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COMPARATIVE STUDY ON THE INFLUENCE OF DIFFERENT DAMPING VALUES IN SEISMIC SITE RESPONSE ANALYSIS

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Abstract. *The methods of damping selection are different, therefore there will be a great difference in the results of time domain analysis and frequency domain analysis. The results of time domain analysis and frequency domain equivalent linear analysis are compared by single-layered soil columns with different thicknesses and multi-layered soil column under different input ground motions. How to evaluate Rayleigh damping coefficient is also discussed. The results show that the selection of different target frequency influencing viscous damping coefficient is the main reason leading to different results in time domain analysis. Hysteretic damping can be used more exactly in time domain site response analysis. To reduce the differences of the results of time domain analysis and frequency domain analysis, the full Rayleigh damping formulation and the first and third natural frequency can be employed.*

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1 INTRODUCTION

Seismic site response analysis is an important part to determine the ground motion parameters. The equivalent linear frequency domain analysis method based on one-dimensional site elastic response analysis is commonly performed to account for local site effects on ground motion propagation under an earthquake [1]. Horizontal soil layers represent site stratigraphy, and vertically propagating horizontal shear waves (i.e., SH waves) approximate the ground motion. The damping force is assumed to be proportional to displacement amplitude and in phase with velocity, and the damping is frequency independent. In time domain analysis, it is generally used the Rayleigh damping formulation (i.e., $\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K}$) which results in frequency dependent damping. The only difference between time domain analysis and frequency domain analysis is the selection of damping. The fundamental reason of damping difference is the frequency selected to calculate Rayleigh damping coefficient (i.e., target frequency in this paper). Many scholars have done research on the selection of target frequency [2-13]. In this paper, according to different selection of target frequency, two single-layered soil columns and one multi-layered soil column are calculated under Ninghe wave, Taft wave and El-Centro wave respectively. The results of seismic site response analysis with different damping coefficients are compared, which can provide reference for the selection of target frequency in practical engineering application.

2 EQUATION OF MOTION

For one-dimensional soil model, the equation of nodal motion in frequency domain analysis is expressed as:

$$[(1+2\zeta\, i)\boldsymbol{K} - \omega^2\boldsymbol{M}]\boldsymbol{U} = \boldsymbol{P} \tag{1}$$

where ζ = hysteretic damping ratio, ω= natural circular frequency, $\boldsymbol{M}$ = mass matrix, $\boldsymbol{K}$ =stiffness matrix, $\boldsymbol{U}$ =vector of nodal relative displacements, and $\boldsymbol{P}$ = vector of nodal equivalent seismic load.

The following dynamic equation of motion in time domain analysis is solved:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = -\boldsymbol{M}I\ddot{\boldsymbol{u}}_g \tag{2}$$

where $\boldsymbol{M}$ = mass matrix, $\boldsymbol{C}$ = viscous damping matrix, $\boldsymbol{K}$ = stiffness matrix, $\ddot{\boldsymbol{u}}$ = vector of nodal relative acceleration, $\dot{\boldsymbol{u}}$ = vector of nodal relative velocities, $\boldsymbol{u}$ = vector of nodal relative displacements, $\ddot{\boldsymbol{u}}_g$ is the acceleration at the base of the soil column and I is the unit vector.

3 DETERMINATION OF RAYLEIGH DAMPING COEFFICIENT

For Rayleigh damping is simple and convenient to use, it is widely performed in time domain analysis. The damping matrix $\boldsymbol{C}$ is derived from a linear combination of the mass matrix $\boldsymbol{M}$ and the stiffness matrix $\boldsymbol{K}$:

$$\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K} \tag{3}$$

where $\boldsymbol{C}$, $\boldsymbol{M}$ and $\boldsymbol{K}$ are viscous damping matrix, mass matrix and stiffness matrix respectively, and α, β are the Rayleigh damping coefficients.

In frequency domain analysis, damping ratio is a frequency independent constant. In time domain analysis, the same assumption as in structural dynamic calculation is often used, and it is written as:

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2\omega_i\omega_j}{\omega_j^2 - \omega_i^2} \begin{Bmatrix} \omega_j & -\omega_i \\ -\dfrac{1}{\omega_j} & \dfrac{1}{\omega_i} \end{Bmatrix} \begin{Bmatrix} \zeta_i \\ \zeta_j \end{Bmatrix} \tag{4}$$

where ζ_i and ζ_j are the damping ratio of soil for mode i and j. Viscous damping ratio is often used in seismic site response analysis. Damping ratio ζ_n can be expressed by α, β and corresponding natural circular frequency:

$$\zeta_n = \frac{\alpha}{2\omega_\mathrm{n}} + \frac{\beta\omega_\mathrm{n}}{2} \tag{5}$$

As can be seen from above, damping ratio in time domain analysis is frequency dependent. The relationship between hysteretic damping of frequency domain and Rayleigh damping of time domain can be seen in Fig.1. In general, it is assumed that $\zeta_i = \zeta_i = \zeta$. By Eq. (4), it can be obtained:

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \frac{2\zeta}{\omega_i + \omega_j} \begin{Bmatrix} \omega_i\omega_j \\ 1 \end{Bmatrix} \tag{6}$$

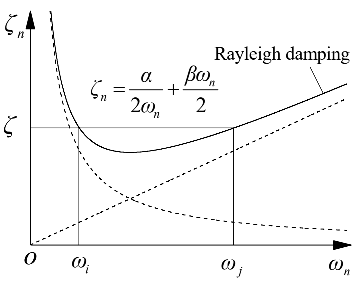

Fig.1: The relationship between hysteretic damping and Rayleigh damping.

Different methods are proposed on the value of Rayleigh damping. The damping matrix is assumed in early formulations to be only stiffness proportional since the value of $\alpha\boldsymbol{M}$ is small compared to $\beta\boldsymbol{K}$. The Eq. (3) can be further simplified to $\boldsymbol{C} = \beta\boldsymbol{K}$, which is called simplified Rayleigh damping formulation. It can be obtained $\beta = 2\zeta / \omega = \zeta / \pi f$ by Eq. (5), defining f as target frequency, and the damping coefficient β will be controlled by the target frequency f. Main methods of selecting the target frequency f are as follows:

(1) The first natural frequency of soil, f_1 (Ref. [2])

(2) The predominant frequency of Fourier spectrum of input ground motion, f_p (Ref. [3])

(3) The centroid frequency of Fourier spectrum of input ground motion, f_c (Ref. [4])

(4) A suitable frequency is obtained by fitting formulation (Ref. [5])

If the full Rayleigh damping formulation $\boldsymbol{C} = \alpha\boldsymbol{M} + \beta\boldsymbol{K}$ is performed, main methods of selecting the target frequency f are as follows:

(1) $\alpha = \zeta\omega$, $\beta = \zeta / \omega$, generally, $\omega = \omega_1$ (the first natural circular frequency), $\omega_1 = 2\pi f_1$ (Ref. [6])

(2) $\alpha_i = \beta_i = \zeta_i \omega_i$, $\zeta = 0.05$, $\omega_i = \dfrac{\pi v_{si}}{2h_i}$, where v_{si} = shear wave velocity (Ref. [7])

(3) $\alpha = 2\zeta(\dfrac{\omega_m \omega_n}{\omega_m + \omega_n})$, $\beta = 2\zeta(\dfrac{1}{\omega_m + \omega_n})$

Two natural frequencies within a significant frequency range of the input ground motion are defined as ω_m and ω_n respectively in Ref. [8]. In Ref. [9], ω_m is the first natural circular frequency, $\omega_n = N\omega_m$, where N is an odd number greater than ω_e / ω_1 (ω_e is the dominant frequency of input ground motion). The first and third natural frequency of soil are defined as ω_m and ω_n respectively in Ref. [10], that is, $\omega_1 = 2\pi f_1$ and $\omega_3 = 2\pi f_3$.

(4) $\alpha = 2\zeta(\dfrac{\omega_a \omega_b}{\omega_a + \omega_b})$, $\beta = 2\zeta(\dfrac{1}{\omega_a + \omega_b})$

where ω_a and ω_b are selected target frequencies. In Ref. [11], if $\dfrac{d\zeta}{d\omega} = -\dfrac{\alpha}{2\omega^2} + \dfrac{\beta}{2} = 0$, it is obtained $\omega = \sqrt{\alpha / \beta}$, and by Eq. (5) the minimum damping ratio can be obtained as follows:

$$\zeta_{\min} = \sqrt{\alpha\beta} \tag{7}$$

The damping ratio at the selected frequency range boundary is:

$$\begin{aligned} \zeta_{\max} &= \frac{\alpha}{2\omega_a} + \frac{\beta\omega_a}{2} \\ \zeta_{\max} &= \frac{\alpha}{2\omega_b} + \frac{\beta\omega_b}{2} \end{aligned} \tag{8}$$

$$\zeta_0 = (\zeta_{\max} + \zeta_{\min})/2 \tag{9}$$

Jean-Francois semblat et al. [12] used the method in Ref. [11] to determine damping coefficient. Degao Zou et al. [13] selected appropriate target frequency to determine damping coefficient based on the Ref. [9] and Ref. [11].

For single-layered soil,

$$f_n = \frac{V_s}{4H}(2n-1) \tag{10}$$

where n is the mode number, f_n is the natural frequency of the corresponding mode, V_s = shear wave velocity, and H = the thickness of soil column.

For multi-layered soil,

$$V_{seq} = \frac{H}{\sum_i (h_i / v_{si})} \tag{11}$$

$$f_n = \frac{V_{seq}}{4H}(2n-1) \tag{12}$$

where $H = \sum_i h_i$, $v_{si} = \sqrt{\frac{G_i}{\rho_i}}$ (v_{si} = shear wave velocity for layer i), h_i, G_i and ρ_i are thickness, shear modulus and density for layer i respectively.

4 INPUT GROUND MOTION AND CALCULATION MODEL

4.1 Input ground motion

Vertically propagating horizontal shear waves approximate the ground motion. Taking processed Ninghe wave, Taft wave and El-Centro wave as input ground motions, acceleration time history, Fourier amplitude spectrum and 5% damped elastic response spectra are shown in Fig. 2.

4.2 Calculation model

30 m single-layered soil column, 50 m single-layered soil column and 50 m multi-layered soil column are calculated respectively. In this paper, the results of the last iteration of the equivalent linear frequency domain analysis method in Ref. [1] are used to define the soil parameters in time domain analysis. The soil parameters are shown in Table 1 to Table 3 (the shear modulus and damping ratio in Table 1 to Table 3 are the final iteration results).

4.3 Target frequency *f*

When determining the Rayleigh damping coefficient, the key is to select the target frequency. The target frequencies obtained using the methods in related references are shown in Table 4.

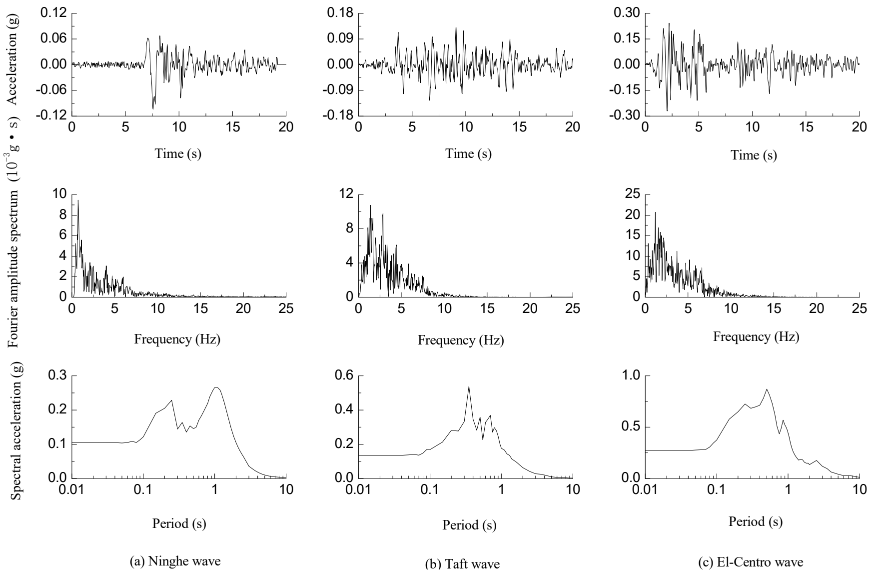

Fig.2: From top to bottom, respectively, acceleration time history, Fourier amplitude spectrum and 5% damped elastic response spectra.

Layer Type	Thickness h/m	Density ρ/(t· m^{-3})	Poisson's Ratio υ	Shear Modulus G / (10^7 Pa)			Damping Ratio ζ		
				Ninghe wave	Taft wave	El-Centro wave	Ninghe wave	Taft wave	El-Centro wave
Clay	10	1.9	0.3	5.26	6.23	3.06	0.097	0.082	0.149
Clay	10	1.9	0.3	1.34	1.92	0.87	0.185	0.174	0.201
Clay	10	1.9	0.3	0.76	1.44	0.59	0.202	0.182	0.205
Bedrock		2.2	0.2	58.3	56.1	64.3	0	0	0

Table 1: Properties of 30 m single-layered soil column.

Layer Type	Thickness h/m	Density ρ/(t • m^{-3})	Poisson's Ratio υ	Shear Modulus G / (10^7 Pa)			Damping Ratio ζ		
				Ninghe wave	Taft wave	El-Centro wave	Ninghe wave	Taft wave	El-Centro wave
Clay	10	1.9	0.3	7.83	8.14	5.17	0.066	0.064	0.099
Clay	10	1.9	0.3	2.20	2.47	1.26	0.172	0.166	0.187
Clay	10	1.9	0.3	1.40	1.92	0.66	0.183	0.174	0.204
Clay	10	1.9	0.3	1.08	1.75	0.59	0.194	0.177	0.205
Clay	10	1.9	0.3	0.99	1.57	0.59	0.198	0.180	0.205
Bedrock		2.2	0.2	55.6	56.1	66.0	0	0	0

Table 2: Properties of 50 m single-layered soil column.

Layer Type	Thick-ness h/m	Density ρ/(t·m^{-3})	Poisson's Ratio υ	Shear Modulus G / (10^7 Pa)			Damping Ratio ζ		
				Ninghe wave	Taft wave	El-Centro wave	Ninghe wave	Taft wave	El-Centro wave
Clay	3	1.99	0.35	1.62	1.74	0.52	0.074	0.069	0.172
Clay	2	1.89	0.35	1.75	1.97	0.56	0.120	0.106	0.184
Silty clay	8	1.99	0.30	3.47	4.16	0.33	0.092	0.072	0.159
Sandy clay	4	2.00	0.30	1.94	1.95	1.23	0.138	0.135	0.167
Sandy clay	4	2.00	0.30	1.82	1.96	1.09	0.142	0.137	0.176
Silt	4	2.07	0.30	0.19	9.23	0.19	0.156	0.089	0.156
Sandy clay	5	2.01	0.30	4.60	5.46	3.54	0.131	0.126	0.147
Sandy clay	2	1.96	0.30	20.1	21.1	10.8	0.085	0.081	0.130
Sandy clay	5	2.10	0.30	3.07	3.07	1.73	0.136	0.136	0.175
Silt	2	2.04	0.30	0.16	0.16	0.16	0.156	0.156	0.156
Sandy clay	6	2.00	0.30	3.48	3.47	2.11	0.136	0.135	0.169
Sandy clay	5	2.00	0.30	3.48	3.47	1.95	0.136	0.137	0.175
Bedrock		2.20	0.20	56.1	55.8	69.6	0	0	0

Table 3: Properties of 50 m multi-layered soil column.

5 CALCULATION RESULTS

It is used the form $\boldsymbol{C} = \beta \boldsymbol{K}$, $\beta = 2\zeta / \omega = \zeta / \pi f$ in Ref. [5]. The ratio between the arithmetic mean of the target frequency *f* and the first natural frequency f_1 of soil, and the ratio between the arithmetic mean of the target frequency *f* of soil and the centroid frequency of Fourier spectrum of the input ground motion f_c are calculated under input ground motions, thereby the relationship of *f*, f_1 and f_c is fitted. It can provide reference for determining damping coefficient in time domain analysis, but it is more trouble in application. In Ref. [8, 9, 12, 13], based on appropriate target frequency, Rayleigh damping is determined for given soil column and input ground motion, but a regular and universal conclusion is not given for selection of target frequency. An averaged damping ratio is used in Ref.[11], while the damping ratio is extracted from the equivalent linear frequency domain analysis procedure in this paper, and therefore, the results using the method in Ref. [11] are not compared with that of the equivalent linear frequency domain analysis method in Ref. [1].

In this paper, the results using the methods in Ref. [2-4, 6, 7, 10] are compared with that of Ref. [1] respectively. Taking the results of equivalent linear frequency domain analysis in Ref. [1] as standard, the relative error of the result of time domain analysis and frequency domain analysis is expressed as:

$$e = \frac{|a_{\max}| - |a^*_{\max}|}{|a^*_{\max}|} \times 100\% \tag{13}$$

where $a_{\max}$ = peak ground acceleration in time domain analysis, and $a^*_{\max}$ = peak ground acceleration in frequency domain analysis.
The absolute value of peak ground acceleration is:

$$A_{\max} = |a_{\max}| \; or \; |a^*_{\max}| \tag{14}$$

The absolute values of the peak ground accelerations and the relative errors are shown in Table 5 to Table 7, and the 5% damped elastic response spectrum are shown in Fig. 3.

f /Hz		Ninghe wave	Taft wave	El-Centro wave	References
f_p		0.73	1.42	1.17	[3]
f_c		4.03	3.61	3.71	[4]
30 m single-layered	f_1	0.74	0.93	0.62	[2,6,10]
soil column	f_3	3.71	4.64	3.08	[10]
50 m single-layered	f_1	0.47	0.56	0.35	[2,6,10]
soil column	f_3	2.36	2.78	1.76	[10]
50 m multi-layered	f_1	0.44	0.57	0.31	[2,6,10]
soil column	f_3	2.21	2.84	1.54	[10]

Table 4: Target frequencies to calculate Rayleigh damping coefficient under different ground motions.

Note: f = target frequency, f_p = the predominant frequency of Fourier spectrum of the input ground motion, f_c = the centroid frequency of Fourier spectrum of the input ground motion, f_1 = the first natural frequency, f_3 = the third natural frequency.

Methods in references	Ninghe wave		Taft wave		El-Centro wave	
	$A_{\max}$ /g	e (%)	$A_{\max}$ /g	e (%)	$A_{\max}$ /g	e (%)
Input	0.104	—	0.132	—	0.271	—
[1]	0.231	0	0.223	0	0.278	0
[2]	0.246	6.5	0.211	-5.4	0.223	-19.8
[3]	0.246	6.2	0.236	5.8	0.249	-10.5
[4]	0.324	40.2	0.287	28.6	0.377	35.2
[6]	0.239	3.4	0.215	-3.7	0.219	-21.4
[7]	0.158	-31.8	0.148	-33.6	0.169	-39.2
[10]	0.243	5.0	0.238	6.6	0.268	-3.7

Table 5: The absolute values of peak ground acceleration and relative errors of 30 m single-layered soil column.

Methods in references	Ninghe wave		Taft wave		El-Centro wave	
	A_{max}/g	e (%)	A_{max}/g	e (%)	A_{max}/g	e (%)
Input	0.104	—	0.132	—	0.271	—
[1]	0.151	0	0.155	0	0.200	0
[2]	0.131	-13.4	0.121	-22.5	0.155.	-22.4
[3]	0.149	-1.3	0.157	1.3	0.267	33.7
[4]	0.201	33.2	0.200	28.8	0.420	110.2
[6]	0.141	-6.6	0.130	-16.1	0.162	-18.7
[7]	0.106	-30.0	0.097	-37.3	0.094	-53.1
[10]	0.161	6.6	0.159	2.4	0.214	7.0

Table 6: The absolute values of peak ground acceleration and relative errors of 50 m single-layered soil column.

Methods in references	Ninghe wave		Taft wave		El-Centro wave	
	A_{max}/g	e (%)	A_{max}/g	e (%)	A_{max}/g	e (%)
Input	0.104	—	0.132	—	0.271	—
[1]	0.155	0	0.130	0	0.269	0
[2]	0.118	-23.8	0.095	-26.7	0.195	-27.4
[3]	0.141	-9.1	0.114	-12.7	0.345	28.2
[4]	0.208	34.0	0.147	12.9	0.503	86.8
[6]	0.133	-14.2	0.101	-22.6	0.209	-22.2
[7]	0.086	-44.7	0.089	-32.1	0.102	-62.1
[10]	0.159	2.6	0.135	3.4	0.268	-0.3

Table 7: The absolute values of peak ground acceleration and relative errors of 50 m multi-layered soil column.

6 CONCLUSIONS

In this paper, two single-layered soil columns and one multi-layered soil column are calculated under different input ground motions, and the results of different Rayleigh damping coefficients in time domain analysis are compared with that of frequency domain analysis respectively. Conclusions are obtained as follows:

(1) As can be seen from Fig.1, the damping ratio in the Rayleigh damping formulation is frequency dependent, in contrast to experimental results showing that the damping of soil is mostly frequency independent. The damping coefficients α and β are controlled by the target frequency f. Based on different selection of target frequency f, the damping ratio calculated by the formulation $\boldsymbol{C} = \alpha\boldsymbol{M}$ or $\boldsymbol{C} = \beta\boldsymbol{K}$ and the frequency domain hysteretic damping ratio are quite different.

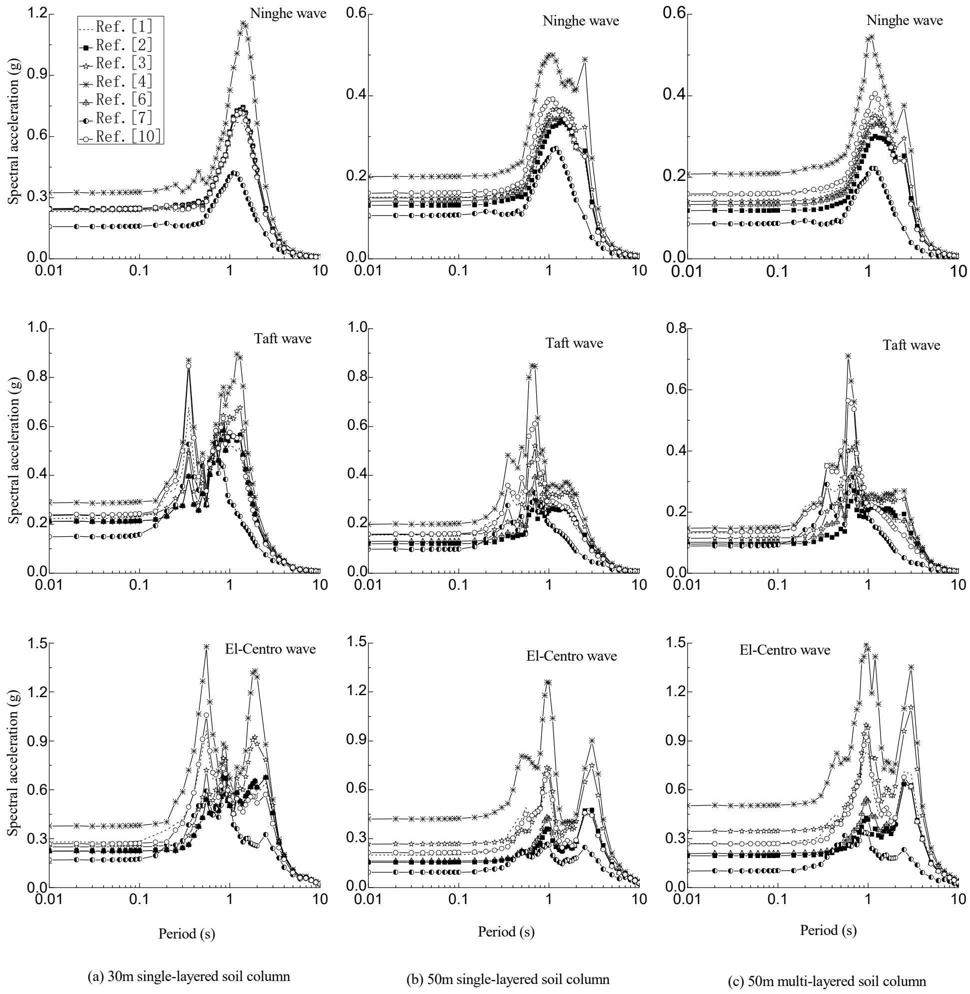

Fig.3: 5% damped elastic response spectra.

(2) From Table 5 to Table 7, conclusions can be obtained as follows. In contrast with the absolute values of the peak ground accelerations calculated using the method in Ref. [1], the errors of the calculation results using the methods in Ref. [2-4, 6, 7] are large, and they are more sensitive to the thickness of soil and the input ground motion, that is, the errors have large discrete with different thicknesses of soil and different input ground motions. To the contrary of the absolute values of the peak ground accelerations calculated using the method in Ref. [1], the errors of the calculation results using the method in Ref. [10] are small, generally less than 10%, and they are insensitive to the thickness of soil and the input ground motion, that is, the errors have small discrete with different thicknesses of soil and different input ground motions. As presented in Fig. 3, the calculation results using the method in Ref. [4] are obviously large, while the calculation results using the method in Ref. [7] are obviously small, which are different from the elastic response spectrum calculated using the method in

Ref. [1]. The errors of the calculation results using the methods in Ref. [2, 3, 6] are relatively large. The calculation results using the method in Ref. [10] agree well with that of Ref. [1].

(3) According to the elastic response spectrum in Fig. 3, unlike the calculation results using the method in Ref. [1], the results using the method in Ref. [10] are large in the corresponding period of the selected target frequency, while the results using the method in Ref. [10] are small in other periods. It can be observed from Fig.1, in the corresponding period of the selected target frequency, the damping ratio calculated using the method in Ref. [10] is smaller than the frequency domain hysteretic damping ratio, so the calculated results are large. In other periods, the damping ratio calculated by the method in Ref. [10] is larger than the frequency domain hysteretic damping ratio, so the calculation results are small.

(4) The determination of Rayleigh damping coefficient in time domain analysis has great influence on the calculation results. When using the method in Ref. [10], that is, using the full Rayleigh damping formulation and selecting the first and second natural frequency as two target frequencies, the results obtained agree well with that of frequency domain analysis. At present, the method in Ref. [10] is an effective approach for selecting damping in time domain analysis, but a better method needs to be studied further.

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