Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2017) 4194-4212

ECCOMAS
Proceedia

COMPDYN 2017
6th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15–17 June 2017

# RATIONAL SEISMIC RETROFITTING OF RC STRUCTURES BASED ON GENETIC ALGORITHMS

**R. Falcone[1], C. Faella[2], C. Lima[3], E. Martinelli[4]**

[1] PhD student, DiCiv – Dept. of Civil Eng., University of Salerno, rfalcone@unisa.it

[2] Full Professor, DiCiv – Dept. of Civil Eng., University of Salerno, c.faella@unisa.it

[3] Research Assistant, PhD, DiCiv – Dept. of Civil Eng., University of Salerno, clima@unisa.it

[4] Associate Professor, PhD, DiCiv – Dept. of Civil Eng., University of Salerno, e.martinelli@unisa.it

**Keywords:** RC frames, Seismic Retrofitting, Optimal Strategy, Soft-Computing.

**Abstract.** *This paper presents a rational strategy developed for optimizing seismic retrofitting of Reinforced Concrete (RC) frames. It is based on combing member- and structure-level techniques in order to achieve optimal design objectives within a multi-level Performance-Based approach. On the one hand, in principle, confinement with composite materials, steel and/or concrete jacketing might be considered as a member-level technique capable to enhance the capacity of under-designed members and, consequently, of the structure as a whole. On the other hand, introducing steel bracing systems or shear walls might be taken into account as structure-level techniques. Generally, member- and structure-level techniques are not employed together in seismic retrofitting or, in the cases in which they are combined, no well-established rules are available for choosing their optimal combination. However, a synergistic use of such techniques by means of well-defined procedures could help designers obtain optimal seismic retrofitting performance. The latest progresses about a procedure developed by the authors for selecting the optimal retrofitting solution among the technically feasible ones, obtained by combining alternative configurations of steel bracing systems and FRP-confinement of critical members, are presented herein. Specifically, the main aspects about formulating a genetic algorithm capable to select the "fittest" retrofitting solution is implemented and summarised. The main assumptions about the representations of "individuals" as part of this genetic algorithm and the main information about the generic operations (i.e. selection, crossover and mutation) are outlined. Finally, the procedure is applied to a 3D frame with the aim to demonstrate its potential.*


Peer-review under responsibility of the organizing committee of COMPDYN 2017.
doi: 10.7712/120117.5716.17175

## 1 INTRODUCTION

Seismic retrofitting is generally requested by the significant number of Reinforced Concrete (RC) buildings existing in earthquake prone areas, as they were built according to past design codes and standards. In fact, existing RC frames are often found vulnerable to seismic actions due to inadequate structural detailing, material deterioration and underestimated earthquake loads, without any considerations of Capacity Design rules introduced by modern seismic codes [1]. The high cost of new construction and historical importance of older buildings has led owners to renovate rather than demolish and rebuild their properties. This has caused governmental institutions to implement mandatory seismic strengthening regulations. Therefore, seismic retrofitting is nowadays a technical challenge for engineers and a societal priority [2]. Plenty of researches have been conducted in the last two decades on seismic assessment and retrofitting of RC frames: they have been mainly aimed at developing both effective methodologies for quantifying seismic vulnerability and consistent technical solutions for seismic retrofitting [3]-[5]. A retrofit scheme can be said to be successful if it results in modifying capacity and demand, making the latter smaller than the former with respect to all the performance objectives of relevance for the structure under consideration.

As a matter of principle, retrofitting solutions can be grouped into two broad classes: on the one hand, the so-called "Member-level" (or so called "local") techniques and, on the other hand, the "Structure-level" (also referred to as "global") techniques [2]. Confining existing members with composite materials [6], steel and/or concrete jacketing can be classified in the first group, as they are capable to enhance capacity (in terms of ductility and/or strength) in under-designed members [7] and, hence, improving the capacity of the structure as a whole in order to meet the requested seismic demand. Conversely, the second group collects all those techniques capable to reducing seismic demand on the existing structure, whose single members' capacity is fairly unchanged [8]. Connecting RC shear walls or steel bracing systems with the existing members or realizing a seismic isolation system are relevant examples of structure-level techniques.

That said, it is clear that member- and structure-level techniques, considered on their own, represent two somehow "extreme" solutions for retrofitting existing structures and generally they are not employed together in seismic retrofitting. Moreover, in that few cases in which they are combined, no well-established rules are currently available for choosing their optimal combination. However, the aforementioned techniques may suitably be combined with the aim to obtain a synergy in increasing seismic capacity of under-designed members and reducing demand on the structure as a whole. In this light, choosing the "fittest" solution among the potentially infinite combinations of member- and structure-level techniques is a task that can be regarded as a constrained optimisation problem [9]. In fact, each one of the combinations leads to different direct costs, life-cycle costs, reliability levels and other quantitative/qualitative parameters describing their "fitness" to be considered as a retrofitting solution. Several researches have been carried out for selecting the most structurally efficient and cost-effective seismic retrofitting solution: the possibility of combining member- and structure-level techniques for minimizing the initial cost of retrofitting is only conceptually explored in previous papers [8]. However, no well-established and completely accepted procedures are available for obtaining the optimal retrofitting solution. Therefore, the definition of a rational strategy for seismic retrofitting is still an open issue. In the Authors' best knowledge, no relevant study is currently available for approaching seismic retrofitting of existing RC frames as an optimisation problem, as recent scientific contributions on optimisation algorithms are restricted to seismic design of new structures [10]-[12]]. Conversely, in the current practice, as well as in most relevant scientific contributions on this topic [3]-[5], [7], [13], [15], seismic

retrofitting of RC frames is addressed as a solely technical problem. Considerations about optimisation (often restricted to the "economic" standpoint) are left to the engineering judgement and, hence, they are not part of a systematic analysis. This is partly due to the complexity of the constrained optimisation problem under consideration, which cannot be duly approached by means of analytical techniques commonly employed in structural engineering, as it can only be solved by means of meta-heuristic techniques [16] that are not in the background of common structural engineers.
Therefore, this paper proposes a genetic algorithm capable of selecting the "fittest" solution (in terms of initial costs) obtained by combining structure-level interventions, based on steel bracing systems, and FRP-based member-level techniques. More specifically, the paper outlines a Genetic Algorithm (GAs) inspired to the well-known Darwin's "evolution of species" and the assumption of the "survival of the fittest" rule [17]. Although some pioneering applications of these techniques are already available in the field of structural engineering, they are mainly restricted to the design of new structures [18]-[21]]. The following sections summarize the main aspects of the proposed genetic algorithm and its application in the rational design of retrofitting interventions. Finally, as an example, the last section presents an application of the proposed procedure.

## 2 THE GENETIC ALGORITHM

### 2.1 Problem statement and formulation

The problem of defining seismic retrofitting interventions on existing RC structures can be conceptually described by the following Limit State (LS) function $g_{LS}$:

$$g_{LS,i} = C_{LS,i} - D_{LS,i} \geq 0 \tag{1}$$

where $C_{LS,i}$ is the capacity of the strengthened structure at the Limit State (LS) or Performance Level under consideration and $D_{LS,i}$ the corresponding demand at the same Limit State. The LS of Damage Limitation (SLD), related to a seismic event whose Probability of Exceedance (PoE) equal to 63% in 50 years, and the Limit State of Life Safety (SLV), corresponding to an event with 10% PoE in 50 years, are taken into account herein [22]. Capacity and demand mentioned in Eq. (1) can be intended in terms of both displacement (for ductile mechanisms) and forces (for brittle failure mechanisms). As a matter of principle, the above inequality, that should be checked at all the relevant LSs, is not met by the structure in its "as built" condition and, more specifically, the capacity is lower than demand in seismically vulnerable structures. Hence, retrofitting intervention aims to enhance their "as built" condition at the i-th relevant LS under consideration and raise the values of $g_{LS,i}$ towards positive values. Then, two opposite "extreme" approaches can be followed to get the inequality (1) valid for the structures under strengthening:

- the capacity of all members could be increased by means of member-level-techniques (i.e., confinement of deficient sections, concrete or steel jacketing of columns, etc.) aimed at obtaining an enhanced global capacity $C_{LS}$ which is not lower than the corresponding seismic demand $D_{LS}$ (which, on the contrary, is almost unaffected by the above mentioned local intervention);
- the exceeding demand on the existing members can be reduced by acting at the structure-level by either introducing a structural sub-system, aimed at working in parallel with the structure and absorbing a significant part of the seismic excitation, or modify-

ing the vibration period of the structure as a whole to reduce the input seismic excitation on the super-structure.

However, as a matter of principle, the retrofitting objective can be achieved by acting on both capacity and demand and, hence, implementing a combination of strengthening techniques belonging to the above mentioned general classes [8]. The choice of the “best” possible combination of such intervention techniques clearly defines an optimisation problem which can be based on the assumption of different types of objective functions. The easiest (and more simplistic) choice is represented by assuming the actual cost of the intervention as the objective function whose minimization represents the optimal solution for the problem of seismic retrofitting under consideration [9]. The optimal retrofitting solution can be found by solving the following constrained optimisation problem stated as follows:

$$\begin{gathered} \overline{x}_{opt} = \arg\min_{x}\left[f\left(\mathbf{x}\right)\right] \\ \text{subjected to} \\ g_{LS,i} \geq 0 \quad \forall i = 1 \ldots n_{LS} \end{gathered} \tag{2}$$

where f(●) is the selected objective function and **x** is the vector of design variables defining the interventions consisting of both FRP confinement of single RC members and concentric steel bracings installed in parallel with the existing structure. The total direct cost is defined by adding the terms in Eq. (3):

$$C_{tot}\left(\mathbf{x}\right) = C_{loc}\left(\mathbf{x}\right) + C_{glob}\left(\mathbf{x}\right) + C_{found}\left(\mathbf{x}\right) \tag{3}$$

where $C_{loc}(x)$ and $C_{glob}(x)$, respectively referred to member- and structure-level techniques. It is worth highlighting that the two cost functions take into account both demolition and reconstruction operations needed for realizing FRP confinement and installing steel bracings. Moreover, $C_{found}(x)$ is the cost of possible interventions on the existing foundations, often needed to respond to the possible increase in vertical and horizontal reactions at the base of the retrofitted structure. Specifically, with the simplified assumption that the intervention is made with micro-piles, whose bearing capacity $q_{lim}$ is known, the cost $C_{found}(x)$ might stem out from a relationship conceptually defined as follows:

$$C_{found}\left(\mathbf{x}\right) = C_{found}\left[\sum_{i=1}^{n_{col}} \frac{\Delta N_i\left(\mathbf{x}\right)}{q_{lim}}\right] \tag{4}$$

where $\Delta N_i$ is the increase of axial force (with respect to the gravitational loads only) due to the seismic actions on the retrofitted structure. Finally, the objective function shown in Eq. (5) include a penalty function $\Phi(\bullet)$:

$$f\left(x\right) = C_{tot}\left(x\right) + \Phi\left\{\max\left(\frac{D_{LS,i}}{C_{LS,i}}\right)\right\} \tag{5}$$

which effectively transform the constrained problem into an unconstrained one by augmenting the objective function with a penalty term whose value determines the amount of constraint violation present in a particular retrofitting solution. The complex nature of the multi-objective optimisation problem addressed here cannot be duly approached by means of analytical techniques commonly employed in structural engineering. Conversely, it can only be solved by means of meta-heuristic techniques. Therefore, the authors propose a rational procedure based on a Genetic Algorithm (GA) for finding the optimal retrofitting solution of the constrained problem described by Eq. (2). The conceptual flow-chart shown in Figure 1 depicts the main steps of the procedure.

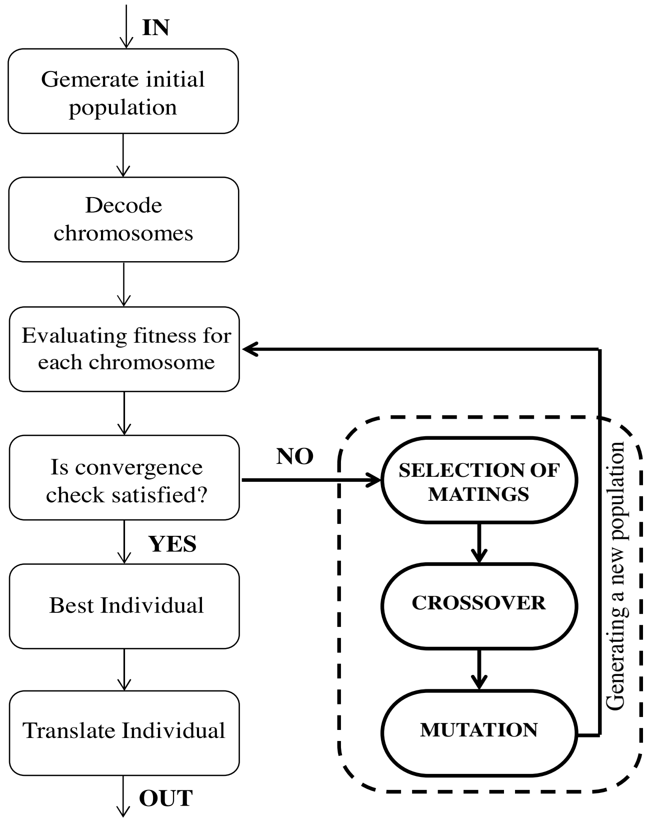



Figure 1: Flow-chart of the optimisation procedure

Goldberg [24] discussed how genetic algorithms have several fundamental advantages over other soft computing tool that allow them to be more robust than other optimisation methods:

1. GAs work with a coding of the parameter set, not the parameters themselves. This is advantageous because often times the parameters are in different units or measurement scales and can be very difficult to model. It is also beneficial when the number of parameters in the multi-objective optimisation problem is very large;
2. GAs search parallel from a population of points, not a single point. This is beneficial when there are multiple local optima because the GA will avoid premature convergence to local optimal solutions or false-peaks;
3. GAs use objective function information, not derivatives or other auxiliary information. This is beneficial if the objective function is not smooth, or is nonlinear, or if there are a large number of parameters to which the gradient information is not known;
4. GAs use probabilistic transition rules, not deterministic ones making them quite robust and able to solve most optimisation problems.

## 2.2 Generation of initial population

The procedure starts with randomly generating a group of $N_{ind}$ individuals known as the population, whose number typically ranges between 50 and 100. Since it is important to have highly diverse composition of the initial population, this step is achieved by generating the required number of individuals by using a random number generator that uniformly distributes numbers in the desired range. The individuale are encoded as strings (chromosomes) composed over some alphabet(s), so that the genotypes (chromosome values) are uniquely mapped onto the decision variable (phenotypic) domain [25]. Several coding methods are currently available, such as binary, gray, non-binary, etc. [26]-[28]]. The binary alphabet {0, 1} is actually adopted in coding for the present work: hence, each individual “x” of such a population is, herein represented through a simple “chromosome-like” array of bits. Hence, in the first step of the proposed procedure a population of 100 individuals is generated by the following random generator (6):

$$\text{pop} = \text{round}\left[\text{rand}\left(N_{\text{ind}}, N_{\text{bits}}\right)\right] \tag{6}$$

where the function rand generates a $N_{ind}$ x $N_{bits}$ matrix (Figure 2) of uniform random numbers between 0 and 1. The function round, instead, rounds the numbers to the closest integer which in this case is either 0 or 1.

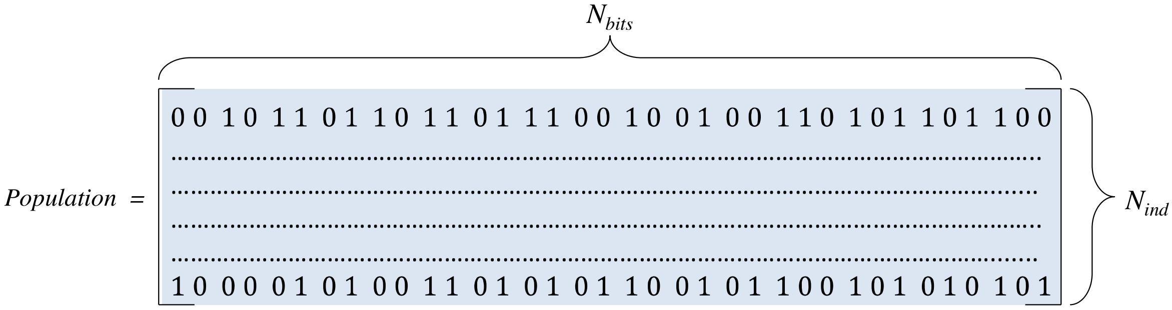



Figure 2: Example of initial population randomly generated

According to Hajela [29] a β-digit binary number representation of a variable allows for $2^{\beta}$ distinct variations of that design variable to be considered. The length of the coded representation of the design variables, i.e. the number of bits, corresponds to their range and precision. The length of the sub-string varies with the desired precision of the results: the longer the string length, the higher the accuracy. The relationship between string length β and precision α is expressed as follows:

$$\left(X_U - X_L\right) \cdot 10^{\alpha} \leq \left(2^{\beta} - 1\right) \tag{7}$$

where $X_U$ and $X_L$ are the upper and the lower bound of the variable, respectively. As described below, the search process operate on this encoding of the decision variables, rather than the decision variables themselves.

## 2.3 Decoding of individual

To proceed with the genetic analogy, the individuals of the population are assumed as chromosomes and the variables corresponds to genes. Thus, a chromosome, i.e. each row of the generated matrix, represent a possible retrofitting solution and is composed of several genes (variables). Examining the chromosome string in isolation yields no information about the problem under consideration. It is only with the decoding of the chromosome into its phenotypic values that any meaning can be found to the representation. Within the present algorithm, the vector **x** is composed of variables describing both member- and structure-level techniques: the string representing the binary coding of one individual [25] is structured by concatenating the set of variables that represent it. Figure 3 depicts an example of the binary genotype adopted in the present algorithm for representing a simple three-storey structure with four beams at each floor (one-bay for each direction in plan): the first part of the string describes the member-level techniques considered in the aforementioned individual, whereas the second part is dedicated to describing the structure-level ones. The variables are not encoded in equal sub-string lengths.

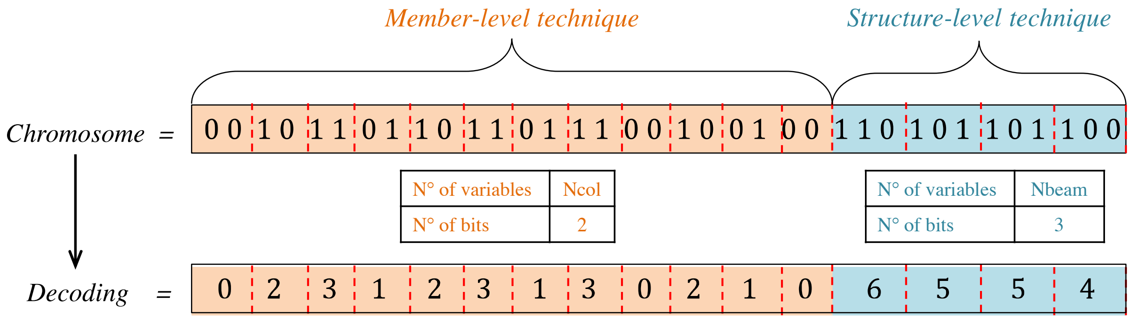



Figure 3: Example of binary genotype for coding a retrofitting intervention

In fact, in the first part of the individual, each couple of bits contains a code related to the number of FRP layers possibly employed for confining the corresponding column of the structure under consideration: hence, a total of $2N_{col}$ bits is contained, $N_{col}$ being the number of column elements in the structural model of the existing frame. The accuracy that can be obtained through two-bit coding is only approximately $1/4^{th}$ of the search space, but as this string length is increased by 1, the obtainable accuracy increases exponentially to $1/8^{th}$ of the search space. In the current implementation, the confinement ranges between zero (as-built configuration denoted by the value “00” assumed in the binary code of the corresponding column) and three layers of FRP (denoted by the value “11” of the binary code representing the i-th column). The information collected in this part of the codified **x** vector are employed for modifying the original (unconfined) mechanical values of the Kent and Park model [30] available in OpenSEES [31] for simulating the behavior of concrete in columns. Particularly, based on various parameters, such as the number of layers, the shape of section transverse section and the type of FRP considered for this application, the original (unconfined) stress-strain relationship describing the material behavior for each column is duly modified are schematically described in Figure 4. As is well-known, the resulting relationship for confined concrete depends on several parameters, such as the number of FRP layers, the shape of transverse section and the type of FRP considered for this application.

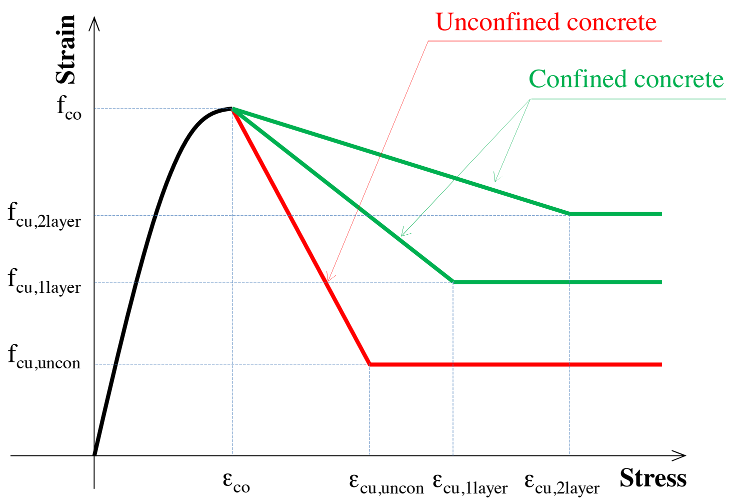



Figure 4: Model by Kent & Park [30] for confined concrete

On the other hand, the following assumption are considered for describing the steel bracings system representing the global intervention codified in the second part of the genotype:

1. the transverse section of each couple of diagonals is identified by means of a label corresponding to a commercial steel profile;
2. only the profiles of the first storey level are reported in the codified "x" vector, whereas those adopted for the upper levels stem out of a consistent design criterion;
3. steel bracings are supposed to be possibly realised between each couple of columns connected by a beam: therefore, the maximum number of bracings is equal to the number of beam $N_{beam}$ at the first floor.

The aforementioned assumptions are intended at keeping the set of design variables as small as possible. Therefore, the current implementation of the proposed procedure employs three bits for each variable related to steel diagonal bracings and, hence, only $2^3 = 8$ codes are available to describe the section of diagonals possibly installed at the first storey level, in correspondence of a generic beam (see assumption n. 3). They can range from 0 (absence of bracing) to 7: each code included between "001" and "111" points to a position in a commercial steel profile table (assumption n. 1) and, hence, the relevant properties of bracing diagonals are available therein (Figure 5).

| **LABEL** | HEB | b [mm] | h [mm] | a [mm] | e [mm] | r [mm] | $A_1$ [cm$^2$] |
|---|---|---|---|---|---|---|---|
| **1** | 100 | 100 | 100 | 6,0 | 10,0 | 12 | 26,04 |
| **2** | 120 | 120 | 120 | 6,5 | 11,0 | 12 | 34,01 |
| **3** | 140 | 140 | 140 | 7,0 | 12,0 | 12 | 42,96 |
| **4** | 160 | 160 | 160 | 8,0 | 13,0 | 15 | 54,25 |
| **5** | 180 | 180 | 180 | 8,5 | 14,0 | 15 | 65,25 |
| **6** | 200 | 200 | 200 | 9,0 | 15,0 | 18 | 78,08 |
| **7** | 220 | 220 | 220 | 9,5 | 16,0 | 18 | 91,04 |

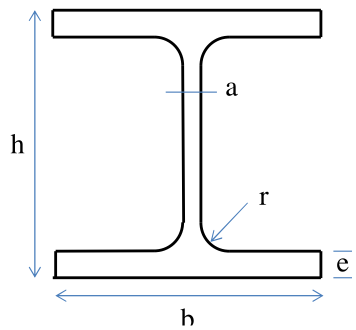



Figure 5: Set of commercial steel profile and corresponding labels

The relationship between the section of steel members at the first level and the section of steel bracings at upper floors is defined according the Eq. (8):

$$A_{k,des} = \frac{\sum_{j=k}^{n} h_j \cdot W_j}{\sum_{i=1}^{n} h_i \cdot W_i} \cdot A_1 \tag{8}$$

where $h_j$ represents the position in height of the floor with respect to the foundation level, n is the total number of floors, $W_j$ is the seismic mass of the j-th floor. Moreover, $A_1$ is the area of the cross section of the bracing at the first level and $A_{k,des}$ is the theoretical area of the bracing cross section required at the k-th floor. It is worth highlighting that any other consistent design criteria, defining the upper level sections depending on the first level ones, might be possibly adopted in lieu of Eq. (8). Finally, the knowledge of theoretical areas allows to select the steel commercial section whose area must be greater than $A_{k,des}$.

## 2.4 Evaluating of fitness

The optimisation target can be measured both in terms of objective f(**x**) and fitness F(**x**) functions, but the latter has a closer association with biology than the former. In fact, fitness in biological sense is a quality value which is a measure of the reproductive efficiency of chromosomes. In the proposed genetic algorithm, fitness is used to allocate reproductive traits to the individuals in the population and thus act as some measure of goodness to be maximized. On the other hand, the objective function is used to provide a measure of how individuals have performed in the problem domain. As already seen, the search of the optimal solution is affected both by the actual cost $C_{tot}$ of the retrofitting solution and the additional information related to the seismic check on Limit State function $g_{LS,i}$. With regards to the latter, Static Non Linear (Pushover) Static Analysis (Figure 6) is considered for determining capacity curves through an incremental static analysis on a nonlinear model of the structure subjected to a triangular distribution of lateral forces, both in x and y directions (further distributions might be easily taken into account for performing the seismic analysis) [22] [32]. A capacity fiber model with plasticity spread along the non-linear elements is taken into account.

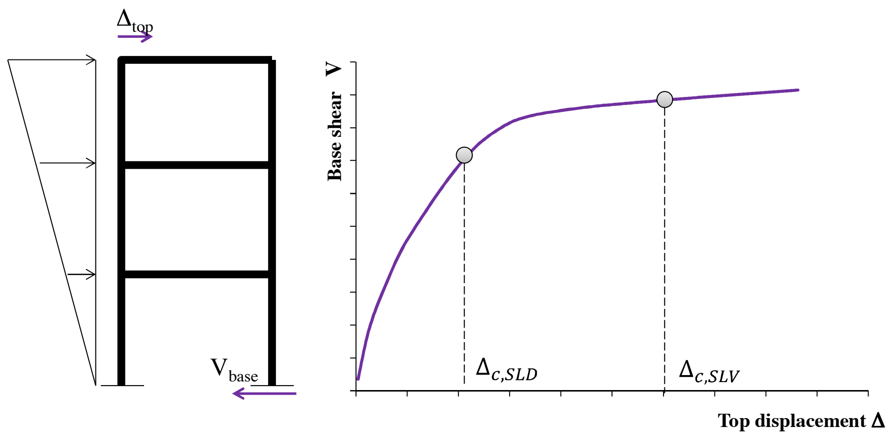



Figure 6: Static Non Linear Analysis and capacity curve

The deformation capacity $C_{LS,i}$ of frame elements, to be verified in accordance with Eq. (1) at Limit State of Damage Limitation (SLD) and Limit State of Life Safety (SLV), is defined in terms of chord rotation θ at yielding and collapse conditions, respectively. The chord rotation is the angle between the tangent to the axis at the yielding end and the chord connecting that end with the end of the shear span ($L_V$ = M/V = moment/shear at the end section). It is also equal to the element drift ratio, i.e., the deflection at the end of the shear span with respect to the tangent to the axis at the yielding end, divided by the shear span. With regard to collapse conditions, the capacity rotation $\theta_{um}$ is herein assessed by the following empirical formulations **Errore. L'origine riferimento non è stata trovata.**:

$$\theta_{um} = \theta_{C,SLV} = \frac{1}{\gamma_{el}} \cdot 0{,}016 \cdot \left(0{,}3^{\nu}\right) \cdot \left[\frac{\max(0{,}01;\omega')}{\max(0{,}01;\omega)} \cdot f_c\right]^{0{,}225} \cdot \left(\frac{L_v}{h}\right)^{0{,}35} \cdot 25^{\left(\alpha\rho_{sx}\frac{f_{yw}}{f_c}\right)} \cdot \left(1{,}25^{100 \cdot \rho_d}\right) \quad (9)$$

Similarly, the chord rotation at yielding $\theta_y$ is determined in accordance with the empirical relationship (10):

$$\theta_y = \left(\theta_{um} - \theta_{um}^{pl}\right) = \theta_{C,SLD}$$

$$\theta_{um}^{pl} = \frac{1}{\gamma_{el}} \cdot 0{,}0145 \cdot \left(0{,}25^{\nu}\right) \cdot \left[\frac{\max(0{,}01;\omega')}{\max(0{,}01;\omega)} \cdot f_c\right]^{0{,}35} \cdot f_c^{0{,}25} \cdot \left(\frac{L_v}{h}\right)^{0{,}35} \cdot 25^{\left(\alpha\rho_{sx}\frac{f_{yw}}{f_c}\right)} \cdot \left(1{,}275^{100\cdot\rho_d}\right) \quad (10)$$

where h is the depth of cross-section, $L_v$ is the ratio moment/shear at the end section, $\alpha$ is the confinement effectiveness factor, $\omega$/$\omega$' is the mechanical reinforcement ratio of the tension and compression, respectively, longitudinal reinforcement, $\nu$ is the dimensionless axial force, $\rho_d$ is the steel ratio of diagonal reinforcement, $f_c$ and $f_{wc}$ are the concrete compressive strength and the stirrup yield strength, respectively. On the one hand, the seismic demand is determined according to the well-known N2-Method [33]: Figure 7 depicts the main operations requested to calculate the "performance-point", whose abscissa is the displacement demand $D_{LS,i}$.

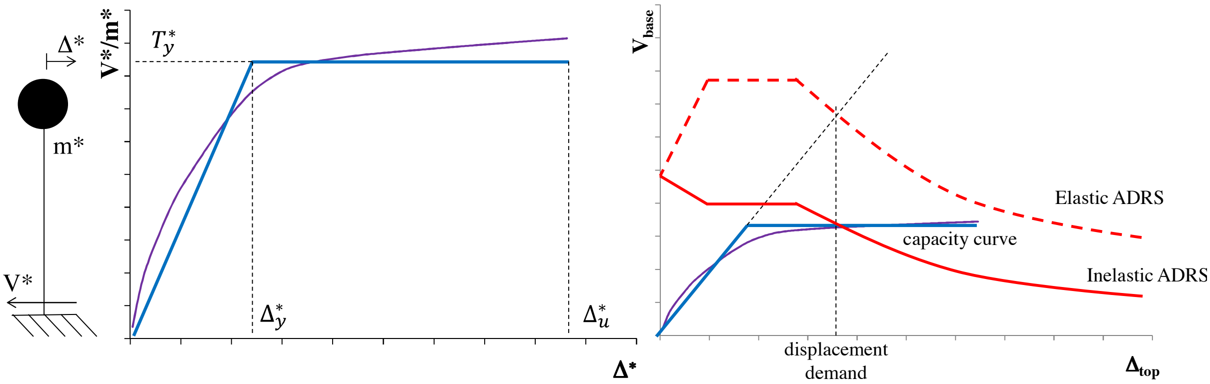



Figure 7: Evaluating seismic displacement demand via N2 Method

Moreover, the proposed heuristic strategy use the penalty function Φ in Eq. (5) with the aim to penalize unsuccessful solutions characterized by seismic demand higher than capacity. The penalty function depends on the maximum value of the capacity to demand ratio. The latter is evaluated through four Non Linear Static Analysis carried out for simulating the seismic response of the structure under consideration both in x and y direction (Table 1).

| | X+ | X- | Y+ | Y- | |
|---|---|---|---|---|---|
| **SLD** | $(D/C)_{SLD,x+}$ | $(D/C)_{SLD,x-}$ | $(D/C)_{SLD,y+}$ | $(D/C)_{SLD,y-}$ | $\max_i\left(\frac{D_{LS,i}}{C_{LS,i}}\right)$ |
| **SLV** | $(D/C)_{SLV,x+}$ | $(D/C)_{SLV,x-}$ | $(D/C)_{SLV,y+}$ | $(D/C)_{SLV,y-}$ | |

Table 1: Evaluating maximum value of limit state function

One of the major challenges in any application of penalty function concerns achieving an appropriate balance of the penalty value. Large penalty values discourage algorithm from exploring infeasible regions and the search is quickly moved inside the feasible region. On the other hand, low penalty values do not prohibit algorithm from searching infeasible regions most of the time. As a result of these findings, the authors propose a "death penalty functions" which immediately reject infeasible solutions [35]. The feasible region is shown graphically in Figure 8. In the domain of technically successful solutions, where no violation is found, a low penalty belonging to the linear branch is imposed on the objective function: it grows monotonically until the threshold value 1 above which the term Φ reaches an almost vertical asymptote.

On the contrary, if the maximum demand to capacity ratio is greater than unit, then the penalty cost becomes very high.

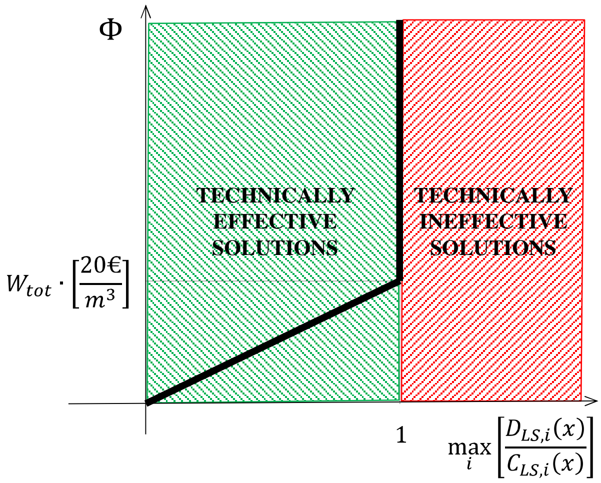



Figure 8: Penalty function

Based on typical retrofitting costs and their possible correlation with the demand to capacity ratio [36], the penalties solution is defined as represented in Figure 8, in which a unit cost of 20 €/m$^3$ is assumed for retrofitting [37]. This definition of penalty function aims at increasing the nominal cost for those solutions that do not meet the constraint condition in Eq. (2). Once the total cost and the limit state function $g_{LS,i}$ are evaluated for all $N_{ind}$ individuals of the population, for the minimization problem under consideration, it is necessary to map the objective function to fitness function form. Within the present work, the authors employ a proportional fitness assignment as transformation. The fitness $F(x_i)$ assigned to each solution "measures" the capability of the i-th individual to "compete" with the other ones of the same generation in achieving the objective of the optimisation problem under consideration. It is computed as the individual's raw performance, $f(x_i)$, relative to the whole population, according to the Eq. (11):

$$F(x_i) = \frac{\min_{i=1..N_{ind}} \left[f(x_i)\right]}{f(x_i)} \tag{11}$$

As can be easily understood from Eq. (11) it is always lower than the unit, which is determined for the individual characterised by the lower value of the objective function. In other words, the fitness function value corresponds to the number of offspring that an individual can expect to produce in the following generation.

## 2.5 Convergence test

Because GA is a stochastic search method, it is difficult to formally specify convergence criteria. As the fitness of a population may remain unchanged for a number of generations before a superior individual is found, the application of conventional termination criteria becomes problematic. A common practice is to terminate the GA after a predefined number of generations. In the present work, starting from the randomly generated initial population, the genetic algorithm evolves through three operators described below until the counter of population reaches a maximum number fixed to 150.

## 2.6 Selection

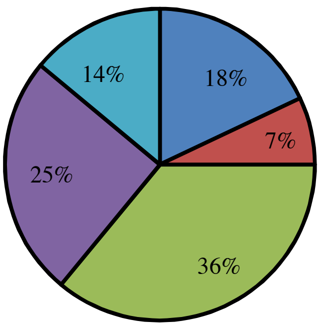
The selection operator selects individuals on the bases of their fitness with the aim of realising the "survival of the fittest" principle, which is the basis of the evolution theory [17]. This principle translates into discarding the chromosomes with the highest cost and into the exclusion of the solutions that don't comply with the seismic check. The essential idea is that individuals are picked out of the current population and, based on probabilistic assumptions, they are included in a mating pool. First, the $N_{ind}$ costs $f(x_i)$ and associated chromosomes are ranked from lowest cost to highest cost. Then, only the best are selected to continue, while the rest are deleted. The selection rate, $X_{rate}$, is the fraction of $N_{ind}$ that survives for the next step of mating. The number of chromosomes that are kept each generation is:

$$N_{keep} = X_{rate} \cdot N_{ind} \tag{12}$$

Of the $N_{ind}$ chromosomes in a generation, only the top $N_{keep}$ survive for mating, and the bottom $N_{ind}$ - $N_{keep}$ are discarded to make room for the new "offspring" solutions. Deciding how many chromosomes to keep is somewhat arbitrary. Keeping too many chromosomes allows bad performers a chance to contribute their traits to the next generation. For this reason, the authors keep 10% ($X_{rate}$ = 0.1) in the selection process. In the present example, $N_{ind}$ = 100. With a 10% selection rate, $N_{keep}$ = 10. Then, 10 individuals with the lowest cost f(x) survive to the next generation and become potential parents. The fitness $F(x_i)$ of each individual is employed in determining the "probability of survival" $p(x_i)$ of the i-th solution as follows:

$$p(x_i) = \frac{F(x_i)}{\sum_{i=1}^{N_{ind}} F(x_i)} \tag{13}$$

It is evident that, by definition, the values of $p(x_i)$ range between 0 and 1 and the sum of probabilities for each individual is equal to the unity. Under the algorithmic standpoint, $p(x_i)$ is the probability that the i-th individual is "selected" within the current generation to be part of the mating pool and, hence, "reproduce" itself into the following generation. The individuals featuring higher values of $p(x_i)$ have the higher probability to be selected as "parents" for generating "offsprings" in the following generation. In the present work, the selection procedure is implemented through the so-called "roulette-wheel" rule [38]. The circular sections within the roulette wheel are marked proportionally to the probability of survival of each individual (Figure 9): in the example, the individual #3 is the fittest individual and, hence, it corresponds to the wider circular sector, whereas individuals #2 is the least fit and, correspondingly, it has a smaller area.




| Individual | Chromosome | Probability | Cumulative P. |
|---|---|---|---|
| 1 | 00101011101011101011010 | 0.18 | 0.18 |
| 2 | 10101010101010101010011 | 0.07 | 0.25 |
| 3 | 00001011001001101001010 | 0.36 | 0.61 |
| 4 | 11101011001011101011111 | 0.25 | 0.86 |
| 5 | 11001011101010101111110 | 0.14 | 1.00 |

Figure 9: Roulette wheel rule in the selection operator

If $N_{ind}$ - $N_{keep}$ individuals have to be selected, $N_{ind}$ - $N_{keep}$ random numbers are generated in the [0,1] range and the individual whose circular segment includes the random number is actually selected for reproduction.

## 2.7 Crossover

The crossover operator aims at creating different individuals in the following generations by combining material extracted from individuals of the previous generation. It is worth highlighting that, in order to preserve some of the fitter individuals, not all the strings in the mating pool are used in crossover. Hence, a threshold combination probability $p_c$ is defined: only $N_{ind}*p_c$ individuals of the population are used in the crossover operation, whereas $N_{ind}*(1-p_c)$ remain unchanged and preserve themselves for following population. Typically, the probability for crossover ranges from 0.6 to 0.95. Because the selection rate $X_{rate}$ is equals to 0.1, the employed probability $p_c$ = 1- $X_{rate}$ results 0.9. Once $0.5*N_{ind}*p_c$ pairs of individuals are selected according to their fitness $F(x_i)$, crossover operator "mixes" the segments of each pair and the information contained between these segments is exchanged. As well as its ideal counterpart in nature, the two genotype strings of the individual participating in the crossover are selected as "parents" and the resulting strings are defined as "children". Figure 10 shows the working principle of the crossover operator: the method used in the present GA is the "multi-point" crossover.

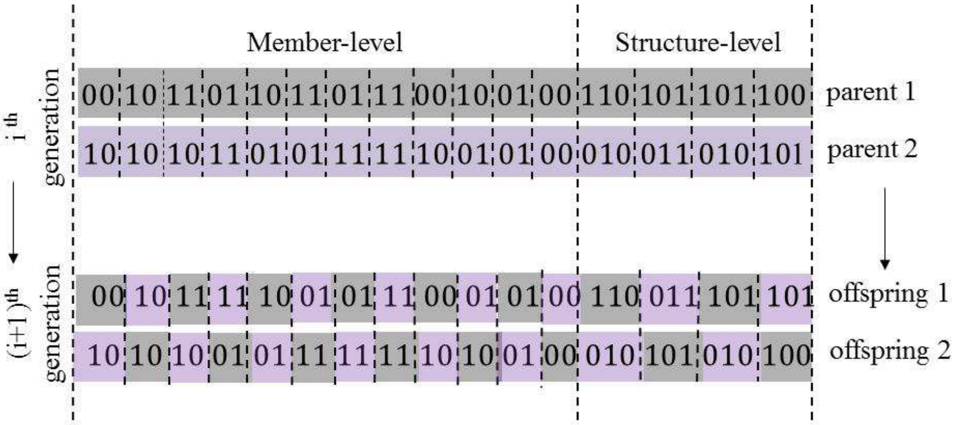



Figure 10: Crossover operator applied to couples of "parent" individuals

In the example shown in Figure 10 the segments are delimited with dashed lines and crossover operator is applied column-by column and bay-by-bay, respectively in the first and second part of the genotype. The influence of the number of crossover points on the resulting efficiency is a key aspect, whose discussion is outside the scope of this paper. Readers may refer to Spears [39]-[40] for further details.

## 2.8 Mutation

In natural evolution, mutation is a random process through which one allele of a gene is replaced by another to produce a new genetic structure. Mutation is the second way a GA explores the solution space. This operator provides a guarantee that the probability of searching any given string will never be zero and acting as a safety net to recover good genetic material that may be lost through the action of selection and crossover [41]. Under the operational standpoint, it introduces diversity not in the original population whenever the population tends to become homogeneous due to repeated use of reproduction and crossover operators. It also tends to distract the GA from converging too fast at local optima before sampling the entire solution space. The need for mutation is in fact to create a point in the neighborhood of the current point, thereby achieving a local search around the current solution. Even though better strings are not guaranteed and/or tested while creating them, it is expected that if weak (or bad) strings are created they will be eliminated by the reproduction operator in the next generation and if good strings are created, they will be increasingly emphasized. Mutation operates at the bit level of the genotype code. To this end, the so-called "mutation probability" $p_m$ is defined as the probability of a single bit to be "inverted" in each offspring individual when the bits are being copied from the current string to the new string. As well as in nature, this probability is usually assumed as a small value, typically in the range 0.001 and 0.01. In the current implementation this parameter is set equal to 0.01. A high value is selected in order to enhance the so-called "ergodicity" of the algorithm, namely its capability to explore the parametric field of the problem under consideration. A coin toss mechanism is employed: if random number between zero and one is less than the mutation probability, then the bit is inverted, so zero becomes one and vice versa (Figure 11). If $N_{bits}$ is the number of bits totally allocated in each string, mutation points are randomly selected from the $(N_{ind}- N_{keep})*N_{bits}$ total number of bits in the population matrix. It is possible that a given binary string may be mutated at more than one point. Figure 11 shows an example of mutation occurring in a generic individual.

Mutation point

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Original string | 0 | 1 | 1 | 0 | **0** | 1 | **1** | 0 | 1 |
| Random number | 0.234 | 0.703 | 0.167 | 0.345 | **0.008** | 0.806 | **0.005** | 0.624 | 0.411 |
| Mutated string | 0 | 1 | 1 | 0 | **1** | 1 | **0** | 0 | 1 |

Figure 11: Examples of mutation points

Finally, the application of selection, crossover and mutation operators on the current population creates a new population. This new population is used to generate subsequent populations and so on, yielding solutions that are closer to the optimum solution. The values of the objective function of the individuals of the new population are again determined by decoding the strings. This completes one cycle of genetic algorithm called a generation. In each generation if the solution is improved, it is stored as the best solution. The process described is repeated till convergence.

## 3 PROPOSED APPLICATION

The working detail and the potential of the presented optimisation procedure are shown in an application: Figure 12 depicts the in-plane configuration of the simple stucture considered herein.

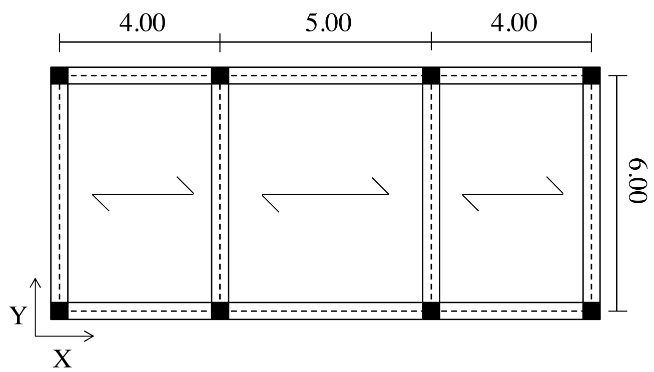



Figure 12: In-plain configuration of the considered structure

A simple 3D three-storey RC frame with three bays in x-direction and one bay along y axis, considered in a construction site caracterized by higl level of seismicity ($PGA_{LD}$ = 0.12g; $PGA_{LV}$ = 0.35g) is taken as preliminary case for applying the proposed retrofitting strategy. The cross sectional area of beams and columns is 30x40 cm$^2$ and 30x30 cm$^2$, respectively. Foundation is not simulated and fixed supports are considered. Rigid joints are used for simulating beam-to-column connections. The total number of design variables considered in the test example (Figure 13) is 34: 3 x 8 member-level variables (8 columns for each floor), plus 10 structure-level variables (6 bays in x-direction and 4 along the y axes).

$$\underbrace{0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0}_{N_{col} = 24}\;\underbrace{0\,0\,0\,0\,0\,0\,0\,0\,0\,0}_{N_{beam} = 10}$$

Figure 13: As-built genotype of the considered structure

Each generation includes 100 individuals and the genetic algorithm stops either 150 after generations or if the objective function keeps unchanged for 20 consecutive generations. Both the LS of Life Safety (SLV) and Damage Limitation (SLD) are considered and, hence, demand and capacity are evaluated in both LSs. A bilinear stress-strain curve with Young modulus equal to 210 GPa and yield stress $F_y$= 220 MPa is adopted for describing the elasto-plastic behaviour of steel. The Kent-Scott-Park model [30] with degraded linear unloading/reloading stiffness and no tensile strength is used in order to describe the constitutive law of existing concrete. The effects of FRP confinement result in increasing the ductility of concrete (Figure 4), according to the afore mentioned model. The live load is taken as Q=2.00 kN/m$^2$ and the permanent load is G=5.00 kN/m$^2$. The gravity loads are contributed from an effective area of 78 m$^2$. A Finite Element model is built in OpenSEES [31] for simulating the seismic response of the structure under consideration. The well-known fiber approach is used to account for material non-linearity by means of the so-called "nonlinear beam-column elements". Five integration sections, located at the Gauss-Lobatto quadrature points, are considered for each beam-column element. Each section is divided into a number of fibers. Similar elements are also employed for modelling the concentric steel bracings in structure-level intervention. An accidental eccentricity is assigned in the middle point according to EN 1993-1-1 [32] in order to obtain the buckling of the bracing in compression. Figure 14 depicts the outcome of the proposed algorithm.

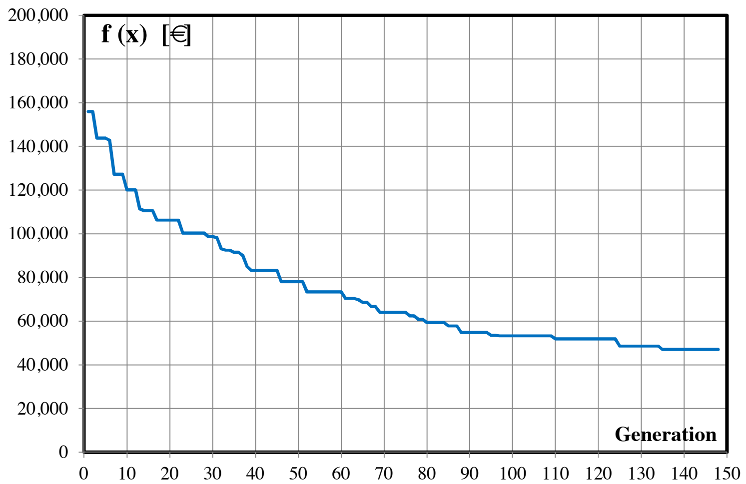



Figure 14: Convergence history: objective function vs generation

The objective function starts from a cost of 156’481 € and decreases progressively. As expected, the curve shows a very steep slope over the first generations and a slower and slower reduction, often characterised by a staircase shape, towards the final convergence, which is supposed to be achieved after 150 generations, as no further relevant improvement is observed in f(x) over the last 15 iterations. Finally, the optimal phenotype in the 150th population (Figure 15) has a cost of 43’703 €, the lowest of all previously processed solutions. In the example proposed herein, the optimal solution came up to consist of a concentric steel bracing (realised in the two plain frame along the y-direction and one in the x direction) and no local FRP interventions are actually required. The optimal section of steel members for the first bracing level is a HE 100 A.

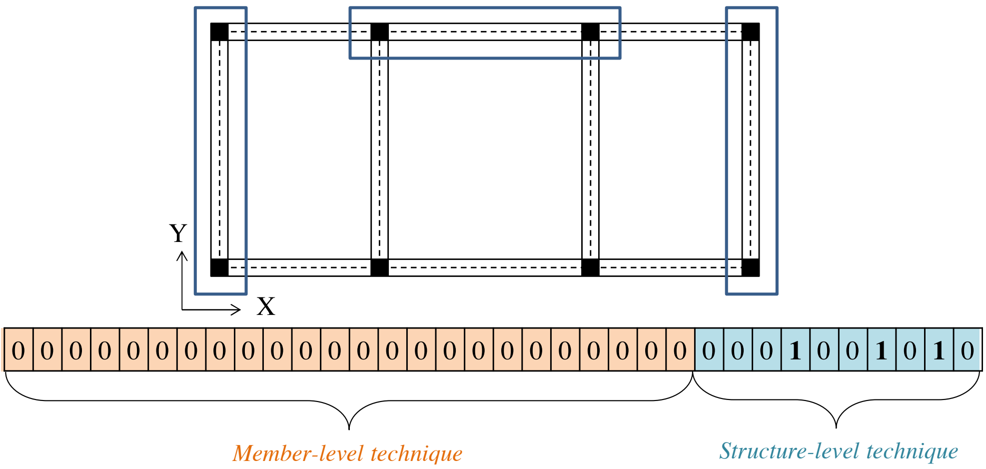



Figure 15: Optimal phenotype and corresponding technical solution

## 4 CONCLUSIONS

This paper outlined the formulation of a rational procedure intended at optimising seismic retrofitting of existing structures by combining member- and structure-level techniques. The proposed genetic algorithm has the potential to support engineering judgement (being far from the ambition to rule it out) in determining the “fittest” seismic retrofitting solution for RC frames. The proposed procedure demonstrates its potential in formally. In fact, the considered application demonstrates that the implemented genetic algorithm is capable of finding a solution characterised by a cost significantly lower than the initially assumed trial solution. Nevertheless, the implementation of this numerical model is still under development: the work ahead should be primarily intended at including the aspects that are not taken into account yet (i.e. multi-criteria objective function, among the others). Moreover, it should also aim at enhancing the computational efficiency of the computer procedure, whose computational cost is one of the main critical issues to be duly addressed for the proposed method be actually feasible in real applications.

## REFERENCES


[1] European Standard EN (Eurocode 8) 1998-1:2005, Design of structures for earthquake resistance, Part 1: General rules, seismic actions and rules for buildings. European Committee for Standardization (CEN), Bruxelles, 2005.

[2] fib 2003. Seismic assessment and retrofit of reinforced concrete buildings. Bulletin No. 24, 138, ISBN: 978-2-88394-054-3.

[3] G.E. Thermou, A.S. Elnashai, Seismic retrofit schemes for RC structures and local–global consequences. *Progress in Structural Engineering and Materials*, 8: 1–15, 2006.

[4] M.G. Ireland, S. Pampanin, D.K. Bull, Experimental Investigations of a Selective Weakening Approach for the Seismic Retrofit of R.C. Walls. Palmerston North (New Zealand), 30 March- 1 April, 2007.

[5] H. Kaplan, S. Yilmaz, N.Cetinkaya, E. Atimtay, Seismic strengthening of RC structures with exterior shear walls, Sadhana, Indian Academy of Sciences, 36 (1): 17–34, 2011.

[6] fib 2006. Retrofitting of concrete structures by externally bonded FRPs, with emphasis on seismic applications, Bulletin No. 35, 220, ISBN: 978-2-88394-075-8.

[7] M. Rodriguez, R. Park, Repair and Strengthening of Reinforced Concrete Buildings for Seismic Resistance. *Earthquake Spectra*, 7(3): 817-841, 1991.

[8] E. Martinelli, C. Lima, C. Faella, Towards a rational strategy for seismic retrofitting of RC frames by combining member- and structure-level techniques. SMAR2015 – *Third Conference on Smart Monitoring, Assessment and Rehabilitation of Civil Structures,* Antalya (Turkey), 6-9 September, 2015.

[9] FEMA 156, Typical Costs for Seismic Rehabilitation of Existing Buildings. Vol 1: Summary (Second Edition), 1994.

[10] G. Quan, M. Barbato, J.P. Conte, P.E. Gill, F. McKenna, OpenSees-SNOP Framework for Finite-Element-Based Optimisation of Structural and Geotechnical Systems. *Journal of Structural Engineering*, 138(6): 822-834, 2012.

[11] S. Gharehbaghia, M.J. Fadaee, Design Optimisation of RC Frames under Earthquake Loads, Int. J. Optim. Civil Eng, vol. 2 n. 4, , pp. 459 – 477, 2012.

[12] A. Guerra, D.P. Kiousis, Design optimisation of reinforced concrete structures, *Computers and Concrete*, 3(5), 313-334, 2006.

[13] A. Kunisue, N. Koshika, Y. Kurokawa, N. Suzuki, J. Agami, M. Sakamoto,. Retrofitting method of existing rein-forced concrete buildings using elastoplastic steel dampers. *Proceedings of the twelfth World Conference on Earth-quake Engineering (12WCEE),* Paper 0648, 2000.

[14] G.E. Thermou, A.S. Elnashai, Seismic retrofit schemes for RC structures and local–global consequences, *Progress in Structural Engineering and Materials*, 8:1–15, 2006.

[15] S. Bordea, D. Dubina, Retrofitting/upgrading of reinforced concrete elements with buckling restrained bracing elements. *Proceedings of the 11th WSEAS International Conference on Sustainability in Science Engineering*, Timisoara (Romania), 27-29 May 2009, 407-402, 2009.

[16] N. Caterino, I. Iervolino, G. Manfredi, E. Cosenza, Comparative Analysis of Multi-Criteria Decision-Making Methods for Seismic Structural Retrofitting, *Computer-Aided Civil and Infrastructure Engineering*, 24(6): 432–445, 2009.

[17] C. Darwin, *On the Origin of Species by means of Natural Selection or the Preservation of Favored Races in the Struggle for Life* (The Origin of Species), London: Murray, 1859.

[18] M. Papadrakakis, N.D. Lagaros, Soft computing methodologies for structural optimisation. *Applied Soft Computing*, 3: 283-300, 2002.

[19] M. Fragiadakis, N.D. Lagaros, M. Papadrakakis, Performance-based multi-objective optimum design of steel structures considering life-cycle cost. *Structural and Multidisciplinary Optimisation*, 32: 1–11, 2006

[20] N.D. Lagaros, Y. Tsompanakis, M. Fragiadakis, M. Papadrakakis, Soft Computing Techniques in Probabilistic Seismic Analysis of Structures. *Intelligent Computational Paradigms in Earthquake Engineering* edited by N. Lagaros and Y. Tsompanakis, Idea Group Publishing ltd, Hershey, PA (USA), 2007.

[21] M. Fragiadakis, M. Papadrakakis, Performance-based optimum seismic design of Reinforced Concrete Structures. *Earthquake Engineering and Structural Dynamics*, 37(6): 825–844, 2008.

[22] M.II.TT. 2008, Norme tecniche per le costruzioni - D.M. 14 gennaio 2008 (in Italian).

[23] C.A. Coello, Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art. *Computer Methods in Applied Mechanics and Engineering*, 191, 1245-1287, 2002.

[24] D.E. Goldberg, B. Korb, K. Deb, Messy genetic algorithms: motivation, analysis, and first results. *Complex Systems*, 3(5), 493-530, 1989.

[25] F. Biondini, Optimal limit states design of concrete structures using genetic algorithms. *Studies & Researches* 20, 1999.

[26] W.M. Jenkins, Towards Structural Optimisation via the Genetic Algorithm. *Computers & Structures*, 40(5): 1321–1327, 1991.

[27] W.M. Jenkins, Structural Optimisation with the Genetic Algorithm. *The Structural Engineer*, 69(24): 418-422, December, 1991.

[28] G.R. Reeves, Modern Heuristic Techniques for Combinatorial Problems. *John Wiley & Sons*, Inc. 1993.

[29] P. Hajela, Stochastic Search in Structural Optimisation: Genetic Algorithms and Simulated Annealing. Chapter 22: 611-635, 1992.

[30] D. C. Kent, R. Park, Flexural members with confined concrete. *Journal of the Structural Division*, 97(7): 1969-1990, 1971.

[31] S. Mazzoni, F. McKenna, M.H. Scott, G.L. Fenves, OpenSEES command language manual. *Pacific Earthquake Engineering Research* (PEER), 2006.

[32] European Standard EN (Eurocode 3) 1993-1-1:2005, Design of steel structures, Part 1-1: General rules and rules for buildings. European Committee for Standardization (CEN), Bruxelles, 2005.

[33] European Standard EN (Eurocode 8) 1998-3-2005, Design of structures for earthquake resistance, Part 3: Assessment and retrofitting of Buildings, European Committee for Standardization (CEN), Bruxelles, 2005.

[34] P. Fajfar, Capacity spectrum method based on inelastic demand spectra. *Earthquake Engineering & Structural Dynamics*, 28(9): 979-993, 1999.

[35] H.P. Schwefel, Numerical optimisation of computer models. *John Wiley & Sons*, Chichester, UK, 1981.

[36] Ordinanza P.C.M. n° 3362 del 08/07/2004 (in Italian) "Modalità di attivazione del Fondo per interventi straordinari della presidenza del Consiglio dei Ministri, istituito ai sensi dell'Art. 32 bis del decreto legge 30 settembre 2003, n° 269, convertito con modificazioni dalla legge 24 novembre 2003, n° 326".

[37] J.T. Richardson, M.R. Palmer, G.E. Liepins, M.R. Hilliard, Some guidelines for genetic algorithms with penalty functions. In J. D. Schaffer (Ed.), *Proceedings of the Third International Conference on Genetic Algorithms* (ICGA'89), Fairfax, VA, USA, 191-197, 1989.

[38] A. Lipowski, D. Lipowska, Roulette-wheel selection via stochastic acceptance. *Physica A: Statistical Mechanics and its Applications,* 6: 2193–2196, 2012.

[39] W.M. Spears, Crossover or mutation? *Proceedings of the Foundations of Genetic Algorithms Workshop*, Vail, Colorado: Morgan Kaufmann, 1992.

[40] W.M. Spears, Adapting crossover in a genetic algorithm. *Naval Research Laboratory AI Center Report* AIC-92-025. Washington, DC 20375 USA, 1992.

[41] D.E. Goldberg, Genetic Algorithms in Search, Optimisation and Machine Learning, *Addison Wesley Publishing Company*, January 1989.