Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2017) 663-674

ECCOMAS
Proceedia

COMPDYN 2017
6th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15–17 June 2017

SCALE MODELS FOR THE EXPERIMENTAL ANALYSIS OF THE COLLAPSE MECHANISMS OF MASONRY BAY WINDOWS UNDER HORIZONTAL ACTIONS

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Keywords: Masonry, Bay windows, Limit analysis, Scale models.

Abstract. *The present contribution illustrates a first set of results obtained within the framework of currently on-going research aimed at evaluating the seismic vulnerability of masonry buildings in Boston's Back Bay area. In particular, the study focuses on the collapse mechanisms of "bay windows", a typical late 19th century architectural element of the Victorian style that characterizes the facades of Back Bay's old buildings.*

With the aim of investigating the main features of bay windows response to horizontal loads, the analysis is focused on a single bay window considering it isolated from the rest of the building and resting on a rigid basement. A series of experimental tests has been conducted on five scale-model bay windows of varying height free from openings. A system of horizontal actions proportional to the model mass density distribution has been imposed during the test by placing the model on a "tilting-table". The recordings of two digital video cameras allowed the identification of the collapse mechanisms.

The results obtained are described in terms of determination of the horizontal loads collapse multiplier and identification of the actual collapse mechanisms. In addition, a first interpretation of the experimental results is provided by means of some simple mechanical schemes that use the typical tools of limit analysis. The agreement between experimental and theoretical results is more than satisfactory, and the proposed theoretical schemes seem able to provide useful information for assessing the mechanical response of bay windows.

Peer-review under responsibility of the organizing committee of COMPDYN 2017.
doi: 10.7712/120117.5448.17065

1 INTRODUCTION

This study is part of the research activities being carried out within the MIT-UNIPI project "Mechanical Models for Masonry Walls under Seismic Actions", funded jointly by the University of Pisa and the Massachusetts Institute of Technology (MIT) in Boston. In particular, in what follows we describe some results obtained from the analysis of the collapse mechanisms of "bay windows", a typical late 19^{th} century architectural element of the Victorian style commonly adopted in Boston, especially in the neighbourhood called Back Bay. This residential area was early developed in 1860 thanks to an unprecedented landfill project that converted the previous milldam into the current area [2].

Seismic concern on this zone is justified by two main reasons. The former is that it can not be excluded that quite strong earthquakes can strike the region as they already did in the past, e.g. the 1755 Cape Ann's earthquake characterized by VII grade on Mercalli's scale. The latter is a not negligible liquefaction risk due to the poor conditions of the ground in which the timber-pile foundations are infilled. As seismic design in this area was not mandatory until 1975, a great percentage of buildings were built without any consideration to horizontal forces.

The determination of the mechanical response of bay windows under horizontal loads is a quite complicated problem to solve, because of the bay window polygonal shape and masonry material nonlinearities. Consequently, the structural role played by the bay windows in the building façade is still uncertain.

A survey made by Boston's team [7] evidenced that the timber floor structures run parallel to the building façade. Therefore, for the sake of simplicity, the connection between the bay window and the horizontal structure of each floor is disregarded; moreover, the seismic forces are schematized as a uniformly distributed load. Within these assumptions, the problem reduces to check the bay window stability by evaluating the ratio between stabilizing and overturning forces, *i.e.* the horizontal load multiplier. At incipient collapse, the load multiplier is assessed here by a simplified approach first proposed by Giuffrè [5]. A set of experiments have been conducted on scaled bay windows using a *tilting table* to evaluate the magnitude of horizontal loads at incipient collapse for models with different slenderness.

The experimental results have been interpreted theoretically using the classical methods and tools of limit analysis, following the approach proposed in [3] and, more generally, in [1]. Masonry has been modelled as a material containing a set of "weakness planes" across which tensile stresses cannot be transmitted. These assumptions led to assess admissible values for the kinematic load multiplier. Quite good agreement is observed between theoretical and experimental results.

2 EXPERIMENTAL INVESTIGATION OF THE COLLAPSE MECHANISMS OF BAY WINDOWS

Five bay windows scale models of varying height H have been built, namely H = 31.1, 34.2, 43.7, 53.1 and 81.3 cm. All the models shared the same plan, which reproduced in a 1/16-scale that of an actual bay window considered representative of Back Bay's style. For the sake of simplicity, bricks in a 1/4-scale have been used and the presence of openings has been disregarded in the models. As a consequence, one single brick in the model corresponded to 64 bricks in the actual bay window. The shape and dimensions of the scale models are shown in Figure 1a, b. Each model had a different slenderness measured by the dimensionless parameter, ξ, defined as the ratio between the frontal length, a, and the height of the model, $\xi = a/H$.

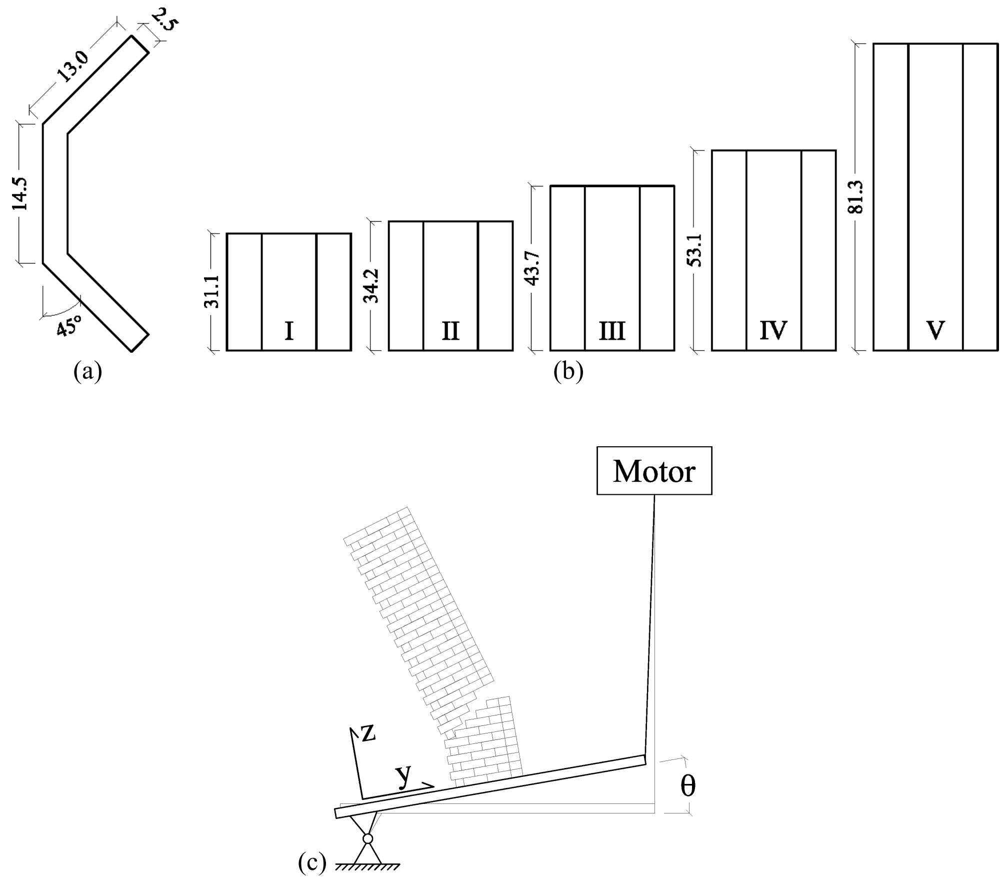

Figure 1: Models plan (a) and elevation (b), dimensions in cm; test setup (c).

The tests were carried out by using the *tilting table* schematized in Figure 1c. The table is progressively inclined with respect to the horizontal [4], [5]. With respect to the reference frame (y, z) that rotates with the model, the horizontal and vertical components of body forces are $b_y = -\gamma \sin\theta$ and $b_z = -\gamma \cos\theta$, where γ is the masonry unit weight and θ is the plane tilt angle. The corresponding horizontal load multiplier is defined as $\lambda = b_y/b_z = \tan\theta$. The tilt angle was measured by a digital inclinometer, while two digital cameras recorded the test. Three tests have been performed for each of the five models.

2.1 Analysis of the experimental collapse mechanisms

All the models collapsed by overturning of some upper part. However, depending on the slenderness of the model it was possible to distinguish two different collapse mechanisms which will be indicated as *global* and *local*. The global collapse mechanisms were characterized by rotation of the upper part of the model around an axis parallel to the tilting table and lying in the extrados of the model front part. A separating plane can be clearly seen between the upper portion and the lower part of the model (Figure 2). This type of mechanisms, typical of high specimens, was defined “global” because it involved more than 50% of the structure.

On the contrary, the local collapse mechanism was typical of short models and was characterized by separation of few brick rows at the top of the model. Such mechanism was defined “local” because it involved less than 50% of the structure. As an example, Figure 3 shows collapse of model “II”.

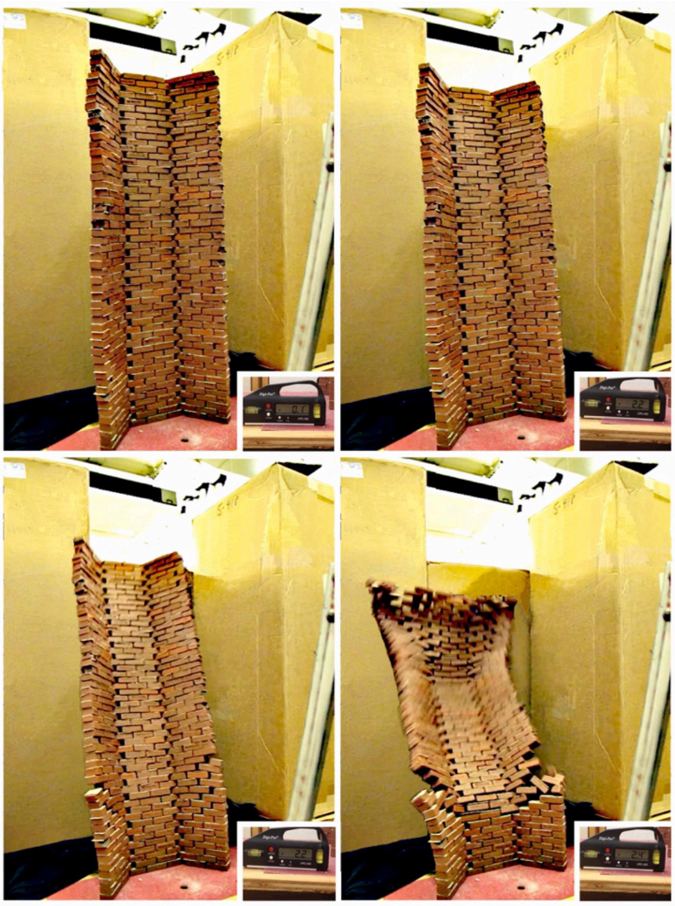

Figure 2: Selected frames of one of the tests conducted on model “V” showing its collapse (global mode).

Figure 3: Incipient collapse (local mode) of a test conducted on model “II”.

Lastly, models characterized by intermediate height values have shown a mixed collapse mode halfway between local and global. Such a finding suggests the existence of a *critical height* that separates local from global collapse modes. In passing, we remark that an odd collapse mode that seems not to fit into the classification illustrated above (Figure 4) was observed for model “I”. It was characterized by opening of the structure in two, somewhat resembling the opening of a “zipper”. In this case, bricks slipped on one another and each lateral wall overturns as if the front wall did not restrain it.

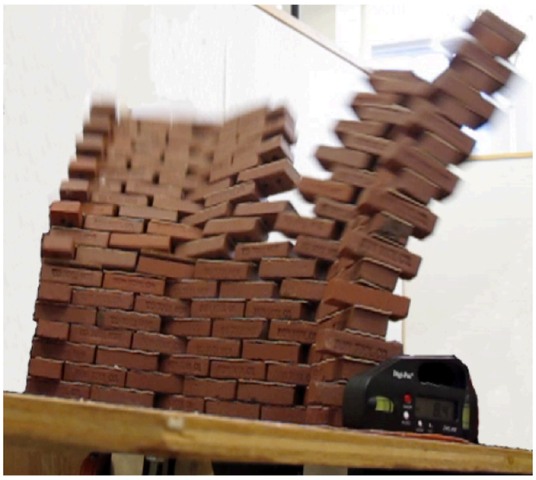

Figure 4: Odd collapse shown during a test of model "I"

2.2 Experimental results

For a given model, the mean values θ_{sp} of the corresponding three inclination angles at collapse are listed in Table 1, together with the horizontal load multipliers $\lambda_{sp} = \tan\theta_{sp}$, the observed collapse mechanism, and the model slenderness, ξ. The results obtained are plotted in Figure 5. In the same diagram, the regression lines of the experimental results are shown for the two sets of data corresponding to global (III – IV – V) and local (I – II – III) collapse mode, respectively.

Model	ξ	$\theta_1/\theta_2/\theta_3$	θ_{sp}	λ_{sp}	Collapse mechanism
I	0.466	9.1° / 8.3° / 7.9°	8.43°	0.148	Local
II	0.424	8.0° / 7.6° / 6.6°	7.40°	0.130	Local
III	0.332	6.4° / 6.3° / 6.2°	6.30°	0.110	Mixed
IV	0.273	6.8° / 5.6° / 4.7°	5.70°	0.100	Global
V	0.178	2.5° / 2.4° / 2.2°	2.37°	0.041	Global

Table 1: Collapse angles θ_i, their mean values θ_{sp}, and load multiplier λ_{sp}.

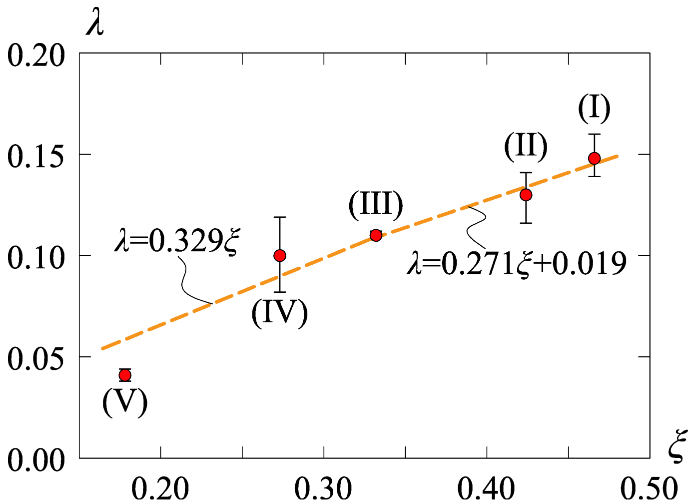

Figure 5: Experimental results

3 ASSESSMENT OF COLLAPSE LOAD BY LIMIT ANALYSIS

Limit analysis enables predicting some kinematically admissible values of the load multiplier. Here, masonry is considered as a material containing a prescribed set of weakness planes across which tensile stresses cannot be transmitted. Furthermore, masonry compressive strength is assumed unbounded and sliding collapse modes are not allowed. As is well known, under such hypotheses both the kinematic and static theorem of limit analysis hold [6].

In the following, assessment of upper bounds of the horizontal load multiplier by means of the kinematic theorem is illustrated. In this regard, a suitable set of kinematically admissible collapse mechanisms is considered. Each mechanism is characterised by the rotation of some upper part of the model bounded from below by a separating plane surface. As is shown in Figure 6, the collapse mechanism is individuated by the two heights, h_1 and h_2, measured from the top base. In the following, such collapse mechanisms will be indicated as "*Rotational Collapse Modes*" (RCM).

In the case under consideration, the internal energy dissipation is null and the upper-bound theorem becomes equivalent to the virtual work principle for a rigid body. The structure is loaded by only its self-weight, and respect to the reference frame (y, z) that rotates with the model, the horizontal and vertical components of body forces are $b_y = -\gamma \sin\theta$ and $b_z = -\gamma \cos\theta$. Equilibrium of the upper part of the model in limit condition yields the result $y_A/z_A = \tan\theta = \lambda$ where y_A and z_A are the centroid coordinates measured from the axis rotation and λ is the kinematically admissible value of the horizontal load multiplier.

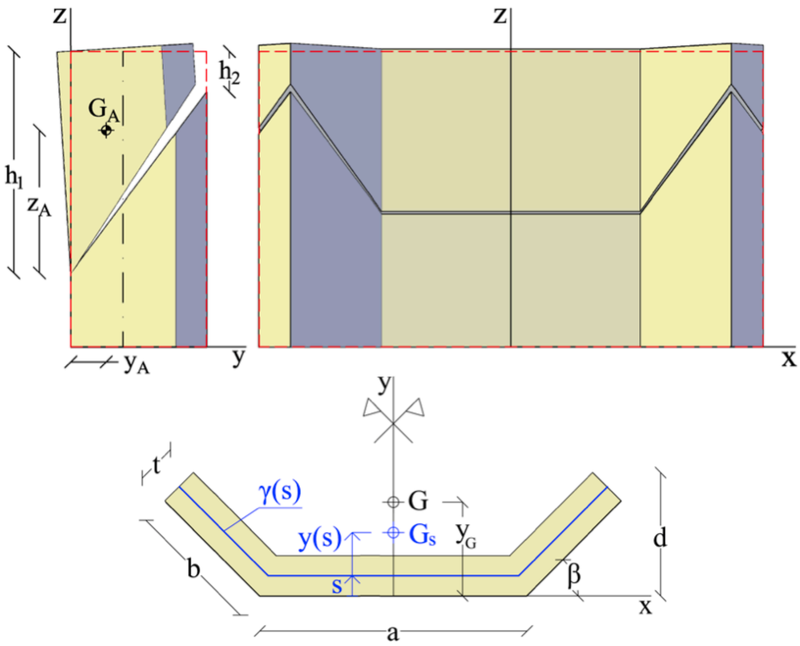

Figure 6: Typical RCM collapse mechanism considered in the limit analysis; geometrical scheme and notations.

Simple but lengthy calculations, here omitted for the sake of brevity, enable obtaining the centroid coordinates (y_A, z_A) of the rotating upper part of the model. The corresponding horizontal load collapse multiplier can be expressed as:

$$\lambda\left(\eta_1,\eta_2\right) = 2\xi\left(rp + cq\right)\frac{D_1\eta_1 + \eta_2}{D_2\eta_1^2 + 2\eta_1\eta_2 - \eta_2^2}\,. \tag{1}$$

Expression (1) makes use of the dimensionless parameters:

$$\xi = \frac{a}{H}, \qquad r = \frac{b}{a}, \qquad c = \frac{t}{a}, \qquad \eta_1 = \frac{h_1}{H}, \qquad \eta_2 = \frac{h_2}{H}$$
$$m = \tan\frac{\beta}{2}, \qquad p = \sin\beta, \qquad q = \cos\beta, \tag{2}$$

together with the two constants:

$$D_1(r,c,\beta) = \frac{(rp+cq)[6r^2p + 6rcq + 3c - 2c^2m(3+q)]}{4r^3p^2 + 6r^2cpq + 4rc^2q^2 + 2c^2 - c^3m(4+q+q^2)} - 1,$$
$$D_2(r,c,\beta) = \frac{6(rp+cq)^2(1+2r-2cm)}{4r^3p^2 + 6r^2cpq + 4rc^2q^2 + 2c^2 - c^3m(4+q+q^2)} - 1. \tag{3}$$

It is possible to demonstrate that (1) is a function that grows monotonically with η_2 and decreases monotonically with η_1. Hence, when $\eta_1 = 1$ and $\eta_2 = 0$, expression (1) takes its minimum and the corresponding value of λ yields the better estimation from above of the load multiplier, according to the upper-bound theorem of limit analysis [8]. Such considerations lead to the expression:

$$\lambda_{RCM} = \lambda(1,0) = 2\xi(rp+cq)\frac{D_1(r,c,\beta)}{D_2(r,c,\beta)} = \xi f(r,c,\beta). \tag{4}$$

3.1 Parametric study of analytical results: influence of β and r on the kinematically admissible collapse values

By means of analytical expression (4), a parametric study on the influence of the main parameters model on the kinematically admissible value of horizontal load multiplier, λ_{RCM}, can be performed. As it can be easily verified, the load multiplier is proportional to the model slenderness, ξ. For a given value of ξ, it could be of some interest to investigate the influence of the inclination of the lateral parts, β, and that of the lateral-to-frontal length ratio, r, as well.

Figure 7 shows the diagram of values $\lambda/\xi = f(r, c_0, \beta)$ as a function of angle β for different values of r. The value of the dimensionless parameter $c_0 = t/a = 0.175$ is chosen accordingly to the dimensions of the models tested. In general, $f(r, c_0, \beta)$ reaches a maximum in correspondence of the inclination $\bar{\beta} = \text{atan}(r/c) < \pi/2$. This angle has a straightforward geometrical interpretation, as it corresponds to the maximum y-direction width of the bay window, for r fixed.

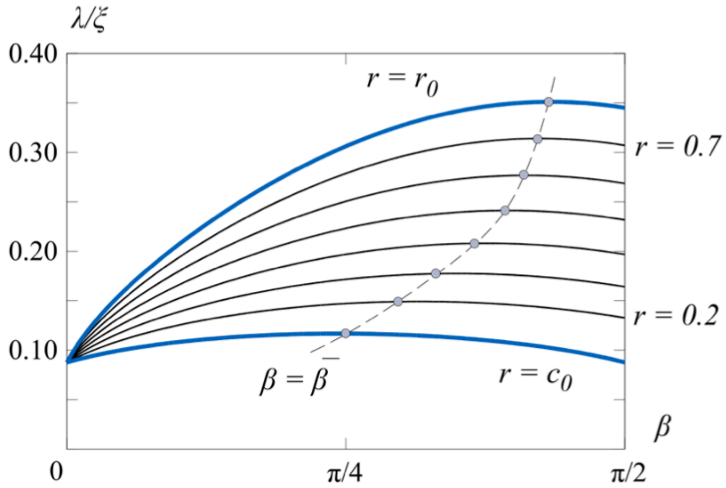

Figure 7: values of function $f\left(r, c_0, \beta\right) = \lambda_{RCM}/\xi$ vs. angle β for different values of r.

In Figure 8 the values $\lambda/\xi = f(r, c_0, \beta)$ are plotted vs. parameter $r = b/a$ for different values of c and for $\beta = \beta_0 = \pi/4$. If the bay window slenderness, ξ, is kept constant, an almost linear dependence is observed upon both parameters $c = t/a$ and $r = b/a$, for $r > 0.3$.

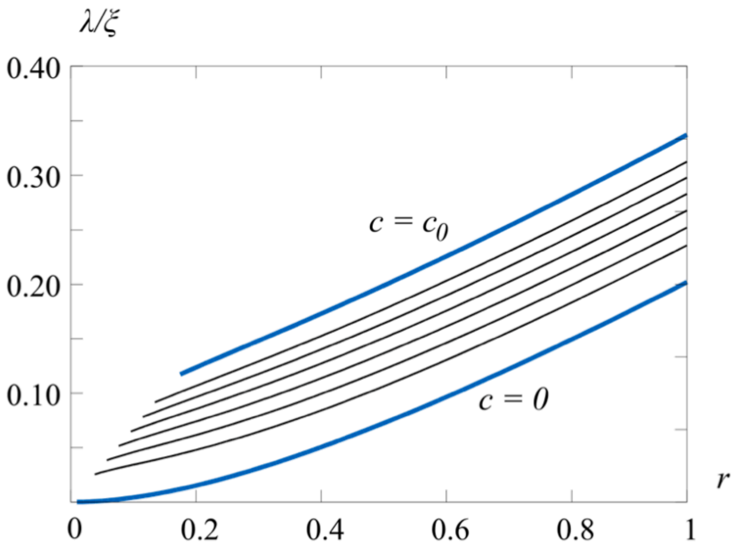

Figure 8: $f\left(r, c, \beta_0\right) = \lambda_{RCM}/\xi$ varying with r for different values of c.

4 COMPARISON BETWEEN EXPERIMENTAL AND THEORETICAL RESULTS

A first comparison between experimental and theoretical results can be made by using expression (4). Such expression enables assessing upper bound estimates of the collapse values of the horizontal load multiplier, λ_{RCM}, for given shape and dimensions of the bay window. The experimental results of the tests performed in this study are compared to theoretical estimates provided by (4) by setting:

$$\lambda_{RCM} = \xi f(r_0, c_0, \beta_0) \simeq 0.306\, \xi, \tag{5}$$

in which $r_0 = 0.896$, $c_0 = 0.175$ and $\beta_0 = \pi/4$.

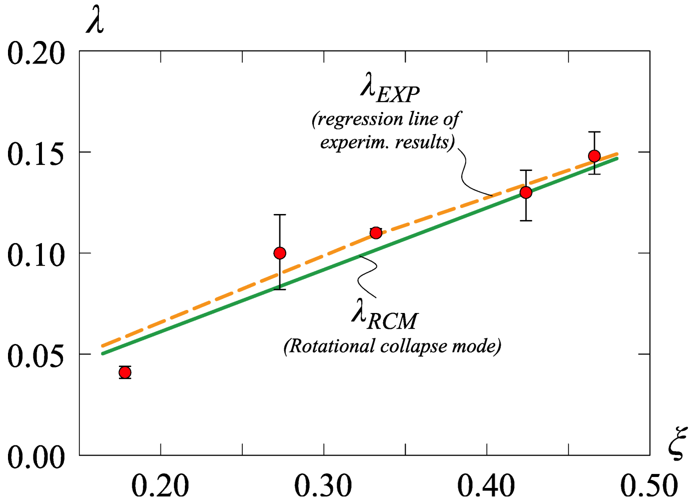

Figure 9: Comparison between experimental, λ_{EXP} (orange dashed line), and theoretical, λ_{RCM} (green continuous line), results.

Figure 9 shows such a comparison in graphical form. First of all, it should be observed that the theoretical values of λ_{RCM} and the regression line of the experimental results are very close to each other. However, the same figure shows that in some cases the theoretical values of λ_{RCM} deduced from the kinematic theorem are smaller than the corresponding experimental values, thus violating the lower-bound theorem of limit analysis. Moreover, the collapse mechanisms assumed in the analysis differ from those observed experimentally (Figure 10). In the analysis, the collapse mechanism is characterised by a separating plane surface that goes from the bottom-left end to the top-right end of the structure, regardless the model slenderness. On the contrary, during the experiments two different collapse modes had been observed, local and global, for large and small model slenderness, respectively. Local collapse mechanisms are very different from the theoretical ones. Global mechanisms somewhat resemble the theoretical ones, although the experimental separating surface usually follows the brick pattern.

Although the first results illustrated here seem promising, further analysis is needed to achieve a better interpretation of the experimental results. In this regard, the discrepancies between theoretical and experimental results could be a direct consequence of the hypotheses adopted to simplify the analysis. Masonry is modelled as a material totally unable to transmit tensile stresses along any surface. Hence, rotation of the upper part of the model at collapse may take place without any internal energy dissipation. On the contrary, collapse by rotation of the actual models can develop free from any dissipation as long as the separating surface inclination is lower than a given limit angle, α_m, which depends on the brick pattern (Figure 11). If the surface inclination is higher than α_m, bricks interlocking and friction would produce some dissipation, thus increasing the corresponding value of the horizontal load multiplier.

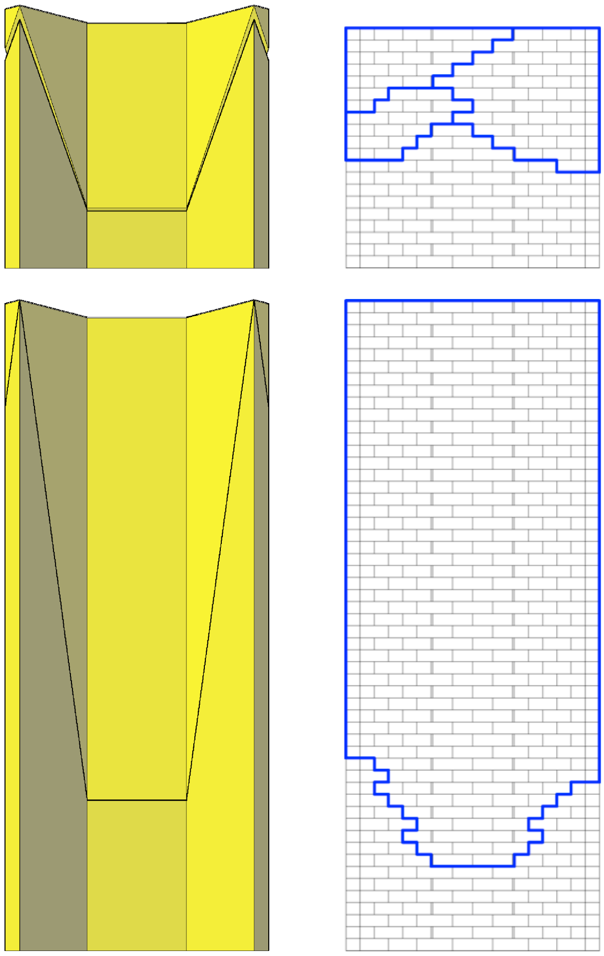

Figure 10: Comparison between theoretical (left) and experimental (right) separation surface.
Top: model I (ξ = 0.466); bottom: model V (ξ = 0.178).

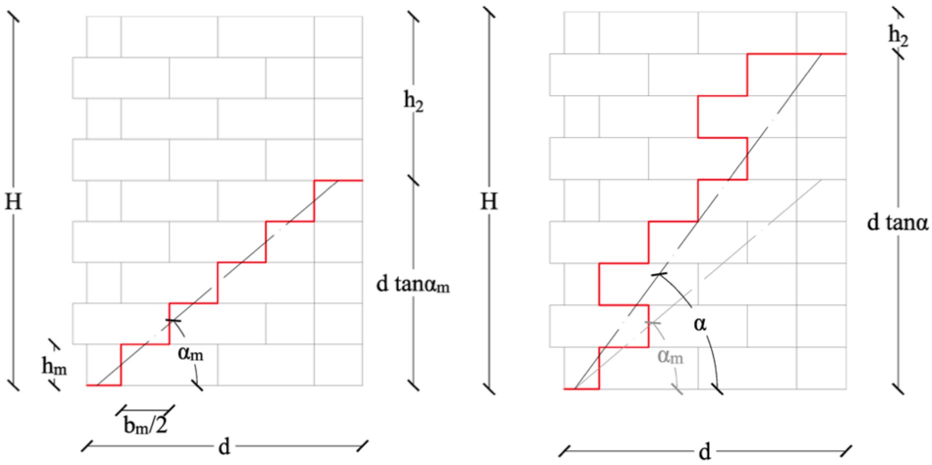

Figure 11: effect of the brick pattern

It can therefore be argued that (4) holds until the surface inclination, α, is lower than α_m, which can be set equal to:

$$\tan \alpha_m = \frac{2h_m}{b_m}, \tag{6}$$

where b_m and h_m are the length and height of a single brick. Under the limitation $0 \leq \alpha \leq \alpha_m$, the minimum kinematic value of the horizontal load multiplier for given shape and dimensions of the bay window can be expressed as:

$$\lambda_{LRCM} = 2\xi(rp+cq)\frac{D_1(r,c,\beta)+1-(rp+cq)\tan\alpha_m}{D_2(r,c,\beta)+1-(rp+cq)^2\tan^2\alpha_m}\,, \tag{7}$$

where r, c, β, p, q are defined by (2). Collapse mechanisms considered in (7) are indicated as "*Limited Rotational Collapse Modes*" (LRCM). By substituting in (7) the parameters values corresponding to the tests, we obtain:

$$\lambda_{LRCM}(r_0, c_0, \beta_0) \simeq 0.462\,\xi. \tag{8}$$

As Figure 12 shows, kinematic load multipliers determined according to (8) are not too far from experimental values, especially for small slenderness. Furthermore, what is more important is that all these values are strictly greater than collapse multipliers, in agreement with the upper-bound theorem. Hence, horizontal load multipliers assessed by this way can legitimately be considered kinematical admissible values. However, further analyses are needed to refine the estimations from above and bring them closer to experimental results.

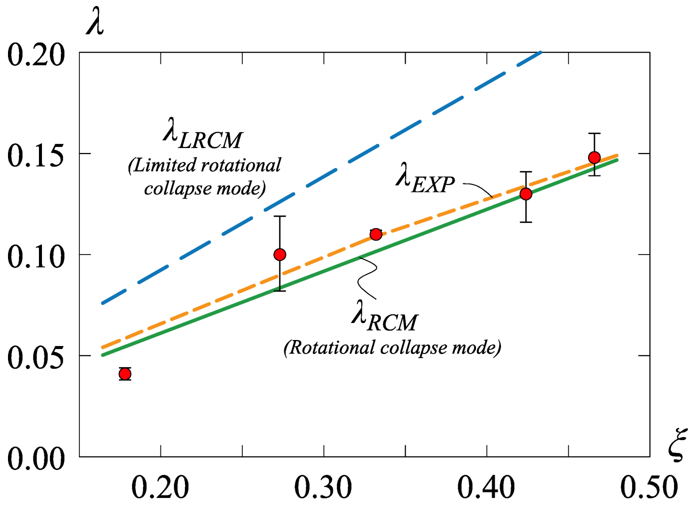

Figure 12: Comparison between experimental and theoretical results

5 CONCLUSIONS

- The collapse of masonry bay windows elements has been addressed, both experimentally and theoretically.
- Tests on scale models provided a first set of experimental results regarding the collapse load multiplier and mode of bay windows. In particular, two main collapse modes, indicated respectively as local and global, had been observed during the tests.
- The experimental results had been interpreted theoretically by using standard tools of limit analysis. By means of the kinematic theorem and by taking into consideration a suitable set of collapse mechanisms it had been possible to obtain an analytical expression of kinematical admissible values of the horizontal load multiplier. Parametric study of this expression enabled a first analysis of the influence on the collapse of the main geometrical parameters that characterise the bay window. As a whole, theoretical values of the load multiplier proved to be in good agreement with the corresponding experimental results.

- Further work is needed to achieve a full understanding of the experimental results. In particular, the definition of a suitable set of collapse mechanisms able to accounts in some way for the actual brick pattern of the masonry element is currently in progress.

ACKNOWLEDGEMENTS
The financial support of MISTI Global Seed Fund for the MIT-UNIPI research project "Mechanical Models for Masonry Walls under Seismic Actions" (2013) is gratefully acknowledged.

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