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DYNAMICAL PROCESSES ANALYSIS IN THE LOAD BEAMS AFTER PARTIAL DESTRUCTION

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Abstract. *The study of dynamic processes in loaded constructions at sudden changes in their structure and (or) design scheme, due to various reasons, is an urgent task as part of the solution to the problem to ensure reliable and safe operation of facilities. This paper considers the stress-strain state at the ends of a double hinged beam with rectangular cross section in a state of pure bending. It is assumed that at some point the beam is separated from a layer of a certain thickness, which induces a sudden change in the area, moments of inertia and strength of the cross-section. Moreover, the instantaneous partial destruction, changing the calculation scheme of the beam is simulated. Before the formation of the structural damage, the reaction of the construction is defined by a static exposure. The sudden formation of a defect leads to lower overall stiffness of the structure, which no longer provides a static equilibrium of the system. Inertial and dissipative forces, which have suddenly emerged, cause a dynamic response. The beam is set in motion, in which the maximum stresses exceed the stresses developing in a quasi-static layer separation. Mathematically, the problem reduces to the integration of the inhomogeneous differential equation of the 4th order with inhomogeneous boundary conditions for the given initial conditions. The novelty of the approach lies in the fact that the solution is based on the decomposition of the movement on the modes of the natural oscillation of a damaged beam, but the initial conditions are formulated for an undamaged beam. The effect of the layer sudden separation is characterized quantitatively by coefficients such as a ratio of maximum dynamic stress to the maximum static stress and to the stress, developing at the quasi-static transition of a solid beam to a partially damaged one. These coefficients and the time to reach maximum values of stress are determined depending on the degree of damage to a beam (thickness of a separated layer). These quantitative results demonstrate a significant excess (8-fold while reducing by half the height of the cross section) of stresses in a statically loaded beam with a sudden change in the cross-sectional area by reducing its height. The study of the parameters of additional stresses and time of their occurrence can be useful when assessing the vitality of a structure and its elements and time of evacuation in an emergency.*

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1 INTRODUCTION

One of the most important tasks of modern structural mechanics is the problem of sensitivity analysis of developed systems and constructions to changes in their projects, imperfections in manufacture, variations of external influences, structural changes under load, degradation of support devices and other factors of different nature and origin.

Information about the sensitivity is very important and allows solving effectively complex problems of optimization and support of the survivability and reliability of a construction and its elements. The sensitivity analysis in designing the structure means establishing the nature of the relationship between design variables, which are available to an engineer, and state variables, which are determined by the laws of mechanics, i.e. the construction response. The dependence of the construction response measured by such variables as displacement, deformation, tension, etc., from the design variables, such as physical and mechanical properties of materials, dimensions, geometric characteristics and forms of constructions, supporting communication, etc., is implicitly defined by the equations of equilibrium (motion) of the construction mechanics. Sudden failures of constructive elements can lead to changes of the calculating scheme of the construction and create the risk of a progressive (avalanche) collapse [1, 2]. Before the formation of a defect the construction response is determined by the static effects. The sudden formation of a defect leads to a decrease in overall stiffness of the construction, which no longer provides a static equilibrium of the system.

The normative documents for the design of load-bearing structures practically have no recommendations for the calculation of buildings and structures on the survivability in case of a sudden impact, which is not foreseen by the project [3]. Therefore, in the framework of solving the problems of ensuring reliable and safe operation of buildings, the study of dynamic processes in structurally nonlinear systems, which suddenly change, due to various reasons, its structure and the calculating scheme under the load, is an urgent problem. Obtaining such information for real structures requires improvement of known methods and development of new ones for their calculation.

The analysis of modern engineering literature shows that the formulations and methods of solving the problems of durability and survivability, which would take into account sudden changes in a constructive and calculating schemes of the elements of the construction, and operational structural damage, are a few and imperfect. Today, there are a number of works devoted to studies of the dynamic processes initiated in the loaded beams and plates by the sudden formation of various structural changes and damage. Here is a brief overview of some of them.

In papers [4-6] the problem of stress-deformation state of rods in a sudden shutdown of the supporting connections was solved; the possible mechanism of the avalanche development of damage leading to the loss of bearing capacity or destruction was shown. In result of the sequence of transformations of the boundary conditions the location and magnitude of maximum stresses and deformations are changing. The effect of variable stiffness and cross-sectional shape on a stress-strain state of a beam in the course of a dynamic process caused by a sudden change of resistance conditions was studied in papers [7, 8]. Sphenoid and cone-shaped beams with rectangular, circular and triangular cross section were considered. Vibrations of a single-span beam with clamped and hinged ends loaded by a distributed load at the sudden removal of a hinged support were being studied [9, 10]. The problem was solved taking into account the energy dissipation in the vibrations. The decrease in the level of maximum stresses and strains taking into account the viscosity of the material of the beam was shown. Papers [11-13] analyze the dynamic processes occurring in circular and annular plates of a variable along the radius thickness with a central rigid inclusion at a sudden conversion of pinching the inner contour into the free support. The research used the analytical method of integration of differential equations in Cartesian and polar coordinates with arbitrary variable coefficients [14-16]. Papers [17-19] are dedicated to the solution to the task of defining dynamic additional load in the elements of space frame-rod systems, modeling spatial frames of multi-storey buildings and their fragments at a sudden change of their structure. The method of analyzing the dynamic phenomena in constructions subjected to sudden structural rearrangements covers the study of the processes in the loaded elastic beams in case of sudden formation of transverse cracks [20, 21] and longitudinal stratifications [22-24]. The physical model of a beam with a crack experiencing bending vibrations is a construction consisting of two beam segments connected by a torsion spring located in the section with a crack [25]. Longitudinal stratification is interpreted as a sudden distraction of shear connections between layers while maintaining cross connections. Instead of a monolithic rod (plate), a layered package of rods (plates) is formed, and it allows mutual sliding of layers. In this case, the bending rigidity of an object is decreasing and vibration amplitudes are increasing. Paper [26] investigates the redistribution of stresses in the stretched-reinforced rods in a sudden breakage of the reinforcement or the destruction of the matrix. The mechanism of vibration stimulation in case of an accident is due to the fact that at the destruction of one of the components of the composite, the tensile strength, which is statically attached thereto, goes to the preserved material by impact (blow). Mathematical models for the transverse vibrations of a beam on an elastic base, which are initiated by full or partial destruction of the base [27-28]. Spectrum of natural frequencies of bending vibrations of a beam on a partially destructed base is determined by the frequency equation, obtained by the procedure similar to that used in the finite element method in constructing the stiffness matrix of the finite element. Four-vectors of the section condition are introduced: the deflection, the angle of the rotation of the cross section, the bending moment and the cross section force. In a matrix form, dependences of state vectors in an arbitrary section from a vector of initial parameters are obtained. State vectors are a set of blocks, including kinematic and power parameters. Due to this, the matrix of the initial cross-section effect on an arbitrary one becomes a cell matrix, always of size 4x4. The use of state vectors and initial parameters reduces the order of matrix effects up to the fourth at any number of the conjugating parts of the beam, which significantly reduces the calculations complexity. All these papers, with the participation of the authors of this article use the same method for solving homogeneous and nonhomogeneous differential equations describing dynamic processes in loaded elastic bodies, when developing pulse changes in loads, support conditions, dimensions and shapes of the cross sections, the partial destruction of a beam and (or) ground at a sudden cracking and other structural rearrangements of the construction. The solutions are constructed using the Fourier method of separation of variables, decomposition of dynamic movements, the initial static deformations and loads in the modes of natural vibrations of damaged constructions with the use of Duhamel's integral.

The problem of estimating the tension-strain state of a loaded elastic beam is being considered below, and its calculating scheme changes suddenly due to the reduction in cross sectional area. This change is accompanied by a dynamic process, during which deformation and tension greatly exceeds the static performance and in general can lead to loss of bearing capacity and destruction.

2 QUASI-STATIC SEPARATION OF LAYER

Let us consider the tension-deformation state of a double hinged beam with rectangular cross section which is loaded at the ends by concentrated moments μ. The calculating scheme and the size of the structure are shown in figure 1.

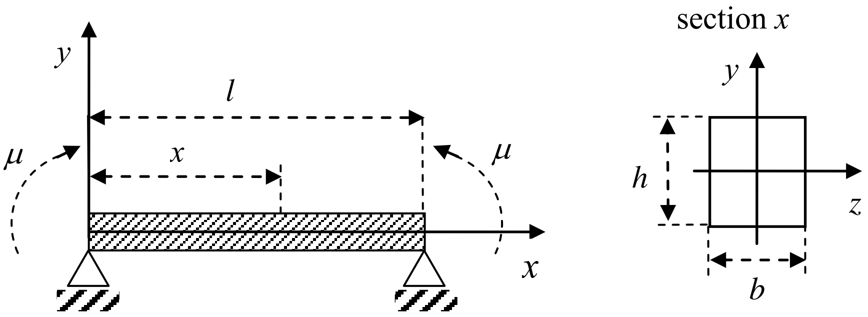

Figure 1: The beam calculating scheme.

The beam is in pure bending, the bending moment in an arbitrary cross-section x is equal to μ, and the maximum pressure

$$\sigma_{st_{\max}} = \frac{\mu}{W_z},$$

where the resistance moment of the cross section is $W_z = \dfrac{2J_z}{h}$, the axial moment of the section inertia is $J_z = \dfrac{bh^3}{12}$.

The deflection of the beam w in the result of the static effect of moments μ are determined by the solution of the differential equation

$$\frac{d^2\overline{w}_{st}}{d\xi^2} = \overline{\mu}, \tag{2.1}$$

where $\xi = \dfrac{x}{l}$, $\overline{w}_{st} = \dfrac{w_{st}}{l}$, $\overline{\mu} = \dfrac{\mu\, l}{E\, J_t}$ are dimensionless variables and parameters.

The solution of equation (2.1) with boundary conditions

$$\overline{w}_{st}(0) = 0, \quad \overline{w}_{st}(1) = 0 \tag{2.2}$$

has the form

$$w_{st} = -\frac{\overline{\mu}}{2}\xi(1-\xi), \ 0 \le \xi \le 1. \tag{2.3}$$

It is further assumed that at some point the layer with thickness αh $(0 \le \alpha \le 1)$ suddenly separates from the beam, and it means a sudden change in the height of the cross section to the value of h_1 (figure 2).

$$h_1 = (1-\alpha)h. \tag{2.4}$$

The moments of the section inertia and resistance are respectively

$$J_{z_1} = J_z(1-\alpha)^3;$$
$$W_{z_1} = W_z(1-\alpha)^2.$$

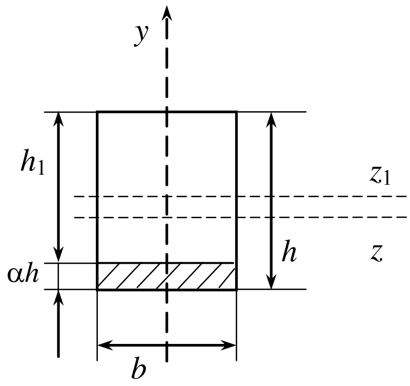

Figure 2: The cross section of the damaged beam.

The maximum tension $\sigma_{st_{\max 1}}$ at the quasi-static separation of the layer αh is the following

$$\sigma_{st_{\max 1}} = \frac{\sigma_{st_{\max}}}{(1-\alpha)^2}$$

or, in the dimensionless form

$$\bar{\sigma}_{st_{\max 1}} = \frac{\sigma_{st_{\max 1}}}{\sigma_{st_{\max}}} = \frac{1}{(1-\alpha)^2}.$$

The dependence $\bar{\sigma}_{st_{\max 1}}$ on parameter α is shown in figure 3.

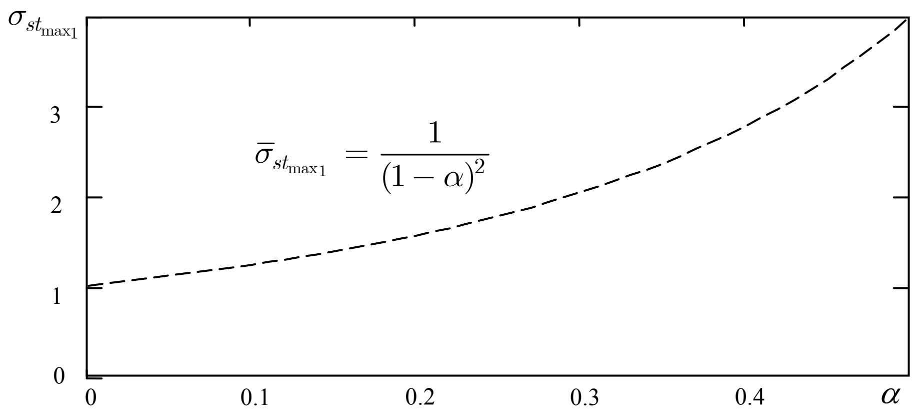

Figure 3: The influence of the thickness of the quasi-statically separated layer at the maximum tension.

3 FORMULATION OF PROBLEM OF THE SUDDEN SEPARATION OF LAYER

In case of the sudden separation of layer αh the transition dynamic process occurs in which the maximum stress at some point exceeds the stress developing under the layer quasi-static separation. The following is the calculation of the parameters of this process in the ideal case, i.e. without attenuation of resulting elastic vibrations.

Bending elastic vibrations emerging after the separation of the layer αh are described by the differential equation

$$EJ_{z_1} \frac{\partial^4 w}{\partial x^4} + \rho A_1 \frac{\partial^2 w}{\partial t^2} = 0, \tag{3.1}$$

where $w = w(x,t)$ are the deflections of the beam vibrations;

t – time;

ρ, E – are respectively, the density and the modulus of the material elasticity;

$A_1 = bh_1 = bh(1-\alpha)$ is the beam cross-sectional area after the separation of layer αh.

The unknown function $w = w(x,t)$ must satisfy the limit states:

1) boundary

$$w(0,t) = 0, \qquad EJ_{z_1} w''(0,t) = \mu;$$
$$w(l,t) = 0, \qquad EJ_{z_1} w''(l,t) = \mu. \tag{3.2}$$

2) initial

$$w(x,0) = w_{st}(x), \qquad \left.\frac{\partial w}{\partial t}\right|_{x,0} = 0. \tag{3.3}$$

Conditions (3.3) imply that the motion of a beam in case of the sudden separation of layer αh starts from the condition (2.3) created by the static deformation (kinematic stimulation of vibrations).

Thus, the mathematical problem is reduced to the integration of the homogeneous equation (3.1) with inhomogeneous boundary conditions (3.2) at initial conditions (3.3).

Let's write down problem (3.1)-(3.3) in the dimensionless form, in addition, introducing to (2.1) the following variables and parameters

$$\tau = \frac{t(1-\alpha)}{l^2}\sqrt{\frac{EJ_z}{\rho A}}, \qquad \bar{w} = \frac{w}{l}, \qquad \bar{\mu}_1 = \frac{\mu l}{EJ_z(1-\alpha)^3}.$$

Then the equation (3.1) takes a form

$$\frac{\partial^4 \bar{w}}{\partial \xi^4} + \frac{\partial^2 \bar{w}}{\partial \tau^2} = 0, \tag{3.4}$$

boundary conditions

$$\bar{w}(0,\tau) = 0, \qquad \left.\frac{\partial^2 \bar{w}}{\partial \xi^2}\right|_{0,\tau} = \bar{\mu}_1;$$
$$\bar{w}(1,\tau) = 0, \qquad \left.\frac{\partial^2 \bar{w}}{\partial \xi^2}\right|_{1,\tau} = \bar{\mu}_1. \tag{3.5}$$

initial conditions

$$\bar{w}(\xi,0) = -\frac{\bar{\mu}}{2}\xi(1-\xi), \qquad \left.\frac{\partial \bar{w}}{\partial \tau}\right|_{\xi,0} = 0. \tag{3.6}$$

4 METHOD FOR SOLVING THE DYNAMIC PROBLEM

We seek a solution of equation (3.4) in the form

$$\bar{w} = \bar{w}_0(\xi,\tau) + f(\xi). \tag{4.1}$$

Substituting the submission (4.1) in equation (3.4), we obtain an inhomogeneous differential equation for the function $\bar{w}_0(\xi,\tau)$

$$\frac{\partial^4 \bar{w}_0}{\partial \xi^4}+\frac{\partial^2 \bar{w}_0}{\partial \tau^2}=-\frac{d^4 f}{d\xi^4}. \tag{4.2}$$

The function $f(\xi)$ is chosen so that the boundary conditions for equation (4.2) would become homogeneous

$$\begin{aligned} &\bar{w}_0(0,\tau)=0, \qquad \left.\frac{\partial^2 \bar{w}_0}{\partial \xi^2}\right|_{0,\tau}=0; \\ &\bar{w}_0(1,\tau)=0, \qquad \left.\frac{\partial^2 \bar{w}_0}{\partial \xi^2}\right|_{1,\tau}=0. \end{aligned} \tag{4.3}$$

For this, function $f(\xi)$ must satisfy the following requirements

$$\begin{aligned} &f(0)=0, \qquad \left.\frac{d^2 f}{d\xi^2}\right|_{0}=\bar{\mu}_1; \\ &f(1)=0, \qquad \left.\frac{d^2 f}{d\xi^2}\right|_{1}=\bar{\mu}_1. \end{aligned} \tag{4.4}$$

The function that satisfies requirements (4.4) has the form

$$f(\xi)=-\frac{\bar{\mu}_1}{2}\xi(1-\xi)=\frac{\bar{w}_{cm}(\xi)}{(1-\alpha)^3}. \tag{4.5}$$

It follows that the right part of equation (4.2) is equal to 0, that is equation (4.2) is homogeneous. Separating the variables in equation (4.2) by submission

$$\bar{w}_0=\bar{W}_0(\xi)T(\tau), \tag{4.6}$$

we get two ordinary differential equations for the functions $\bar{W}_0(\xi)$ and $T(\xi)$

$$\frac{d^4 \bar{W}_0}{d\xi^4}-w^2\bar{W}_0=0. \tag{4.7}$$

$$\frac{d^2 T}{d\tau^2}+w^2 T=0, \tag{4.8}$$

where w is the dimensionless frequency of a beam proper vibrations.

The general solutions to equations (4.7) and (4.8) have the form, respectively

$$\bar{W}_0=C_1\sin\sqrt{w}\xi+C_2\cos\sqrt{w}\xi+C_3 sh\sqrt{w}\xi+C_4 ch\sqrt{w}\xi. \tag{4.9}$$

$$T=A\sin w\tau+B\cos w\tau, \tag{4.10}$$

where A, B, C_i $(i=\overline{1,4})$ are the constants of integration.

Separating variables in the boundary conditions (4.3), we obtain

$$\bar{W}_0(0) = 0, \qquad \left.\frac{d^2\bar{W}_0}{d\,\xi^2}\right|_0 = 0;$$
$$\bar{W}_0(1) = 0, \qquad \left.\frac{d^2\bar{W}_0}{d\,\xi^2}\right|_1 = 0. \tag{4.11}$$

Satisfying the boundary conditions (4.11), we obtain from (4.9) the system of algebraic equations for C_i $(i = \overline{1,4})$

$$\begin{cases} C_2 + C_4 = 0 \\ -C_2 + C_4 = 0 \\ C_1 \sin\sqrt{w} + C_2 \cos\sqrt{w} + C_3 sh\sqrt{w} + C_4 ch\sqrt{w} = 0 \\ -C_1 \sin\sqrt{w} - C_2 \cos\sqrt{w} + C_3 sh\sqrt{w} + C_4 ch\sqrt{w} = 0, \end{cases} \tag{4.12}$$

whence it follows that $C_2 = C_4$ and

$$\begin{cases} C_1 \sin\sqrt{w} + C_3 sh\sqrt{w} = 0 \\ -C_1 \sin\sqrt{w} + C_3 sh\sqrt{w} = 0. \end{cases} \tag{4.13}$$

The condition for the existence of nonzero solutions to the system of homogeneous equations (4.13) is its determinant equality to zero

$$\begin{vmatrix} \sin\sqrt{w} & sh\sqrt{w} \\ -\sin\sqrt{w} & sh\sqrt{w} \end{vmatrix} = 0.$$

We obtain the frequency equation after disclosure of the determinant

$$2\sin\sqrt{w}\; sh\sqrt{w} = 0,$$

where

$$\sin\sqrt{w} = 0. \tag{4.14}$$

The roots of the equation (4.14) give the spectrum of frequencies of proper bending vibrations

$$w_n = (n\pi)^2, \quad (n = 1, 2, ...). \tag{4.15}$$

Each proper frequency corresponds to a form of proper vibrations

$$\bar{W}_{0n} = \sin\sqrt{w_n}\xi + \chi_n sh\sqrt{w_n}\xi, \tag{4.16}$$

where it is indicated $\chi_n = -\dfrac{\sin\sqrt{w_n}}{sh\sqrt{w_n}}$.

Further, according to (4.6), taking into account (4.10) and (4.16), we obtain

$$\bar{w}_0 = \sum_{n=1}^{\infty}(A_n \sin w_n\tau + B_n \cos w_n\tau)(\sin\sqrt{w_n}\xi + \chi_n sh\sqrt{w_n}\xi). \tag{4.17}$$

We find coefficients A_n and B_n, thus satisfying initial conditions (3.6)

$$\bar{w}(\xi,0) = \bar{w}_0(\xi,0) + f(\xi) = \sum_{n=1}^{\infty} B_n(\sin\sqrt{w_n}\xi + \chi_n sh\sqrt{w_n}\xi) + \frac{\bar{w}_{st}}{(1-\alpha)^3} = \bar{w}_{st}. \quad (4.18)$$

$$\left.\frac{\partial\,\bar{w}}{\partial\,\tau}\right|_{\xi,0} = \left.\frac{\partial\,\bar{w}_0}{\partial\,\tau}\right|_{\xi,0} = \sum_{n=1}^{\infty} A_n(\sin\sqrt{w_n}\xi + \chi_n sh\sqrt{w_n}\xi) = 0.$$

From the second the ratio (4.18) follows $A_n = 0$. We describe the first the ratio (4.18) in the form

$$\sum_{n=1}^{\infty} B_n \bar{W}_{0n}(\xi) = \beta(\alpha)\bar{w}_{cm}(\xi), \quad (4.19)$$

where it is indicated $\beta(\alpha) = 1 - \dfrac{1}{(1-\alpha)^3}$.

Using the orthogonality property forms of the proper vibrations (4.16)

$$\int_0^1 \bar{W}_{0i}(\xi)\bar{W}_{0j}(\xi)\,d\xi = \begin{cases} 0, & if \quad i \neq j \\ 1, & if \quad i = j, \end{cases}$$

multiplying (4.19) and integrating from 0 to 1, we obtain a formula for calculating coefficients B_i $(i = 1, 2, ..., n, ...)$

$$B_i = \beta(\alpha)\frac{\int_0^1 \bar{w}_{cm}(\xi)\bar{W}_{0i}(\xi)\,d\xi}{\int_0^1 \bar{W}_{0i}^2(\xi)\,d\xi}. \quad (4.20)$$

According to (4.1), we calculate the deflections in the process of vibrations

$$\bar{w}(\xi,\tau) = \sum_{n=1}^{\infty} B_n \bar{W}_{0n}(\xi)\cos w_n\tau - \frac{\bar{\mu}}{2(1-\alpha)^3}\xi(1-\xi). \quad (4.21)$$

By the successive differentiation of a deflections function (4.21), we obtain the rotation angles of cross sections $\dfrac{\partial\,\bar{w}}{\partial\,\xi}$ and curvature $\dfrac{\partial^2\,\bar{w}}{\partial\,\xi^2}$:

$$\frac{\partial\,\bar{w}}{\partial\,\xi} = \sum_{n=1}^{\infty} B_n \frac{d\bar{W}_{0n}}{d\,\xi}\cos w_n\tau - \frac{\bar{\mu}}{2(1-\alpha)^3}(1-2\xi),$$

$$\frac{\partial^2\,\bar{w}}{\partial\,\xi^2} = \sum_{n=1}^{\infty} B_n \frac{d^2\bar{W}_{0n}}{d\,\xi^2}\cos w_n\tau + \frac{\bar{\mu}}{(1-\alpha)^3}. \quad (4.22)$$

5 NUMERICAL ANALYSIS

The highest stresses in the dynamic process after the separation of layer αh because of the symmetry of the problem will develop in the dangerous section $\xi = \dfrac{1}{2}$ at time $\tau = \tau_1$.

$$\gamma_{dyn\cdot_{\max}} = \frac{M\left(\frac{1}{2},\tau_1\right)}{W_z(1-\alpha)^2} = \frac{EJ_z(1-\alpha)}{W_z l}\frac{\partial^2\bar{w}}{\partial\xi^2}\bigg|_{\frac{1}{2},\tau_1}$$

or in a dimensionless form, taking into account (4.22), (4.20) and (2.3)

$$\bar{\sigma}_{dyn\cdot_{\max}} = \frac{1}{(1-\alpha)^2}\left[1-\frac{(1-\alpha)^3-1}{2}\sum_{n=1}^{\infty}\frac{\int\limits_0^1(\xi-1)\bar{W}_{0n}(\xi)d\xi}{\int\limits_0^1\bar{W}_{0n}^2 d\xi}\cdot\frac{d^2\bar{W}_{0n}}{d\xi^2}\bigg|_{\frac{1}{2}}\cos w_n\tau_1\right],$$

where it is indicated $\bar{\sigma}_{dyn\cdot_{\max}} = \frac{\sigma_{dyn\cdot_{\max}}}{\mu}W_z$.

The effect of the sudden separation of layer αh quantifies the ratio $K_{dyn\cdot_1(\alpha)}$, calculated as the ratio of maximum dynamic tension $\bar{\sigma}_{dyn\cdot_{\max}}$ to maximum tension $\bar{\sigma}_{st_{\max 1}}$, which develops at a quasi-static transition of the intact beam to the damaged one

$$K_{dyn\cdot_1}(\alpha) = \frac{\bar{\sigma}_{dyn\cdot_{\max}}}{\bar{\sigma}_{st_{\max 1}}} = 1-\frac{(1-\alpha)^3-1}{2}\sum_{n=1}^{\infty}\frac{\int\limits_0^1\xi(\xi-1)\bar{W}_{0n}(\xi)d\xi}{\int\limits_0^1\bar{W}_{0n}^2(\xi)d\xi}\cdot\frac{d^2\bar{W}_{0n}}{d\xi^2}\bigg|_{\frac{1}{2}}\cdot\cos w_n\tau_1. \quad (5.1)$$

Taking into account the first five summands in the sum (5.1) takes the form

$$K_{dyn\cdot_1}(\alpha) = 1+\frac{4}{\pi}\left(1-(1-\alpha)^3\right)\left(\cos\pi^2\tau_1-\frac{1}{3}\cos 9\pi^2\tau_1+\frac{1}{5}\cos 25\pi^2\tau_1\right). \quad (5.2)$$

We find moment τ_1 as a point of extremum of the function $\bar{\sigma}_{dyn\cdot_{\max}}\left(\frac{1}{2},\tau\right)$ and, consequently, of the function $\frac{\partial^2\bar{w}}{\partial\xi^2}\bigg|_{\frac{1}{2},\tau}$ (4.22). Differentiating (4.22) according to τ and equating the derivative to zero, we find the first nonzero root of the obtained transcendental equation

$$\frac{\partial^3\bar{w}}{\partial\xi^2\partial\tau} = -\sum_{n=1}^{\infty}w_n B_n\frac{d^2\bar{W}_{0n}}{d\xi^2}\bigg|_{\frac{1}{2}}\cdot\sin w_n\tau_1 = 0. \quad (5.3)$$

Taking into account five summands in the left part of equation (5.3) we get

$$\tau_1 = 0{,}010756. \quad (5.4)$$

The comparison of the signs of the derivative of function (4.22) according to the time left (+) and right (–) from the extremum of (5.4) shows that if $\tau_1 = 0{,}010756.$ the function $\bar{\sigma}_{dyn\cdot_{\max}}\left(\frac{1}{2},\tau\right)$, indeed, reaches a maximum.

In addition to the ratio $K_{dyn.1}$ the transition process can be characterized, as well, by a coefficient $K_{dyn.}$, indicating how many times the maximum stress in the dynamic process after the separation of layer αh $\bar{\sigma}_{dyn.\max}$ exceeds the maximum stress in the original static state

$$K_{dyn.} = \frac{\bar{\sigma}_{\max_{dyn.}}}{\sigma_{st_{\max}}} = \frac{K_{dyn.1}}{(1-\alpha)^2}. \tag{5.5}$$

Figures 4 *a* and *b* show the dependence graphics of coefficients $K_{dyn.1}$ (5.2) and (5.5), respectively, on parameter α characterizing the damage degree of the beam.

Note that the true time t_1 to reach the value maximum stress depends on the parameter of damage of the beam α (figure 5).

We also note that the problem has been solved without taking into account energy dissipation during the dynamic process as a result of a sudden change in the size of the beam. It is acceptable for the problem of estimating the degree of stress increase when we need to determine the value of the first maximum stresses at $\tau = \tau_1$.

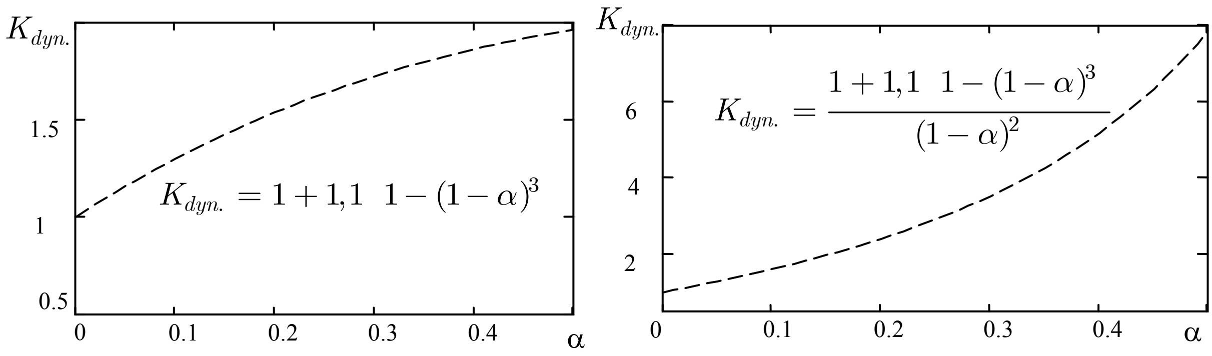

Figure 4: The coefficients characterizing the increase of the maximum stresses depending on the extent of damage to the beam.

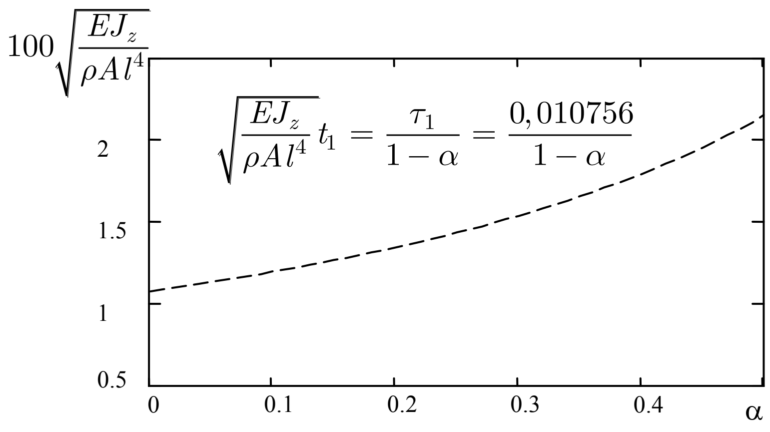

Figure 5: The dependence of the time of the first maximum stress on the parameter of damage α.

6 CONCLUSIONS

Thus, the simplest example a single span simply supported beam has demonstrated significant excess of the stresses in a statically loaded structure at a sudden change in its geometrical parameters (in particular, changes in the area and the axial moment of inertia of the rectangular cross-section by reducing its height). The approach can be applied to the sensitivity analysis for designed building constructions and their elements to sudden structural changes under load.

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