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FAST KINEMATIC LIMIT ANALYSIS OF MASONRY WALLS WITH OUT-OF-PLANE LOADING

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Abstract. *Masonry structures represent one of the commonest structural typologies for buildings worldwide. In particular, masonry walls constitute, by far, the prevailing elementary structural unit in the majority of masonry constructions both modern and historical. This contribution validates an approach proposed by the authors for the limit analysis masonry vaults, now extended to a new general procedure for the structural assessment of masonry walls with out-of-plane loading based on an upper bound formulation. A given masonry panel of arbitrary form possibly with openings is described through its NURBS (Non-Uniform Rational B-Spline) parametric representation in the three-dimensional Euclidean space. An initial set of rigid elements subdividing the original wall geometry is identified through the definition of a suitable lattice of nodes. An upper-bound limit analysis formulation, taking into account the main characteristics of masonry material through homogenization, is deduced where internal dissipation is allowed along element edges only. As shown in a number of examples, a good estimate of the collapse load is obtained, provided that the initial net of yield lines is suitably adjusted by means of a Genetic Algorithm (GA).*

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1 INTRODUCTION

Masonry walls are widely employed both as principal structural members in masonry buildings (see [1]) and as non-structural elements such as infills and partitions (see [2,3]). During earthquakes, masonry walls undergo both in-plane and out-of-plane loading (see, for instance, [4]). In particular, the structural assessment of the ultimate load bearing capacity of out-of-plane loaded masonry walls is a relevant issue for all professionals involved in the rehabilitation and seismic protection of historical masonry constructions (see [5]), which can also rely on innovative techniques ([6–9]).

Since the 70's it is known from experimental tests on laterally loaded brick masonry walls, that failure occurs along a well-defined pattern of lines (see e.g. [10]). This evidence inspired approximate analytical solutions based on both the fracture line theory [11] and the yield line theory [12], which in fact can be considered an application of the kinematic theorem of limit analysis. Despite several approximations and the fact that masonry behavior is not rigid-plastic, limit analysis is still a very reliable computational tool to assess the load bearing capacity of masonry walls (see, e.g., [13]).

The present contribution is aimed at validating through simulations a new adaptive NURBS-based approach to homogenized upper-bound limit analysis of out-of-plane loaded masonry walls presented in [14]. A given masonry wall, with holes having arbitrary shape, is modeled as a rigid block assembly, where dissipation is allowed only along elements edges. Both in- and out-of-plane failure mechanical parameters are obtained through an homogenization procedure, once that a suitable elementary cell is defined.

NURBS (i.e. Non-Rational Uniform Bi-Spline) are special approximating base functions widely used in the field of 3D modeling [15] for their ability of approximating the actual geometry in an extremely accurate way. Recently, some of the Authors have introduced the idea of using NURBS functions as the basis for the limit analysis of masonry arches and vaults [16–22]. By using a suitable genetic algorithm, a mesh of the given surface, usually composed of a relatively small number of NURBS elements but still providing an exact representation of the original surface, can be created. Each element of the mesh is idealized as a rigid body.

Starting from the obtained rigid bodies assembly, an upper bound limit analysis problem with very few optimization variables can be devised. The main aspects of masonry material (i.e. negligible tensile strength, good compressive strength and orthotropy at failure due to bricks arrangement) are taken into account through an homogenization approach based on a Method of Cells-type approach, described in [23].

Due to the very limited number of rigid elements used, the quality of the computed failure mechanism depends on the shape and position of the interfaces, where dissipation is allowed. Mesh adjustments are therefore needed, but the difficulties connected with the utilization of Sequential Linear Programming [24] can be here easily circumvented by adopting a simple meta-heuristic approach of mesh adjustment (i.e. a Genetic Algorithm GA equipped with non-standard optimization tools, see [25,26]). In the GA-NURBS approach proposed in [14], each individual of a given population is represented by a possible rigid body disposition. Thanks to the extremely reduced number of NURBS elements used (and hence the number of variables of the LP problem), the computational effort required at each iteration is almost negligible. It should be noticed that the knowledge of the actual failure mechanism is not required in advance. Furthermore, since NURBS represent a standard in the field of 3D modeling, the proposed method could easily be integrated within existing commercial CAD software packages, which are popular in the community of practitioners.

In this contribution, a number of technically meaningful examples are shown to validate the GA-NURBS approach introduced in [14].

2 NUMERICAL EXAMPLES

For the sake of brevity, we refer to [14] for the mathematical details on the adaptive homogenized NURBS-based upper-bound limit analysis formulation adopted. In this Section we propose a number of numerical examples specifically chosen to validate the GA-NURBS approach. The first example is taken from experimental literature ([27]), whereas the second and the third are taken from existing masonry churches hit by recent earthquakes ([28,29]). Mechanical properties assumed for masonry material in the three examples are summarized in Table 1.

Case study	Masonry material parameters			Specific weight [kN/m^3]
	Compressive strength (f_k) [MPa]	Tensile strength (f_t) [MPa]	Shear strength (f_{vk0}) [MPa]	
SB02 [27]	8.0	0.32	0.32	0.0
S. Pietro in Coppito, façade [28]	2.0	0.05	0.30	18.5
S. Giacomo Maggiore in Polesine, apse [29]	2.4	0.20	0.20	18.0

Table 1: Material parameters used in the simulations.

2.1 Panel with a rectangular opening in two way bending

As a first example, the panel SB02 with a rectangular opening experimentally tested in [27] is analyzed. The panel has dimensions 5615x2475x102 mm and was built in stretcher bond between two stiff abutments with the vertical edges simply supported and the top edge free. The panel was loaded by air bags until failure with increasing out-of-plane pressure *p*. During the tests, the air pressure and the displacement *d* for the middle point of the free edge were monitored. The rectangular opening has dimensions 2260x1125 mm. Masonry parameters adopted are reported in Table 1 and a null value of specific weight is assumed to replicate experimental condition.

The initial NURBS mesh of the panel surface is composed of sixteen quadrangular elements, obtained by subdividing the parameters space starting from a 5x5 lattice of nodes. Four nodes, corresponding to the four external vertexes of the wall, are fixed. The genetic algorithm allows evaluating the optimal position of the remaining twenty-one nodes, in order to minimize the collapse load multiplier and therefore obtain the actual failure mechanism. The position of every node is governed by two parameters, except for edge-nodes that are ruled by a single parameter. Thus, the general problem so, is governed by a total of thirty parameters. It must be observed that, for the specific load configuration here considered, symmetry allows to reduce the number of governing parameters to fourteen. In the genetic algorithm an initial population of 20 individuals have been chosen, each individual being a fourteen-element vector. A collapse load of 2.19 kN/m2 has been obtained.

Fig. 1(a) show the initial NURBS mesh of the masonry wall, whereas Fig. 1(b) depicts the final collapse mechanism. Good agreement can be observed by comparing the computed results using the proposed GA-NURBS procedure with the outcomes of both original experiments and different numerical procedures found in literature (i.e. [30,31]). In Fig. 1(c), the failure mechanism computed in [30] through a homogenized finite-element limit analysis approach is shown whereas in Fig. 1(d) experimental and numerical out-of-plane force-displacement curves referred to Specimen SB02 are reported.

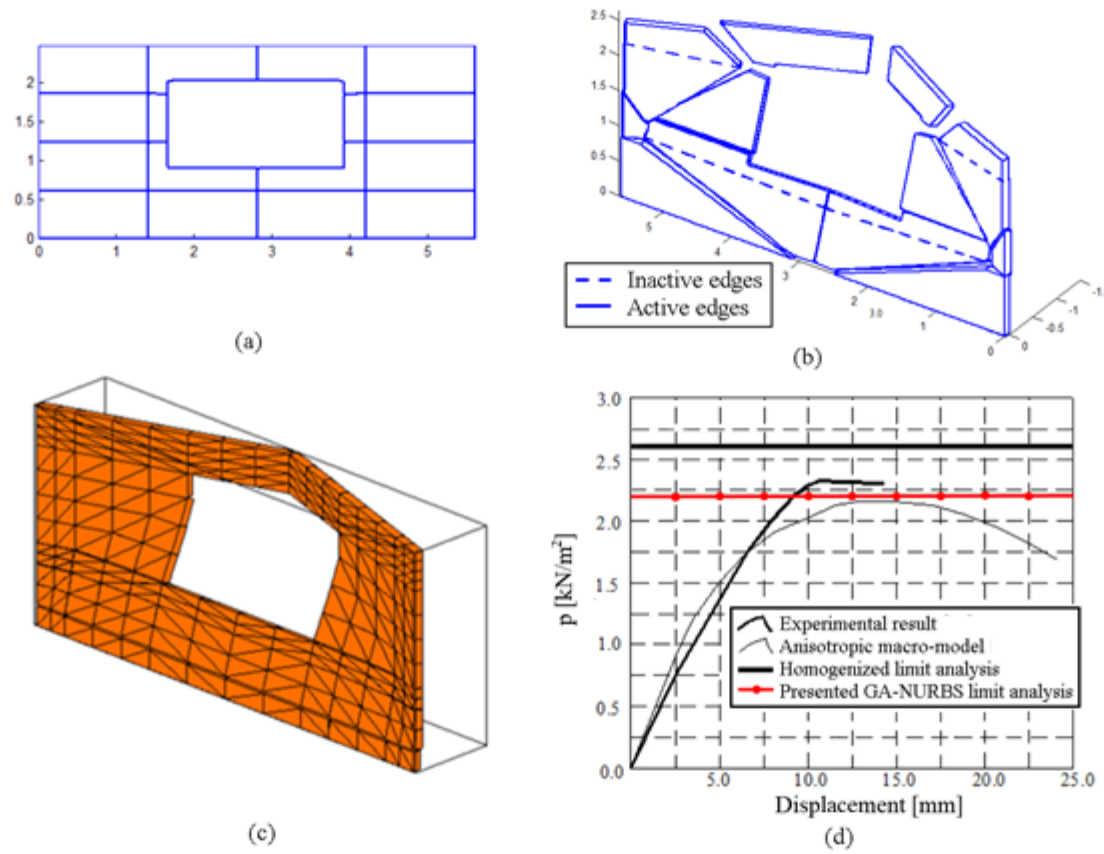

Figure 1: SB02 masonry panel with a rectangular opening tested in [27]. (a) Initial mesh. (b) Failure mechanism through the proposed GA-NURBS procedure. (c) Failure mechanism obtained by FEM homogenized limit analysis [30]. (f) Comparison in terms of collapse load with experiments and other numerical approaches [30,31].

2.2 San Pietro in Coppito façade

The second example considered relates to the masonry façade of the S. Pietro in Coppito church [28]. The church of San Pietro di Coppito in L'Aquila, damaged by the April 6, 2009 earthquake, dates to the mid-13th century; it was progressively transformed up to the 19th century and then restored to its initial appearance in 1969–1972. The church features a central nave and a single aisle, and is therefore highly asymmetrical along its length, the transept is split into two parts of different shapes and heights. The most significant and visible damage the church incurred during the earthquake involves the façade and the bell tower; the top portion of the façade is unconstrained, since the back portion of the church roof does not reach the same height. The façade of the church and its geometry, characterized by complex openings of circular and semicircular shape, are depicted in Fig. 2(a-b).

A NURBS description of the surface comes particularly handy in the present case, since NURBS allows for an exact representation of openings of arbitrarily complex geometry. Traversal walls have been considered as well. All the walls are assumed with fixed constraints at the base. Material parameters are reported in Table 1. Two load conditions have been applied: a triangular out-of-plane pressure linearly increasing with height (Fig. 2(a)), and an asymmetric linear out-of-plane pressure distribution increasing with both height and length (Fig. 2(b)).

The initial NURBS mesh of the façade is composed of sixteen quadrangular elements, obtained by subdividing the parameters space starting from a 5x5 lattice of nodes. Lateral walls have been subdivided into quadrangular elements starting from a 3x4 lattice of nodes.

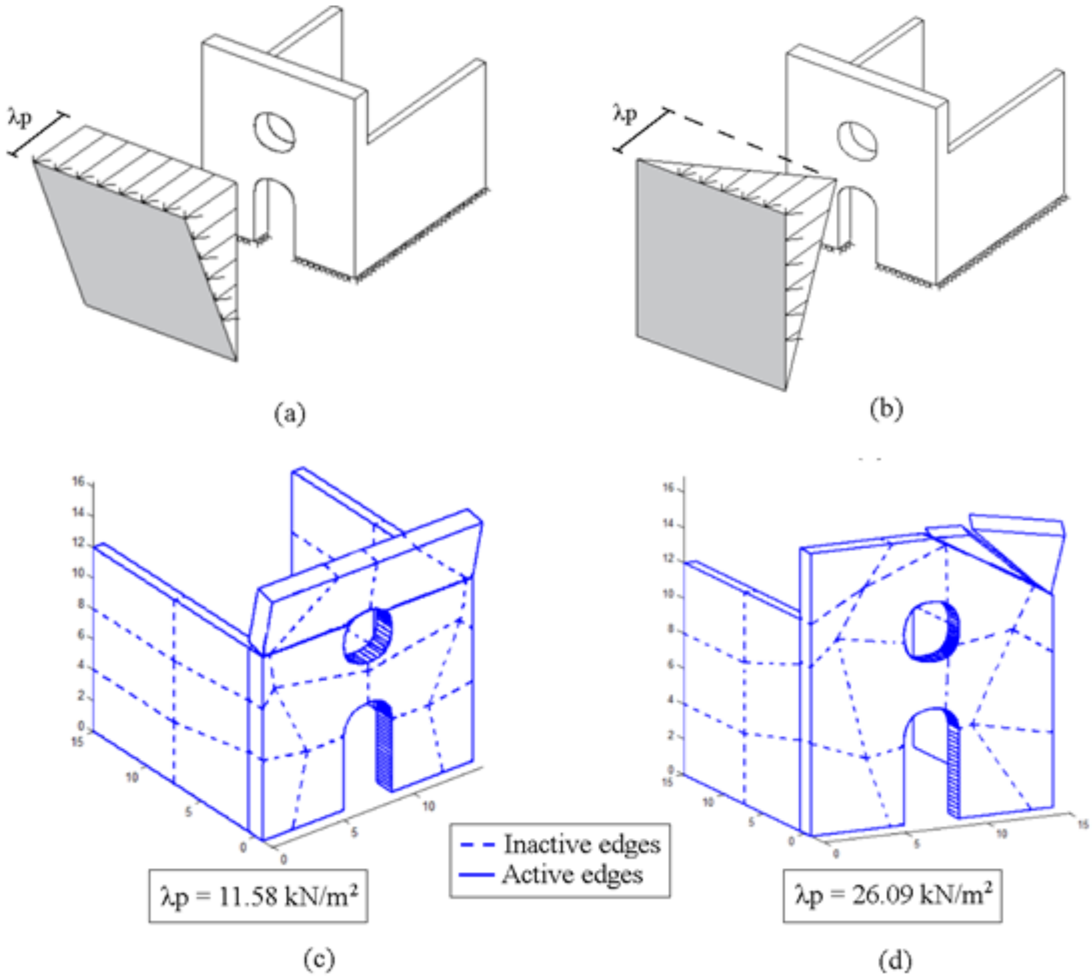

Figure 2: S. Pietro in Coppito façade. (a-b) Geometry and two load conditions. (c) Failure mechanism obtained through GA-NURBS procedure for the first load condition. (d) Failure mechanism obtained through GA-NURBS procedure for the second load condition.

The position of every node is governed by two parameters, except for edge-nodes that are ruled by a single parameter. Therefore, the general problem is governed by a total of thirty parameters. Fig. 2(c-d) show the collapse mechanism obtained through the proposed GA-NURBS approach.

Finally, it should be noticed that there is a good agreement between the failure mechanism shown in 2(d) and the actual collapse mechanism observed after L'Aquila 2009 earthquake and depicted in [28], i.e. the local collapse of the upper left corner of façade.

2.3 S. Giacomo Maggiore in Polesine apse

The last example analyzes to the masonry apse of the S. Giacomo Maggiore in Polesine (Mantua) church [29]. The church of S. Giacomo Maggiore in Polesine was damaged by the May 2012 Emilia seismic swarm and dates back to the 14th century; it was progressively transformed up to the 18th century when it acquired the present form. The church features a central nave. The most significant and visible damage the church incurred during the earthquake involves the lateral walls and the bell tower; the apse did not undergo significant damage, nevertheless it has been chosen to demonstrate the capability of the proposed NURBS-based approach to deal with very complex (even curved) geometries. The semicircular apse of the church, with rectangular openings, and its geometry, is depicted in Fig. 3(a).

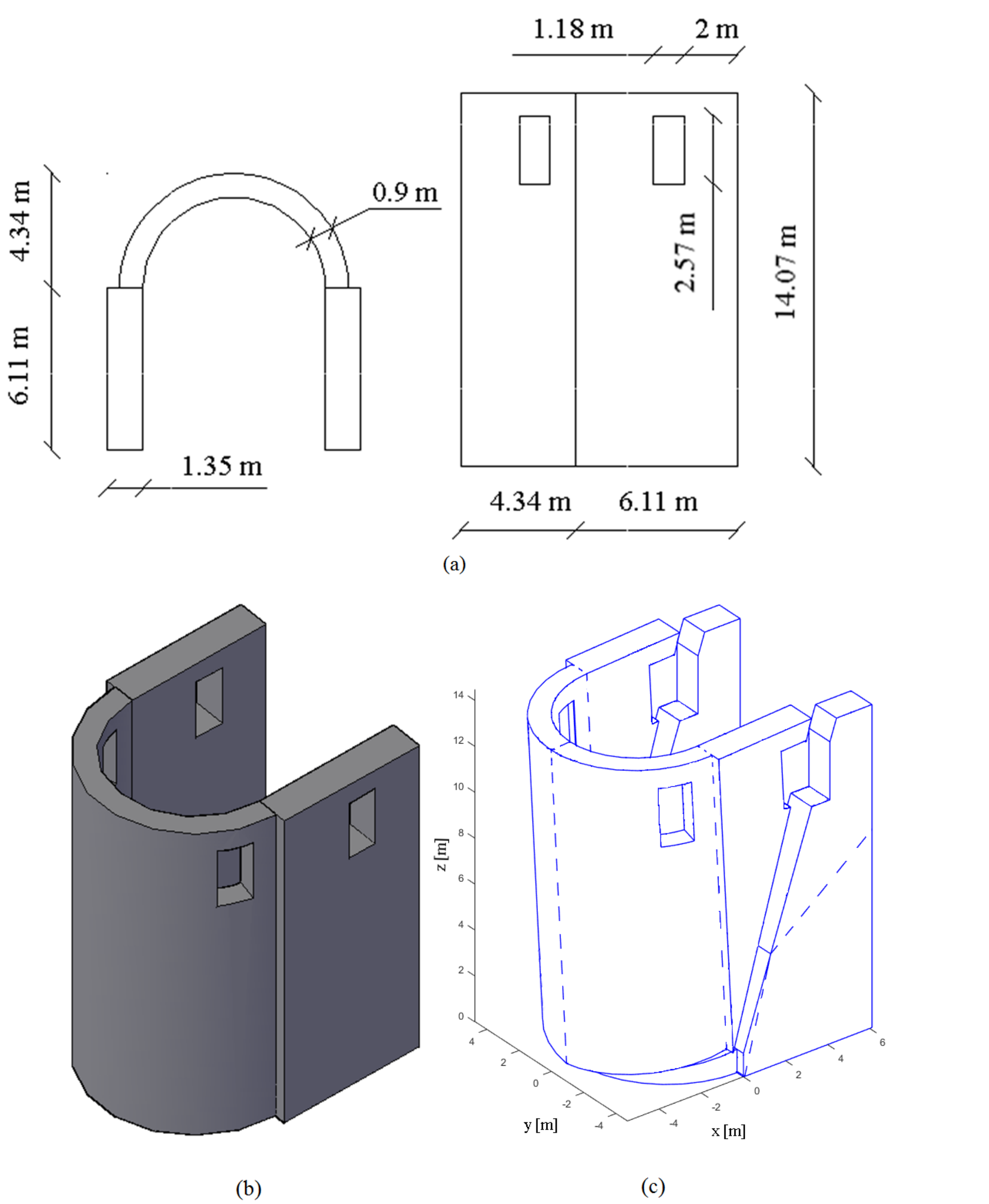

Figure 3: S. Giacomo in Polesine apse. (a) Geometry. (b) NURBS geometric model. (c) Failure mechanism obtained through GA-NURBS procedure. (d) Failure mechanism obtained through GA-NURBS procedure.

Again, the NURBS description of the surface (see Fig. 3(b)) is particularly helpful, since NURBS allows for an exact representation of the semicircular wall and openings. The walls are assumed with fixed constraints at the base. Material parameters are reported in Table 1. A uniform unitary horizontal acceleration has been applied as a live load.

The initial NURBS mesh of the apse is composed by three NURBS patches (a semicircular and two planar lateral walls) subdivided in a 4x4 lattice of nodes each. The position of every node is governed by two parameters, except for edge-nodes that are ruled by a single parameter. Symmetry and compatibility conditions between patches reduce the total number of parameters governing the problem to seven.

Fig. 3(c) shows the collapse mechanism obtained through the proposed GA-NURBS approach. The failure mechanism found is compatible with many observations of damaged apses in masonry churches hit by earthquakes.

3 CONCLUSIONS

Through a number of technically meaningful examples and comparisons with both numerical and experimental results, the GA-NURBS approach presented in [14] has shown to be able to well predicting the load bearing capacity and failure mechanism of out-of-plane loaded masonry walls with complex geometries. The strength of the method lies in the fact that it is possible to get accurate results by using very few elements. Such peculiarity allows to maximize both accuracy and computational speed.

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