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AN INTEGRATED PROCEDURE FOR SEISMIC RESPONSE ANALYSIS OF STRUCTURES USING DIFFERENTIAL QUADRATURE METHOD

Hongjing Li, Xijun Kong, and Haojie Xing

College of Civil Engineering, Nanjing Tech University
30 South Puzhu Road, Nanjing 211816, China
e-mail: hjing@njtech.edu.cn

Keywords: Seismic response, Time-stepping procedure, Differential quadrature method, Multi-degree-of-freedom system, Computational effort.

Abstract. *The current linear and nonlinear seismic response analysis approaches are based generally on the numerical approximation step-by-step procedures in practice, and huge computational effort is usually required in order to achieve the desired level of accuracy over the range of time for which the response is needed. However, higher efficiency is the final objective developing the analysis procedures in the same requirements of numerical stability and numerical accuracy. In this paper, a numerical time-stepping procedure is developed for the solution of equations governing the motion of a multi-degree-of-freedom system subjected to earthquake induced ground acceleration on the basis of the differential quadrature (DQ) rule. Initially the history of the seismic ground motion and the structural dynamic responses are divided into a sequence of time intervals in which the DQ analysis procedure will be carried out for evaluation of the seismic response, and the ground acceleration function is constructed over each interval by piecewise linear interpolations. Then each interval is divided into a set of time step once again, and the final responses of velocity and acceleration are expressed approximately as a weighted linear sum of the initial conditions and the response of displacement at the discrete points over the interval by using the DQ rule. The formulae for the quadrature solution of the differential equations of motion are deduced so that the history of the dynamic response of the system could be calculated in the time-stepping procedure using the DQ method. Finally a two-story shear frame structure is employed for numerical illustration, results show that this DQ analysis procedure may achieve a high order of accuracy when the lengths of the time intervals and the discrete points over each interval are chosen properly.*

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1 INTRODUCTION

It is important to evaluate the dynamic behavior of a code-designed structure during a strong earthquake, and the time-stepping procedures may provide a tool for implementation of this purpose. Among the different time-stepping methods, Newmark beta methods are more general and applied frequently in the field of earthquake engineering. It is noted that a practical structure is usually system with a huge number of degree of freedom, and the seismic response must be calculated within a very short time interval because of demands of the numerical stability and accuracy. This means that the time history of the seismic response can only be obtained by computations repeatedly many times according to the different time-stepping approaches established generally on the numerical approximation step-by-step procedures, so huge computational efforts are usually required. However, the higher efficiency is the final objective developing the analysis procedures in the same requirements of numerical stability and numerical accuracy.

Differential quadrature method (DQM) proposed by Bellman and his associates [1, 2] is a numerical solution technique for the initial and/or boundary value problems of ordinary and partial differential equations. The main idea of the DQM is that the values of derivatives at each sampling grid point are expressed approximately as weighted linear sums of the function values at all sampling grid points within the domain under consideration, and it is recognized production of the high accurate solutions with a minimal computational effort [3]. In engineering field, the DQM is usually applied for solving of time-invariant problems such as static analysis or natural vibration analysis [4-17], that means the DQM is used for the approximation of the derivatives to spatial coordinates and the solutions of various engineering boundary value problems have been provided by this method. Earthquake induced ground motion is a kind of time varying excitation for structures, and obviously the problem of seismic response analysis belongs to the initial value problem after the discretization of spatial domain and the boundary conditions of the structure are carried out by certain numerical method such as the finite element method and the differential equations governing the motions of the structure are established. A method solving the initial value problems using the differential quadrature rule has been developed by Fung [18, 19] and Liu et al. [20], in this paper, it is extended to evaluate the dynamic response of the structure subjected to excitation of ground acceleration induced by an earthquake. Because the seismic ground acceleration is a wave process with complicated frequency components and practically expressed as a discrete time function with thousands of sampling grid points during duration of the earthquake, the differential quadrature rule cannot be used directly for the time derivatives approximation to achieve the solution of the governing equations of the structure. Based on reference [21], an integrated procedure that the seismic time history analysis is transformed into a sequence of initial value problems and the differential quadrature rule is applied step by step within a short time interval to avoid distortion of the weighting coefficient matrix is presented, the purpose is to provide a high accuracy numerical approach for structural response analysis due to earthquake excitation.

2 DIFFERENTIAL QUADRATURE RULE

The DQM approximates the values of the derivatives at each sampling grid point for a function as a weighted linear sum of function values at all given sampling grid points. A differential quadrature approximation at the i-th discrete point is given by

$$f_i' = \sum_{j=0}^{n} w_{ij} f_j \quad \text{for } i = 0,\ 1,\ 2,\ \cdots,\ n \tag{1}$$

in which n denotes the number of discrete points considered in the domain; $f_i = f(x_i)$ and $f_i' = f'(x_i)$ are the function value and its derivative at the point $x = x_i$, respectively; w_{ik} is the corresponding weighting coefficient, they are written in matrix form as

$$\boldsymbol{w} = \begin{bmatrix} w_{00} & w_{01} & \cdots & w_{0n} \\ w_{10} & w_{11} & \cdots & w_{1n} \\ \vdots & \vdots & & \vdots \\ w_{n0} & w_{n1} & \cdots & w_{nn} \end{bmatrix} \tag{2}$$

Generally the form of the function $f(x)$ is taken as

$$f(x) = x^k, \ k = 0, 1, 2, \cdots, n \tag{3}$$

then the weighting coefficient matrix can be calculated, i.e.

$$\boldsymbol{w} = \begin{bmatrix} 1 & x_0 & \cdots & x_0^n \\ 1 & x_1 & \cdots & x_1^n \\ \vdots & \vdots & & \vdots \\ 1 & x_n & \cdots & x_n^n \end{bmatrix} \begin{bmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & n \\ & & & 0 \end{bmatrix} \begin{bmatrix} 1 & x_0 & \cdots & x_0^n \\ 1 & x_1 & \cdots & x_1^n \\ \vdots & \vdots & & \vdots \\ 1 & x_n & \cdots & x_n^n \end{bmatrix}^{-1} \tag{4}$$

or

$$\boldsymbol{w} = \boldsymbol{VAV}^{-1} \tag{5}$$

where $x_0, x_1, x_2, \cdots, x_n$ are the co-ordinates of the n+1 discrete points; $\boldsymbol{V}$ represents a Vandermonde matrix which consists of elements of x_j^k and always has an inverse.

For numerical computational problems, it is convenient to choose the discrete points with equal spacing, and this choice is performed practically in most time-stepping procedures. But in DQM, the accuracy and the stability characteristic of the quadrature solutions are dictated by the choice of the locations of the sampling grid points, and basically the non-uniform grids have been considered instead of the equally spaced grids. However, an issue of the sampling grid points for differential quadrature solution to give unconditionally stable higher-order accurate time step integration algorithms is introduced by Fung [18], in which independent variables x may be determined by solving the following equation:

$$x^m - W_1 - W_2 x - W_3 x^2 - \cdots - W_m x^m = 0 \tag{6}$$

where

$$W_k = \frac{(-1)^{m-k} m!\, m!\, (m+k-2)!}{(k-1)!\,(k-1)!\,(m+1-k)!\,(2m)!} \frac{2[m+\mu(k-1)]}{1+\mu} \tag{7}$$

3 SEISMIC RESPONSE ANALYSIS BY DQ RULE

3.1 Dynamic response of the MDOF systems

The differential equations governing the response of a multi- degree-of-freedom system to earthquake-induced ground motion can be expressed as

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{c}\dot{\boldsymbol{u}} + \boldsymbol{k}\boldsymbol{u} = -\boldsymbol{m}\boldsymbol{\iota}\, \ddot{u}_g \tag{8}$$

that satisfies the initial conditions

$$\boldsymbol{u} = \boldsymbol{u}(0), \ \dot{\boldsymbol{u}} = \dot{\boldsymbol{u}}(0) \tag{9}$$

where $\boldsymbol{k}$, $\boldsymbol{c}$ and $\boldsymbol{m}$ are the stiffness, damping and mass matrices, respectively; $\boldsymbol{\iota}$ is the influence vector; $\ddot{u}_g(t)$ is the ground acceleration induced by the earthquake.

The displacement $\boldsymbol{u}$ of an N-DOF system can be expressed as the superposition of the modal contributions, that is

$$\boldsymbol{u}(t) = \sum_{j=1}^{N} \boldsymbol{\Phi}_j q_j(t) \tag{10}$$

in which $\boldsymbol{\Phi}_j$ is j-th natural mode of the system without damping, and q_j corresponding modal co-ordinates.

Using this equation, the coupled equations (8) in $\boldsymbol{u}(t)$ can be transformed to a set of uncoupled equations with $q_j(t)$ as the unknowns if the system has classical damping. Substituting Eq. (10) in Eq. (8) gives

$$\sum_{i=1}^{N} \boldsymbol{m}\boldsymbol{\Phi}_i \ddot{q}_i(t) + \sum_{i=1}^{N} \boldsymbol{c}\boldsymbol{\Phi}_i \dot{q}_i(t) + \sum_{i=1}^{N} \boldsymbol{k}\boldsymbol{\Phi}_i q_i(t) = -\boldsymbol{m}\boldsymbol{\iota}\, \ddot{u}_g \tag{11}$$

Premultiplying each term in this equation by $\boldsymbol{\Phi}_j^T$ and using the orthogonality property of the modes gives

$$\ddot{q}_j + 2\zeta_j \omega_j \dot{q}_j + \omega_j^2 q_j = -\eta_j \ddot{u}_g \text{ for } j = 1, 2, \cdots, N \tag{12}$$

where ω_j and ζ_j are the natural frequency and damping ratio of the j-th mode of the MDOF system, respectively. η_j is the modal participation factor, calculated by

$$\eta_j = \frac{\boldsymbol{\Phi}_j^T \boldsymbol{m}\boldsymbol{\iota}}{\boldsymbol{\Phi}_j^T \boldsymbol{m}\boldsymbol{\Phi}_j} \tag{13}$$

It is noted that the N uncoupled equations in (12) are independent each other, therefore they can be solved individually by any analytical or numerical methods. Once the modal coordinates $q_j(t)$ have been determined, the solutions of the system, i.e. $\boldsymbol{u}$, $\dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$, are also obtained.

3.2 Seismic response analysis within a time step by DQ rule

For the dynamic equation excited by the earthquake, it is practical to determine the solution by numerical methods. Considering the seismic response within a time interval of $[t_i,\ t_j]$, in which t_i and t_j are two different discrete time instants at time axis. For differential quadrature analysis of seismic response, the time interval of $[t_i,\ t_j]$ is defined as a time step in which the responses will be determined. Obviously the length of the step, Δt_{ij}, can be obtained as

$$\Delta t_{ij} = t_j - t_i \tag{14}$$

It is unnecessary to take Δt_{ij} be constant, in fact, the length of each step is usually not equal in differential quadrature analysis because of the property of higher accuracy of this method, so that benefits of less computational efforts can be obtained.

Establishing a local coordinate system shown in Fig.1 over the step, then Eq. (12) becomes a set of the ordinary differential equations to coordinate τ , it is rewritten by omitting the subscript j as

$$\ddot{q}(\tau) + 2\zeta\omega\dot{q}(\tau) + \omega^2 q(\tau) = -\eta\ddot{u}_g(\tau) \tag{15}$$

Introducing a state vector consisting of $q(\tau)$ and $\dot{q}(\tau)$ as follows:

$$\boldsymbol{y} = \begin{Bmatrix} q \\ \dot{q} \end{Bmatrix} \tag{16}$$

then Eq. (15) may be formed in the state space, that is

$$\boldsymbol{M}\dot{\boldsymbol{y}} + \boldsymbol{K}\boldsymbol{y} = -\boldsymbol{M}\boldsymbol{I}\,\ddot{u}_g \tag{17}$$

with the initial condition

$$\boldsymbol{y} = \boldsymbol{y}(0) = \begin{Bmatrix} q_0 \\ \dot{q}_0 \end{Bmatrix} \tag{18}$$

where

$$\boldsymbol{M} = \begin{bmatrix} -\omega^2 & 0 \\ 0 & 1 \end{bmatrix},\ \boldsymbol{K} = \begin{bmatrix} 0 & \omega^2 \\ \omega^2 & 2\zeta\omega \end{bmatrix},\ \boldsymbol{I} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \tag{19}$$

It is noted that both $\boldsymbol{M}$ and $\boldsymbol{K}$ are symmetrical in Eq. (17) and an identical equation has been introduced. Compared with Eq. (15), the order of Eq. (17) reduced to one order but number of equations increased twice.

The DQ rule proposed in Eq. (1) will be used to achieve the solutions of Eq. (17). In order to apply the DQ rule, the time step under consideration is divided into n parts including separate nodes with coordinates of τ_k $(k = 0, 1, 2, \cdots, n)$. Let the initial point τ_0 and the final point τ_n be at the two terminals of the time step, respectively, see Fig. 1. These mini-intervals within the time step are generally not equal in length owing to demands of the numerical accuracy and stability.

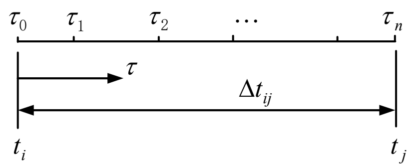


Figure 1: Local coordinate system.

Introducing the following notations:

$$\boldsymbol{y}_k = \boldsymbol{y}(\tau_k),\ \dot{\boldsymbol{y}}_k = \dot{\boldsymbol{y}}(\tau_k),\ \ddot{\boldsymbol{y}}_k = \ddot{\boldsymbol{y}}(\tau_k) \tag{20}$$

therefore

$$\boldsymbol{y}_0 = \boldsymbol{y}(\tau_0) = \begin{Bmatrix} q(t_i) \\ \dot{q}(t_i) \end{Bmatrix},\ \boldsymbol{y}_n = \boldsymbol{y}(\tau_n) = \begin{Bmatrix} q(t_j) \\ \dot{q}(t_j) \end{Bmatrix} \tag{21}$$

It is obvious that $\boldsymbol{y}(\tau)$ is a function about τ, hence the DQ rule formulated in the expression (1) may be used to give approximately the derivatives at each discrete grid points by the weighted sums of the function values at all grid points over the whole time step, i.e.

$$\dot{\boldsymbol{y}}_k = \sum_{r=0}^{n} w_{kr}\boldsymbol{y}_r \text{ for } k = 0, 1, 2, \cdots, n \tag{22}$$

It is noted that total n equations are included in Eqs. (22), they can be also expressed in form of matrix, this leads to

$$\dot{\boldsymbol{Y}} = \boldsymbol{Y}\boldsymbol{w}^T \tag{23}$$

where

$$\boldsymbol{Y} = [\boldsymbol{y}_0, \boldsymbol{y}_1, \cdots, \boldsymbol{y}_n] \tag{24}$$

Substituting Eq. (23) in Eq. (17) gives

$$\boldsymbol{AX} + \boldsymbol{XB} = \boldsymbol{C} \tag{25}$$

in which $\boldsymbol{X}$ is a matrix assembled compactly by a set of unknown vectors, $\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n$, which would be solved for the solution of seismic response. The form of matrix $\boldsymbol{X}$ is

$$\boldsymbol{X} = [\boldsymbol{y}_1, \cdots, \boldsymbol{y}_n] \tag{26}$$

$\boldsymbol{A}$ and $\boldsymbol{B}$ are the coefficient matrice, $\boldsymbol{C}$ is also a matrix corresponding to discrete values of earthquake-induced ground acceleration at different time instants. They can be calculated by

$$\boldsymbol{A} = \boldsymbol{M}^{-1}\boldsymbol{K} = \begin{bmatrix} 0 & -1 \\ \omega^2 & 2\zeta\omega \end{bmatrix}, \tag{27a}$$

$$\boldsymbol{B}^T = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nn} \end{bmatrix} = \begin{bmatrix} 1 & \tau_1 & \cdots & \tau_1^{n-1} \\ 1 & \tau_2 & \cdots & \tau_2^{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \tau_n & \cdots & \tau_n^{n-1} \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 2 & & \\ & & \ddots & \\ & & & n \end{bmatrix} \begin{bmatrix} \tau_1 & \tau_1^2 & \cdots & \tau_1^n \\ \tau_2 & \tau_2^2 & \cdots & \tau_2^n \\ \vdots & \vdots & \ddots & \vdots \\ \tau_n & \tau_n^2 & \cdots & \tau_n^n \end{bmatrix}^{-1} \tag{27b}$$

$$\boldsymbol{C} = -\boldsymbol{I}\ddot{\boldsymbol{u}}_{\boldsymbol{g}} - \boldsymbol{y}_0\boldsymbol{w}_0 \tag{27c}$$

$$\ddot{\boldsymbol{u}}_{\boldsymbol{g}} = \{\ddot{u}_{g1}, \ddot{u}_{g2}, \cdots, \ddot{u}_{gn}\}, \ \boldsymbol{w}_0 = \{w_{10}, w_{20}, \cdots, w_{n0}\}$$

It may be observed that the coefficient matrice of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are definite in the Eq. (25), and the modal coordinates q_k and $\dot{q}_k$ will be determined by solving the matrix equation (25), then the seismic response vector $\boldsymbol{y}$ is also obtained in discrete form

4 EXPRESSION OF GROUND ACCELERATION

The earthquake induced ground acceleration is a complicated time history process with abundant frequency components, and it can be only defined by numerical value at discrete time instants. If the accelerogram of the strong ground shaking within certain duration T has been instrumented during an earthquake, the discrete values of the ground acceleration will be known as

$$\ddot{u}_{gl} = \ddot{u}_g(t_l), \ t_l \in [0, T], \ l = 1, \cdots, m \tag{28}$$

where T is duration of the strong ground motion, m is the number of sampling grid points within the domain interval under consideration, usually the accelerogram is recorded with equal spacing, that means all the length of each step within the time interval of $[0, T]$, i.e. $\Delta t = t_{l+1} - t_l$, is equal.

But in DQM, the unequally spaced grids has generally been chosen to enhance the accuracy of the quadrature solutions, this will lead to the result that the discrete points used in the DQ rule must not be coincident with the sampling grid points at the recorded accelerogram, so the values of the ground acceleration at some discrete points will be unknown probably when the DQM is employed to evaluate the seismic response of the system according to Eq. (15). Obviously, the sampling values at each discrete grid point within the given time step may be utilized to construct the time function of the ground acceleration by any interpolation approach. In fact, it is natural and convenient to form the acceleration function based on the piecewise linear interpolations. Over each time step $t_i \leq t \leq t_j$, the acceleration function is given by

$$\ddot{u}_g(\tau) = \ddot{u}_{gi} + \sum_{l=1}^{k} \alpha_l \Delta t + \alpha_{k+1}(\tau - k\Delta t) \tag{29}$$

in which α_k is the constant slope of the assumed linearly varying acceleration over the k-th mini-interval during the time step under consideration, it can be calculated according to

$$\alpha_k = \frac{\ddot{u}_{g,k+1} - \ddot{u}_{gk}}{\Delta t} \tag{30}$$

Thus, the earthquake induced ground acceleration $\ddot{u}_g(t)$ is expressed by a time function constituted from a sequence of piecewise linear functions. This treatment process is shown as Figure 2.

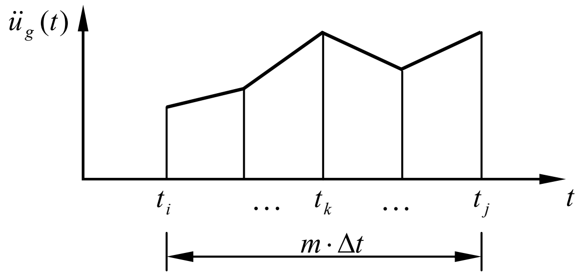


Figure 2: Function of earthquake induced ground acceleration.

5 THE DQ PROCEDURE FOR SEISMIC RESPONSE ANALYSIS

It is assumed that there totally are *m* sampling grid points spaced equally for time history of the ground acceleration within certain time step of Δt_{ij}, obviously we obtain

$$\Delta t_{ij} = m \cdot \Delta t \tag{31}$$

Over the time step ΔT_i, the non-uniform discrete points serving the DQ rule will probably not coincide with the original sampling points spaced equally for the recorded accelerogram. After the function expression of the ground acceleration is formed according to formula (29), the value of the ground acceleration at any time instant will be determined.

The DQ analysis procedure for seismic response may be established as follows:

(1) Determining the duration defining the time domain for seismic response analysis;

(2) Constucting the function expression of the earthquake induced ground acceleration within the duration domain by the piecewise linear interpolation method;

(3) Dividing the duration domain into a sequence of time steps, it is unnecessary that the beginning and terminal points of the interval coincide with the original sampling points for the record of strong ground motion;

(4) Over each time step, a sequence of grid points is chosen for dissection of the interval by using any proper discrete scheme, for example, the choice of the Chebyshev-Gauss-Lobatto points;

(5) Evaluating the seismic response over each time step according to the DQ analysis method mentioned above, so the time history of seismic response may be obtained by combining computational results during all the steps.

This procedure is shown briefly in Figure 3.

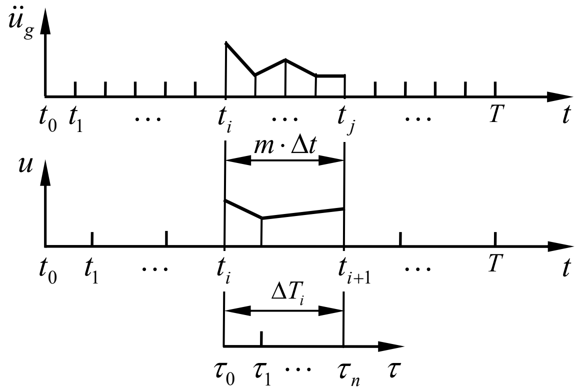


Figure 3: Illustration of the DQ analysis procedure.

It is noticed that re-divided time steps over the duration domain (step 3) are significant in the DQ analysis procedure. It is unnecessary to divide the duration domain in equal length for each step, but the dissection should be implemented by considerations of both characteristics of the seismic ground motion and dynamic properties of the system.

6 NUMERICAL EXAMPLE

In this section, a two-story shear frame shown in Fig. 4 is employed for numerical illustration of the DQ analysis procedure. The dynamic properties of the two-DOF system are listed in Table 1, in which the damping properties are kind of Rayleigh damping with 5 percent modal damping ratio.

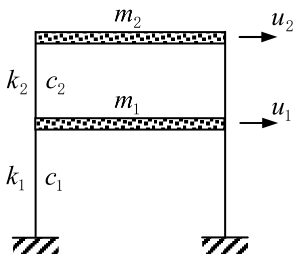


Figure 4: Case structure.

Stiffness properties [kN·m^{-1}]		Mass properties [kN·s^2·m^{-1}]		Damping properties [kN·s·m^{-1}]		Natural periods [s]	
k_1	k_2	m_1	m_2	c_1	c_2	T_1	T_2
9.0×10^4	6.75×10^4	4.0×10^2	3.0×10^2	6.0×10^2	4.5×10^2	0.6378	0.2751

Table 1: Characteristics of the case structure.

The accelerogram recorded at #9 El Centro Array during the Imperial Valley earthquake, America of 19th May, 1940 is selected for excitation of the system. The time history of the ground acceleration and corresponding earthquake response spectrum are shown in Fig. 5 and Fig. 6.

In order to verify effect of the DQ procedure for seismic response analysis, the dynamic responses of the system due to given earthquake excitation are computed first by modal superposition method. All terms of modal contributions are calculated via Duhamel's integral and the results will be the standard for comparison, and called it 'exact' solution.

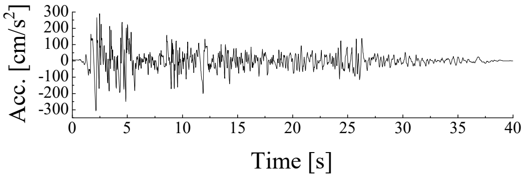


Figure 5: Time history of the ground acceleration.

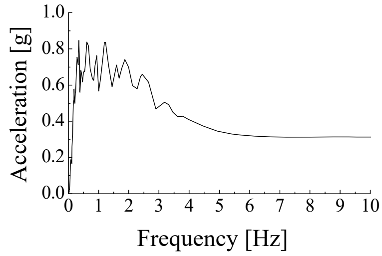


Figure 6: Response spectrum of the ground acceleration

The seismic responses of the system are calculated by the time-stepping analysis procedure using DQ rule, the calculated results are shown in Fig. 7 in which exact solution is expressed by the actual line and the DQ results the discrete circles. It is noticed that the sampling periods for the ground acceleration is taken in 0.01 second, and the intervals with same length are also chosen for quadrature integration computation. Only the responses within first 25 seconds are drawn because the main dynamic behaviors of the system have been included. It has been founded that the calculated dynamic responses by DQ analysis procedure are very close to the exact solution when the length of the time interval is selected short enough.

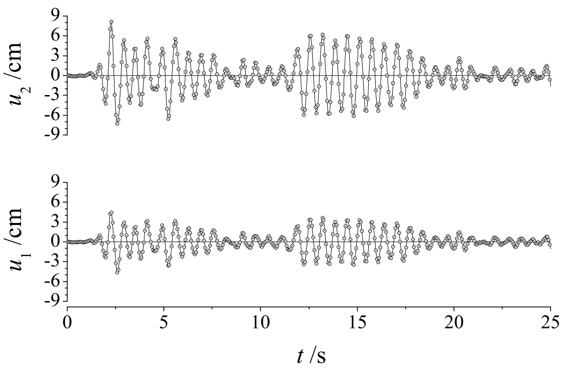
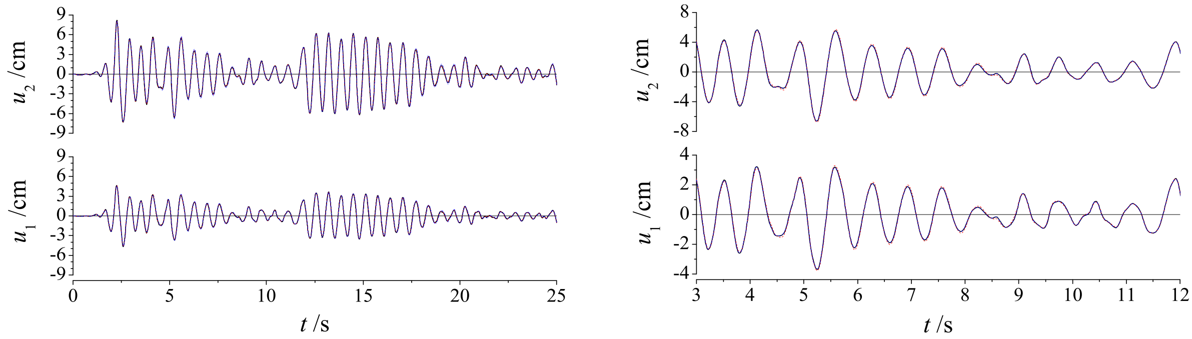


Figure 7: Calculated results of seismic responses during 0-25 seconds.

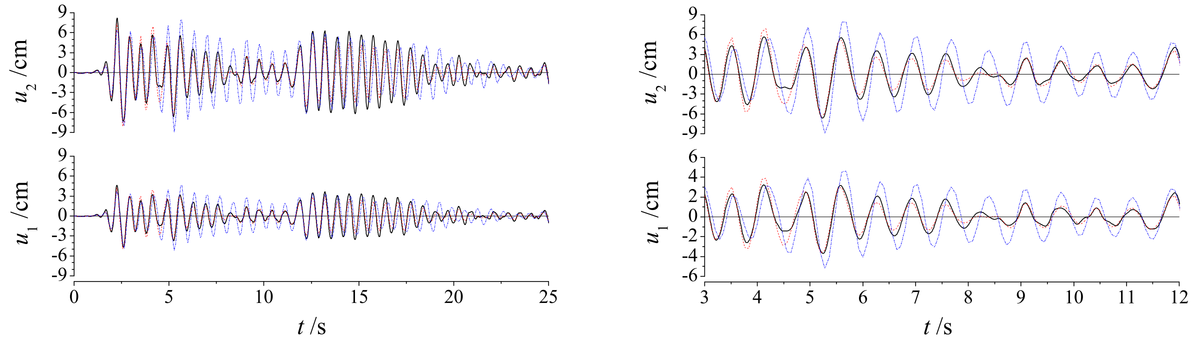
If the length of the time step is enlarged, for example, from 0.01 second to 0.02, 0.04 and 0.08 second, and information according to original sampling period is omitted, the computational efforts will be reduced. Fig. 8 and Fig. 9 show the calculated results of dynamic response using DQ analysis procedure based on the time step with the length of 0.02 second and 0.08 second, respectively. Furthermore, Newmark beta method generally applied for linear and nonlinear seismic response analysis is employed under the same condition with DQ analysis procedure to evaluate the dynamic responses of this system for comparison. The actual line is still the exact solution calculated in the sampling period of 0.01 second, and the results by DQ procedure and Newmark beta method are expressed in the line of dashes and the dotted line, respectively.

It can be founded from Fig. 8 and Fig. 9 that difference between the calculated results from DQ analysis and Newmark method, respectively is very small if the time steps are taken short enough (the length is 0.02 second shown in Fig. 8), all the two approaches can achieve good accuracy. But in the condition of large interval, the calculated results by DQ analysis procedure are obviously better than those by Newmark method.



(a) (b)

Figure 8: Comparison of calculated results using DQM and Newmark-β method while Δt = 0.02s, where (b) is the local zoom of (a).



(a) (b)

Figure 9: Comparison of calculated results using DQM and Newmark-β method while Δt = 0.08s, where (b) is the local zoom of (a).

7 CONCLUSIONS

In this paper, the differential quadrature rule is applied to discretization of time domain and an integrated time-stepping procedure is presented for dynamic response analysis of system subjected to earthquake induced ground motion. The time history of the ground acceleration expressed by a set of discrete values is constructed as a function to time using piecewise linear interpolations, so that all the values of the ground acceleration at any time instants may be obtained. This leads to DQ rule could be used over a time interval with not long length and finite discrete points selected unnecessarily the original sampling grid points for the ground acceleration. Because the time coordinate is one dimensional system, difficulties existed in the settlement of boundary conditions are naturally overcome, and accuracy for computation will be increased through proper choice of positions of the discrete points within each interval. Results from numerical analysis show that the proposed quadrature procedure can still achieve an excellent analysis for the time history of the seismic response even when the length of the time interval is selected a little larger. So the computational efficiency would be improved by using the proposed quadrature procedure for seismic response analysis in time domain, because a relative larger length of the discrete interval may be selected in practice so that the actual computer efforts will decrease greatly.

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