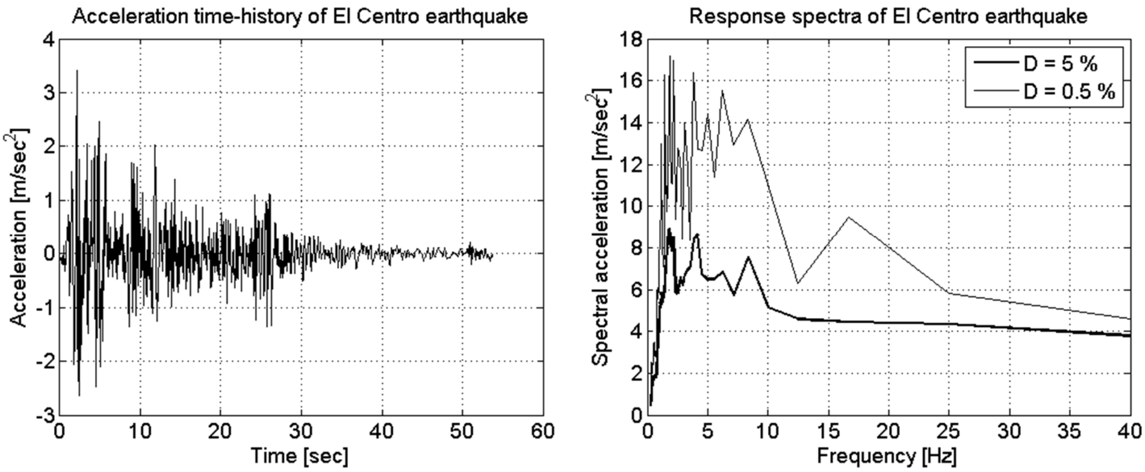

Figure 2: El Centro earthquake

For the seismic input, data recorded from the El Centro earthquake (1940) with a PGA of 3.417 m/sec^2 at 2.14 sec was used. Acceleration time-history of the earthquake and its response spectra for proportional damping of 0,5 % (liquid) and 5 % (structure) are presented in Fig. 2.

3.1 Impulsive vibration

Impulsive mode of oscillation corresponds to the lateral mode of a tank-liquid system. It refers to a situation when the tank oscillates in unison with its content (liquid). The lateral force acting on the tank depends on the respective impulsive frequency. Tank flexibility affects the impulsive component of hydrodynamic effects, hence impulsive frequency of the assumed system as well. In Eurocode 8, there is a procedure for the determination of impulsive frequency at which this unfavourable response occurs. The equation was proposed by Malhotra [9] and is expressed as

$$f_i = \frac{1}{C_i} \frac{\sqrt{t/R}\sqrt{E}}{\sqrt{\rho_L} H_L} \tag{1}$$

where ρ_L is the density of the liquid, E is Young's modulus of elasticity of the tank material and C_i represents coefficients for respective slenderness parameter H_L / R which are presented e.g. in [9, 10].

Slenderness parameter γ	Eurocode 8	ANSYS Multiphysics
0.5	23.68 Hz	21.06 Hz
1	14.41 Hz	14.20 Hz
2	7.38 Hz	7.28 Hz

Table 1: Comparison of impulsive natural frequencies

Tab. 1 presents calculated impulsive natural frequencies for investigated tank-liquid systems by analytical approach (Eurocode 8) and compared by numerical computations in ANSYS. Fig. 3 represents computed unfavourable responses for various tanks containing liquid at natural frequencies listed in Tab. 1.

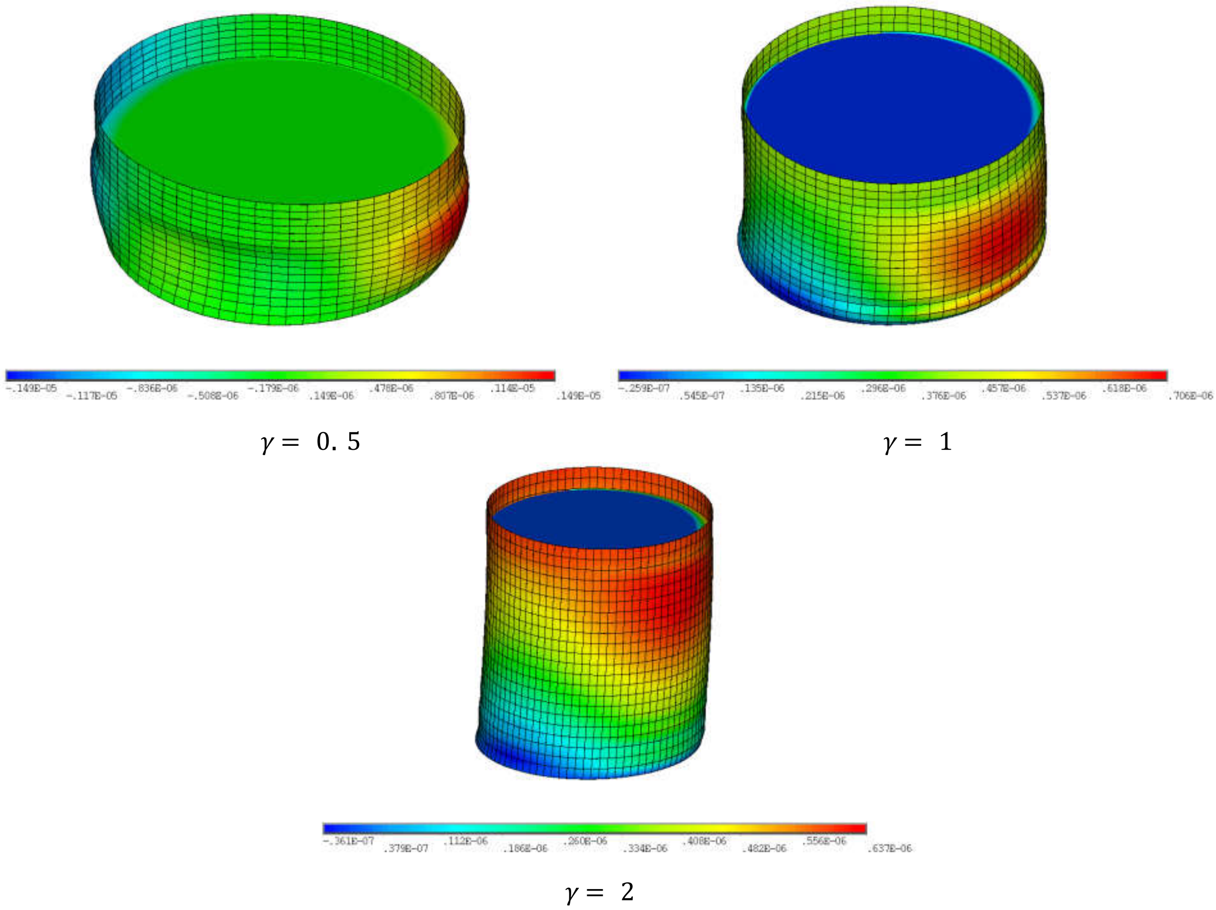

Figure 3: Impulsive modes of oscillation

For tanks supported on softer soils, the dynamic response may be significantly different from those supported on the rigid foundation. In addition to the lateral component of motion, the foundation motion may include a rocking component. If the response of the structure is dominated by its impulsive mode, the soil-structure interaction effects may result in a decrease in the natural impulsive frequency of investigated tank-liquid system. This decrease depends on the flexibility of the foundation soils.

Slenderness parameter γ	ANSYS Multiphysics
0.5	19.40 Hz
1	9.24 Hz
2	3.12 Hz

Table 2: Impulsive natural frequencies influenced by foundation flexibility

In Tab. 2, impulsive natural frequencies of investigated tank-liquid systems supported on a flexible foundation are presented. In FE analysis, the Winkler's foundation model is used with foundation stiffness of $k = 10\mathrm{e}7\,\mathrm{N/m^3}$. When comparing results with frequencies from Tab. 1, soil-structure interaction decreased values of original impulsive natural frequencies.

3.2 Convective vibration

The upper (convective) portion of the liquid (Fig. 1) does not move as a rigid body with the tank walls but experiences a sloshing motion. Convective effects are associated with oscillations of much shorter frequency than the impulsive effects. These two components are weakly coupled and each effect is insensitive in characteristics of the other.

Convective oscillations can be expressed as a linear combination of the corresponding natural modes of the liquid. Slosh modes of the free surface in a vertical cylindrical vessel of a circular cross-section are usually formulated analytically by the Helmholtz equation. Using polar coordinates, the Helmholtz equation can be expressed as

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + k_m^2\right) S_m = 0 \tag{2}$$

where k_m is a wave number of a respective mode of oscillation and S_m is a function describing the amplitude of the response depending on the location of the oscillating free surface. The solution for each slosh mode of (2) can be expressed by the Bessel function of the first kind

$$S_{n,m} = J_n\left(\lambda_{n,m}\frac{r}{R}\right)\cos n\theta \tag{3}$$

where J_n represents the Bessel function of the first kind and $\lambda_{n,m}$ is the *m*th zero of $J_n'(\lambda_{n,m})$.

The natural frequency of the convective liquid with respective wave number may be calculated as follows

$$f_{\mathrm{c}(n,m)} = \frac{1}{2\pi}\sqrt{\lambda_{n,m}\frac{g}{R}\tanh\left(\lambda_{n,m}\frac{H_{\mathrm{L}}}{R}\right)} \tag{4}$$

Eurocode 8 standard introduces (4) for calculation of natural convective frequencies but the expression is modified only for modes antisymmetric about the axis of rotation, i.e. modes of oscillation are described by the Bessel function of the first kind and first order.

Fig. 4 shows three selected slosh modes of oscillation which are computed analytically using (4) for the investigated model of the liquid storage tank (with slenderness ratio equal to one) and subsequently compared with sloshing waves computed by FEM in ANSYS.

The natural frequencies corresponding to the selected modes of oscillation are introduced in Tab. 3. Analytically calculated convective frequencies are compared with those obtained from FE analysis.

Mode of oscillation	Analytical approach	ANSYS Multiphysics
(1,1)	0.30 Hz	0.30 Hz
(1,2)	0.51 Hz	0.52 Hz
(2,3)	0.70 Hz	0.71 Hz

Table 3: Comparison of selected convective natural frequencies

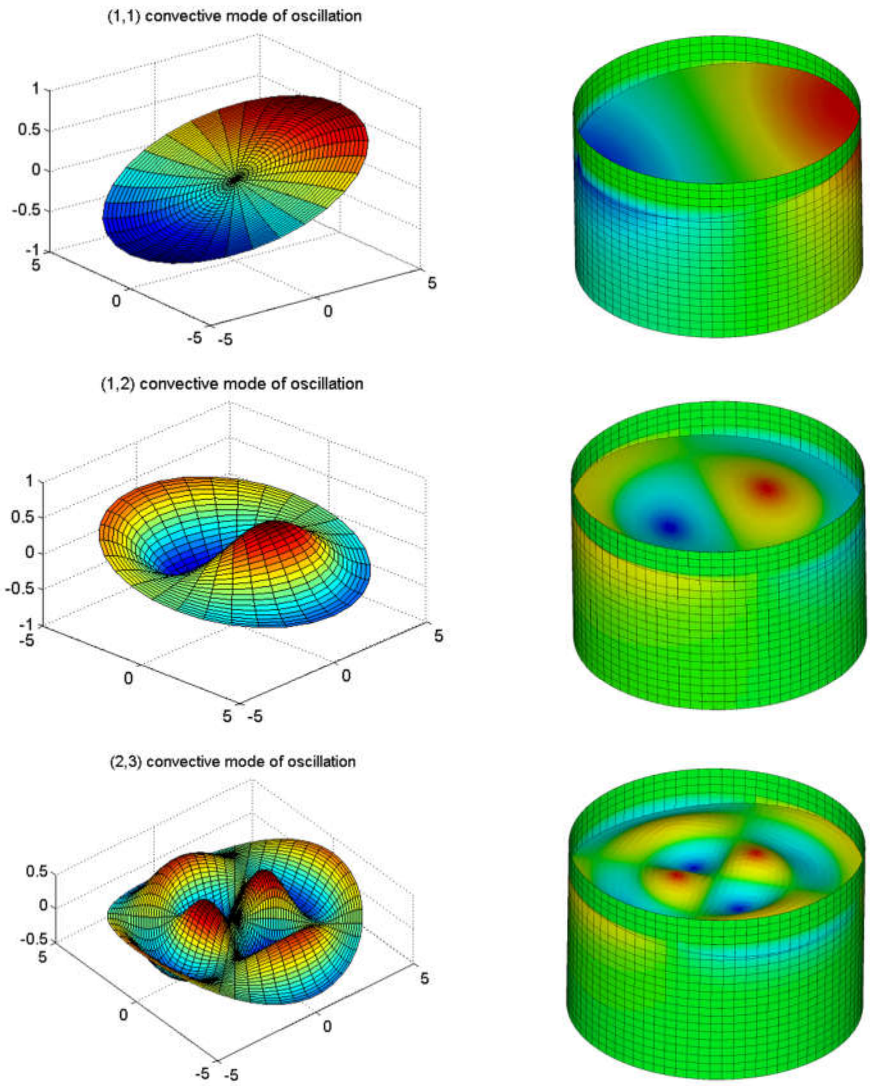

Figure 4: Selected convective modes of oscillation

For most tanks, the first impulsive and first convective modes account for 85–98% of the total liquid mass in the tank. The remaining mass of the liquid vibrates primarily in higher impulsive modes in tall tanks, and higher convective modes for broad tanks [9]. Therefore, only first oscillating frequency and mode is considered for design purposes. Eurocode 8 introduces the expression for the calculation of the first convective frequency in the simplified procedure for fixed base cylindrical tanks.

$$f_c = \frac{1}{C_c\sqrt{R}} \tag{5}$$

where C_c represents coefficients for respective slenderness (e.g., in [9, 10]).

Fig. 5 gives the values of the first convective frequency as a function of the most used slenderness parameters γ. The figure presents the sloshing frequencies for γ assuming constant radii R of the investigated tank (5 m) whilst the liquid heights are changing. As it can be seen, frequencies become almost independent of γ (for γ larger than about 1). Frequencies show good correlation between analytical and numerical calculations.

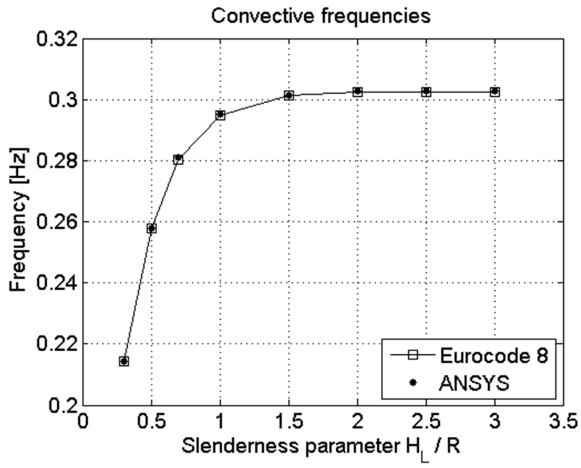

Figure 5: Convective frequencies for various slenderness parameters

When investigating an influence of flexible foundation on convective components, effects of soil-structure interaction are expected to be small due to weak coupling between convective and impulsive components (the convective effects are associated with oscillations of much shorter frequency than those characterizing the impulsive effects) and may be neglected [7].

3.3 Sloshing wave height

When investigating open tanks filled with liquid, the maximum vertical displacement of the free liquid surface is observed. Oscillation of the convective liquid in the containers may lead to negative effects, such as deformations of the tank walls (closed tanks) or liquid spilling (tanks without roofs). Therefore, sufficient freeboard between the free surface and the top of the tank must be designed.

The sloshing wave height may be given from the following expression

$$d(r,\theta,t) = R\sum_{n,m=1}^{\infty} \frac{2}{\lambda_{n,m}^2 - 1} \frac{J_n\left(\lambda_{n,m}\frac{r}{R}\right)}{J_n\left(\lambda_{n,m}\right)} \frac{S_e(f_{c(n,m)})}{g} \cos n\theta \tag{6}$$

where $S_e(f_c)$ is the convective spectral acceleration, obtained from a 0,5 %-damped elastic response spectrum.

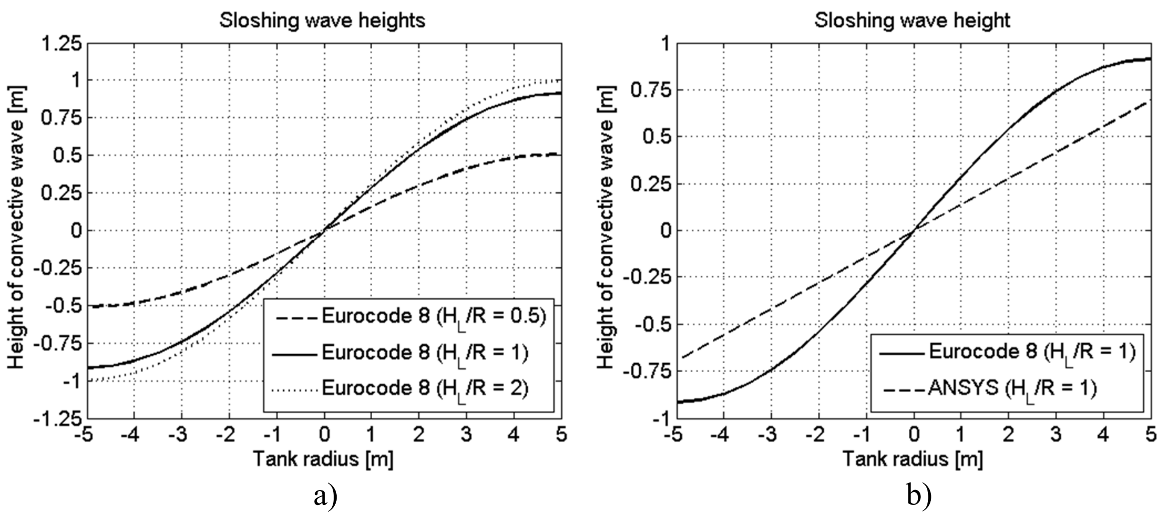

Figure 6: Sloshing wave height

In Eurocode 8, the wave height is calculated by assuming only the first mode of oscillation and the procedure is reduced for specifying the maximum height at the tank wall. Peak value of the wave height at the edge may be expressed as

$$d_{\mathrm{max}} = 0.84 \frac{RS_{\mathrm{e}}(f_{\mathrm{c}1,1})}{g} \tag{7}$$

For calculation of the vertical displacements for tank models, only the first convective mode is considered with spectral acceleration values from the respective response spectrum (Fig. 2). In Fig. 6a surface displacements along tank radius are shown. For the tank of γ equal to one, maximum vertical displacement is 0.70 m using (6) or (7) and 0.91 m using ANSYS (Fig. 6b).

3.4 Response spectrum analysis

Response spectrum method (spectrum analysis) is a significant method for seismic analysis of structures and equipment. It is mainly used in place of a time-history analysis to determine the response of structures to time-dependent loading. A spectrum analysis is one in which the results of a modal analysis are used with a response spectrum to calculate displacements and stresses in the model. The structures and equipment components are modeled usually as multi-degree-of-freedom systems on the base of finite element method [14].

The modal solution is required because the structure's mode shapes and frequencies must be available to calculate the spectrum solution. The sufficient number of modes characterizing the structure's response in the frequency range of interest must be extracted. The ratio of the effective mass to the total mass greater than 0.9 (more than 90% of the mass is included) is generally considered acceptable [15].

When investigating tank-liquid system using FEM and considering a fluid-structure interaction, the modal analysis uses as the mode-extraction method unsymmetric matrix formulation. If the unsymmetric eigensolver is used, right and left eigenvectors must be computed. The effective mass for the *i*th mode can be expressed as

$$\mathbf{M}_{\mathrm{e}i} = (\mathbf{d}_{\mathrm{s}})^{\mathrm{T}} \mathbf{K}_{\mathrm{s}} \boldsymbol{\phi}_{si}^{\mathrm{R}} \frac{(\boldsymbol{\phi}_i^{\mathrm{L}})^{\mathrm{T}} \mathbf{Md}}{\omega_i^2} \tag{8}$$

where $\mathbf{d}_{\mathrm{s}}$ is the direction vector of the excitation (structural DOFs only), $\mathbf{K}_{\mathrm{S}}$ is the stiffness matrix of the structural part, $\boldsymbol{\phi}_{si}^{\mathrm{R}}$ is the *i*th right eigenvector (structural DOFs only).

The equation of motion for the system subjected to the ground motion $\ddot{u}_{\mathrm{g}}(t)$ is written as

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{Md}\ddot{u}_{\mathrm{g}}(t) \tag{9}$$

where $\mathbf{d}$ is a vector of excitation direction (direction cosines).

The response of the structure is represented in terms of a linear superposition of mode shapes

$$\mathbf{u}(t) = \textstyle\sum_{i=1}^{n} \boldsymbol{\phi}_i \mathrm{w}_i\,(t) \tag{10}$$

where $\mathrm{w}_i(t)$ are called normal coordinates and are functions of the time variable t.

When using the unsymmetric eigensolver, the matrices are unsymmetric. Both left and right normalized eigenmodes are used to decouple the modal equations as follows

$$(\boldsymbol{\phi}^{\mathrm{L}})^{\mathrm{T}}\mathbf{M}(\boldsymbol{\phi})\ddot{\mathbf{w}}(t) + (\boldsymbol{\phi}^{\mathrm{L}})^{\mathrm{T}}\mathbf{C}(\boldsymbol{\phi})\dot{\mathbf{w}}(t) + (\boldsymbol{\phi}^{\mathrm{L}})^{\mathrm{T}}\mathbf{K}(\boldsymbol{\phi})\mathbf{w}(t) = -(\boldsymbol{\phi}^{\mathrm{L}})^{\mathrm{T}}\mathbf{Md}\ddot{u}_{\mathrm{g}}(t) \tag{11}$$

The participation factor for a given excitation direction, if the unsymmetric eigensolver is used, is defined as

$$\gamma_i = \left(\boldsymbol{\phi}_i^{\mathrm{L}}\right)^{\mathrm{T}} \mathbf{Md} \tag{12}$$

Equation (11) leads to the modal coordinate equation

$$\ddot{\mathrm{w}}_i(t) + 2\omega_i \xi_i \dot{\mathrm{w}}_i(t) + \omega_i^2 \mathrm{w}_i(t) = -\gamma_i \ddot{u}_{\mathrm{g}}(t) \tag{13}$$

In the modal superposition method, (12) is solved to obtain the time histories of the normal coordinates $\mathrm{w}_i(t)$ which with (10) gives the history of the relative displacement vector $\mathbf{u}(t)$.

This concept may be used to apply the response spectrum method to the MDOF structure. If the response spectrum of the given earthquake is known (e.g. acceleration response spectrum) it can be used in the calculating the maximum response of the system. Maximum displacement vector in the ith mode can be written

$$\mathbf{u}_{i\,\max} = \frac{\gamma_i S_{\mathrm{A}i}}{\omega_i^2} \boldsymbol{\phi}_i \tag{14}$$

where $\mathrm{S}_{\mathrm{A}i}$ represents the spectral acceleration for the ith mode.

Given displacement vector (14) represents the maximum value of any response of interest. The overall response of the system is calculated by combining the maximum modal responses specified by one of the mode combination methods (SRSS, CQC, GRP, etc.).

A general rule for modal response combination can be defined as

$$R = \pm\sqrt{\varepsilon_{i,j} R_i R_j} \tag{15}$$

where R_i, R_j are the ith and jth modal response respectively and $\varepsilon_{i,j}$ is a combination matrix whose shape and values are based on the chosen modal response combination.

A response spectrum method for determining the overall responses of the investigated tank-liquid systems fixed to a rigid foundation was performed. the seismic response was described by two response spectrum curves, one with 5 % proportional damping of the structure while the latter had 0,5 % for the liquid (Fig. 2). Fig. 7 shows the overall responses of liquid storage tanks to the El Centro earthquake. Tab. 4 presents results of the response spectrum method for the investigated liquid storage tanks.

Slenderness parameter γ	Maximum displacement	Maximum sloshing wave
0.5	0.304e-3 m	0.526 m
1	0.683e-3 m	0.694 m
2	0.005 m	0.712 m

Table 4: Summary of results from response spectrum method

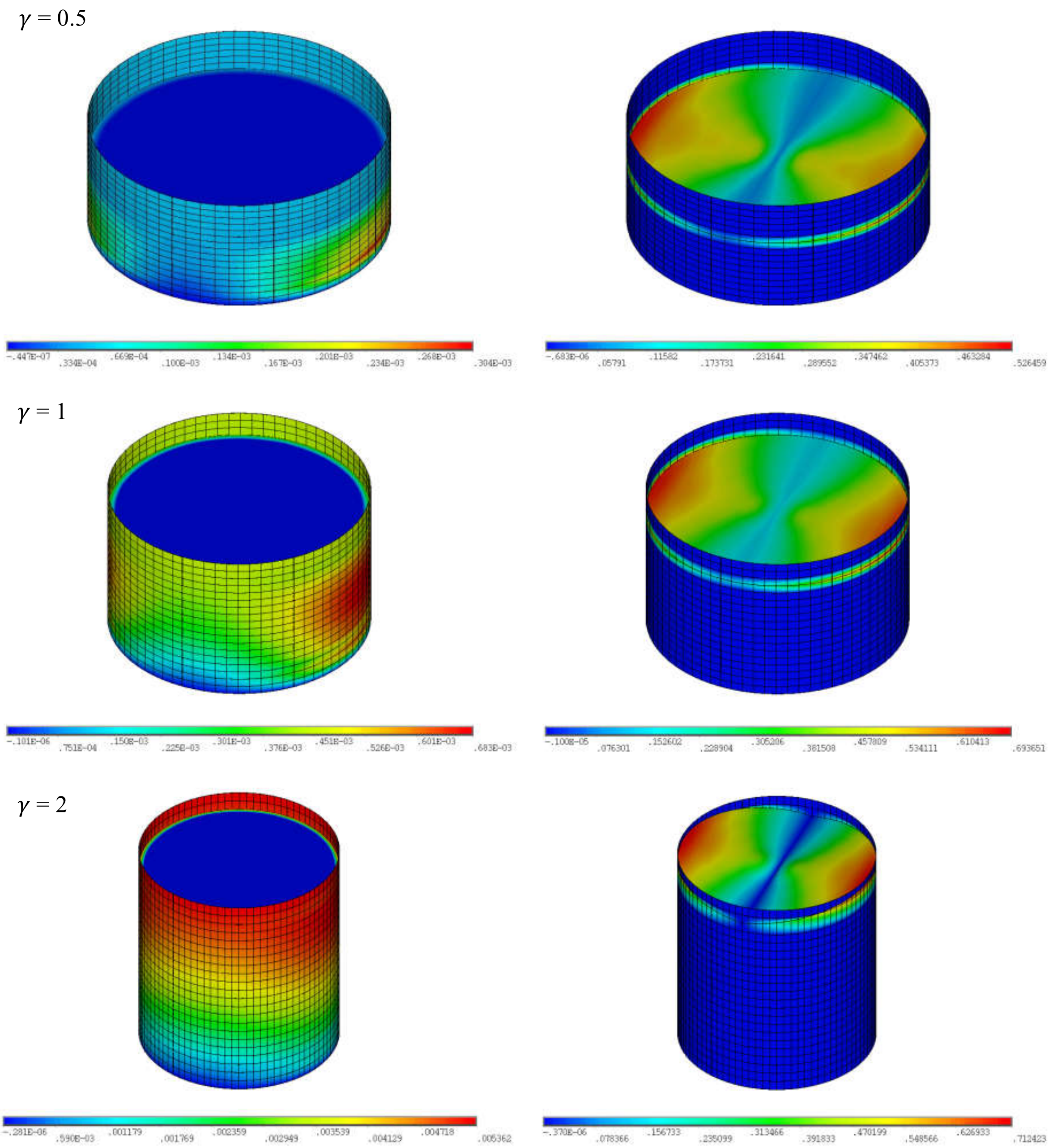

Figure 7: Overall responses of liquid storage tanks to El Centro earthquake

4 CONCLUSIONS

The continuing development of technologies in each industry requires increased attention to operational safety and protection of the environment. Liquid storage tanks serve as reservoirs for a variety of liquids (e.g., storage tanks in nuclear power plants), which are usually vulnerable to seismic effects (past earthquakes e.g., in Fukushima, San Fernando, etc.). Therefore, their safe operation is desirable. Seismic analysis is one method which should be carried out to provide satisfactory performance of tanks, especially in earthquake prone regions.

The aim of the paper was to perform a seismic design for various tank-liquid systems (from broad to tall tanks) and determine the dynamic effects such as impulsive and convective frequencies and modes of oscillation, which may result in unison vibration and sloshing respectively, and the overall response to the seismic loading. In addition to the aforementioned dynamic effects, some seismic characteristics (e.g., base shear, overturning moment, hydrodynamic pressure etc.) used in the seismic design were not covered in this paper, since they were given attention in other publications of the authors, e.g. in [11, 16, 17].

For the calculation of dynamic characteristics, analytical models according to Eurocode 8 standard and numerical approach (finite element method) were used. Some seismic characteristics calculated analytically were compared with results obtained in ANSYS Multiphysics and Matlab software. Results between each solution represented good conformity.

5 ACKNOWLEDGMENTS

This work was supported by the grant from the Grant Agency of VEGA no. 1/0742/15 entitled *Analysis for Seismic Resistance of Liquid Storage Tanks with Nonlinear and Time-Dependent Parameters* and by the Slovak Research and Development Agency under the contract no. APVV-15-0630 entitled *Extension of the Validity of the Computation Standards for the Seismically Resistant Liquid Storage Tanks, in terms of Safety at NPPs and Other Industrial Areas*.

REFERENCES

[1] M. Musil, M. Sivy, O. Chlebo and O. Prokop, Seismic Response of Elevated Liquid Storage Tanks, *Aplimat 2017 Proceedings of the 16th Conference on Applied Mathematics*, Bratislava, Slovakia, January 31 – February 2, 2017.

[2] P. Peciar, R. Fekete, M. Peciar, *Process Engineering II*, Vydavatelstvo STU, 2016.

[3] S. Ziaran, M. Cekan, O. Chlebo, M. Musil, Analysis of Seismic Response on the Excitation of Support Structures, *Proceedings of the 43rd International Congress on Noise Control Engineering* (*Inter-Noise 2014*), Toowong DC, The Australian Acoustical Society, 2014.

[4] G. W. Housner, Dynamic Pressures on Accelerated Fluid Containers, *Bulletin of the Seismological Society of America*, **47**(1), 1987.

[5] G. W. Housner, *Earthquake Pressures on Fluid Containers*, Eighth Technical Report under Office of Naval Research, California Institute of Technology, Pasadena, California, 1954.

[6] M. A. Haroun, G. W. Housner, Seismic Design of Liquid Storage Tanks, *Journal of the Technical Councils of ASCE*, **107**(1), pp. 191-207, 1981.

[7] A. S. Veletsos, *Seismic Response and Design of Liquid Storage Tanks, Guidelines for the Seismic Design of Oil and Gas Pipeline Systems*, Technical Council on Lifeline Earthquake Engineering (ASCE), pp. 255-370, 1984.

[8] A. S. Veletsos, J. Y. Yang, Earthquake Response of Liquid-Storage Tanks, *Advances in Civil Engineering through Engineering Mechanics*, North Carolina State University, Raleigh, North Carolina, USA, 1977.

[9] P. K. Malhotra, T. Wenk, M. Wieland, Simple Procedure for Seismic Analysis of Liquid-Storage Tanks, *Structural Engineering International*, **10**(3), pp. 197-201, 2000.

[10] EN 1998-4, *Eurocode 8 – Design of Structures for Earthquakes Resistance, Part 4 – Silos, Tanks and Pipelines*, CEN, 2006.

[11] M. Sivy, M. Musil, Seismic Resistance of Storage Tanks Containing Liquid in Accordance with Principles of Eurocode 8 Standard, *Strojnicky casopis – Journal of Mechanical Engineering*, **66**(2), pp. 79-88, 2016.

[12] R. A. Ibrahim, *Liquid Sloshing Dynamics Theory and Applications*, Cambridge University Press, 2005.

[13] O. R. Jaiswal, *Review of Code Provisions on Seismic Analysis of Liquid Storage Tanks*, Report No. IITK-GSDMA-EQ-01-V1.0, Indian Institute of Technology Kanpur, India, 2004.

[14] PNM34080183 – *Requirements on Evaluation of Seismic Resistance of Structures, Systems and Components of Mochovce NPP*, 2009.

[15] A. K. Gupta, *Response Spectrum Method in Seismic Analysis and Design of Structures*, Blackwell Scientific Publications, 1990.

[16] M. Sivy, M. Musil, Procedure for Seismic Analysis of Liquid Storage Tanks using FEM Approach and Analytical Models, *Advances in Mechanism Design II: Proceedings of the XII International Conference on the Theory of Machines and Mechanisms*, Springer, 2017.

[17] M. Musil, M. Sivy, Influence of Wall Flexibility of Liquid Storage Tanks on Hydrodynamic Pressures Induced During Seismic Activity, *Engineering Mechanics 2016*: *Book of Full Texts,* 22nd International Conference, Svratka, Czech Republic, May 9 – 12, 2016.