Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2017) 4654-4663

ECCOMAS
Proceedia

COMPDYN 2017
6th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15–17 June 2017

IMPEDANCE FUNCTIONS OF ADJACENT STRIP FOUNDATIONS

Vasiliki G. Terzi[1]

[1] Laboratory teaching staff
Laboratory for Statics and Dynamics, Civil Engineering Department, Aristotle University, Greece
e-mail: terziv@civil.auth.gr

Keywords: Soil-structure interaction, Strip footings, Finite Element Method.

Abstract. *The interaction between two adjacent strip footings resting on an elastic isotropic half-space is studied. The finite element method is used in order to investigate the response of the footings under vertical, lateral and rocking loading. The accuracy of the finite element model is verified by a comparison study with the rigorous results from the boundary element method concerning a single strip footing. Impedance functions are presented over a wide range of frequencies for typical values of the distance ratio S/L, where S is the distance between the footings and L is the width of each footing. The condition under which the interaction between adjacent strip footings should be taken into consideration is further discussed.*

Peer-review under responsibility of the organizing committee of COMPDYN 2017.
doi: 10.7712/120117.5751.17302

1 INTRODUCTION

The assumption of a fixed based structure can be considered appropriate only in limited cases of a hard substratum site. Therefore, the soil-structure interaction (SSI) phenomenon must be taken into account during dynamic structural analysis. However, in practical situations, due to space limitations, structures are usually placed closed to each. Hence, in order to evaluate their dynamic characteristics, it is necessary to investigate not only the interaction between the foundation and the soil medium, but also the interaction between adjacent foundations through the soil [1]. The presence of adjacent foundations may affect the others further in a way such that each foundation which diffracts the incident wave field can be regarded as a disturbance producing a secondary wave field affecting the adjacent ones [2]. The aforementioned phenomenon is known as dynamic cross interaction (DCI) problem [1], structure-soil-structure interaction (SSSI) [3] or foundation-soil-foundation interaction (FSFI) [4].

The present paper investigates the dynamic behavior of an isolated, rigid, massless strip foundation bonded to an elastic half-space as well as the dynamic cross interaction between two strip foundations with the aforementioned characteristics.

Many researchers investigated the dynamic characteristics of the strip foundation-soil interaction. Luco and Westmann (1972) studied the coupled horizontal rocking response of a surface strip foundation under the assumption of welded soil contact. Such response arises when the superstructure is subjected to lateral forces (e.g. wind), when the foundation is exposed to earthquake condition or when it is subjected to machine vibrations [6]. Wang et al. (2015) verified their boundary element model using the results of the aforementioned study [1, 7]. Karasudhi et al. (1968) studied the coupled and uncoupled horizontal rocking response of a surface strip foundation under the assumption of relaxed and welded boundary conditions as well. Hryniewicz (1981) investigated the dynamic response of a strip foundation without taking into account the coupled horizontal-rocking response. Wickham's study (1977) referred to a rigid strip foundation in smooth contact with a semi-infinite elastic solid. Gazetas and Roesset (1976) applied a semi-analytical technique to estimate the dynamic compliances of a rigid strip foundation resting on an elastic, linearly hysteretic, layered half-space. Von Estorff and Schmid (1984) analyzed the dynamic behavior of a strip foundation resting on a soil layer using the BEM. A frequency domain BEM formulation has been used by Dominguez and Abascal (1989) in a 2-D analysis of a strip footing on zoned viscoelastic soils. Ahmad and Bharadwaj (1991) studied the horizontal impedance of embedded strip footing in layered soil. Jendoubi et al. (2008) studied the response of a strip footing resting on a half-space with constant or variable shear wave velocity.

The primary studies on structure-soil-structure interaction referred to the two-dimensional space. A frequency domain BEM formulation was applied by Cruze and Rizzo (1968) in order to study two and three dimensional elastodynamic problems between two rigid surface foundations. The through the soil coupling between adjacent infinitely long shear walls supported on semi-cylindrical foundations under the prism of two-dimensional anti-plane shear problem was studied by Luco and Contesse (1973) for the case of two structures, by Wong (1975) and Wong and Trifunac (1975) for several structures and by Murakami and Luco (1977) for an infinite number of equally spaced structures. Two-dimensional plane-strain models have been studied by Lysmer et al. (1974, 1975), Liang (1974) and Aydinoglu and Cakiroglu (1977) by use of the finite element approach and by Nakai and Fukuwa (1982), Antes and v. Estorff (1986) and Kokkinos and Spyrakos (1991) by a boundary integral equation approach.

The main target of the present study is the estimation of the impedance matrix referring to a system of two strip foundations. The influence of the distance between the two foundations on the impedance values is further investigated.

2 PROBLEM AND MODEL DESCRIPTION

The problem under study is depicted in Fig.1. Two rigid, massless, strip foundations are perfectly bonded to the free surface of an elastic half-space. Perfect bond is defined to be the type of attachment between the foundation and the underlying half-space which is characterized by the complete continuity of displacements in the area of contact. This type of attachment is also referred to as welded contact [9]. The two foundations have the same length (L) and their separation distance is denoted by S. Each foundation is excited by a vertical force (V), a horizontal force (H) and a moment (M). Each external load varies harmonically in time. Subscript 1 refers to the left and subscript 2 refers to the right foundation. The letter w refers to the vertical displacement, u to the horizontal displacement and φ to the rocking angle. The positive directions of external loads and the corresponding positive displacements as well are depicted in Fig. 1.

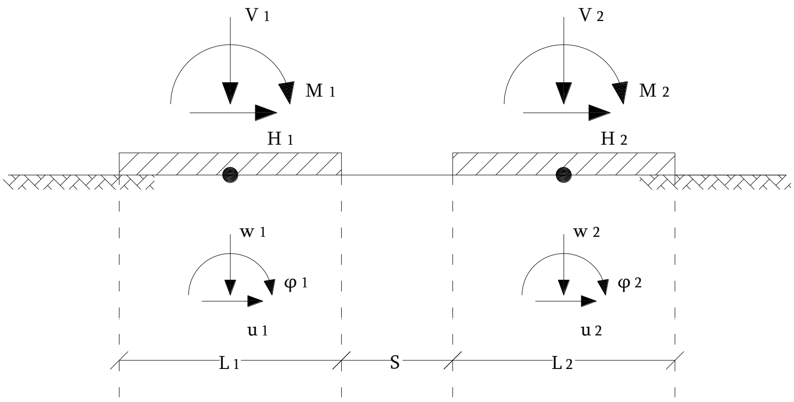

Figure 1: Adjacent strip foundations bonded to an elastic half-space.

The relationship between the external forces and the corresponding displacements of the two foundations is expressed by the following equation.

$$\begin{Bmatrix} V_1 \\ H_1 \\ M_1 \\ V_2 \\ H_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ & & K_{33} & K_{34} & K_{35} & K_{36} \\ & & & K_{44} & K_{45} & K_{46} \\ & \text{symm.} & & & K_{55} & K_{56} \\ & & & & & K_{66} \end{bmatrix} \cdot \begin{Bmatrix} w_1 \\ u_1 \\ \varphi_1 \\ w_2 \\ u_2 \\ \varphi_2 \end{Bmatrix} \tag{1}$$

Each foundation has three degrees of freedom. Therefore, matrix [K] is a 6x6 matrix. Furthermore, matrix [K] is characterized as the impedance matrix. Each matrix term is frequency-dependent and complex. In a simplified model, the real parts represent the springs which are related to the restraining action of the supporting soil or the true stiffness, whereas the imaginary parts represent the dashpots or the out-of-phase components which are related to the effect of energy dissipation by radiation [3,4]. The impedance matrix is symmetrical. During the analyses, it was observed that the impedance matrix is simplified in the following form:

$$\begin{Bmatrix} V_1 \\ H_1 \\ M_1 \\ V_2 \\ H_2 \\ M_2 \end{Bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ & K_{22} & K_{23} & -K_{15} & K_{25} & K_{26} \\ & & K_{33} & -K_{16} & K_{26} & K_{36} \\ & & & K_{11} & -K_{12} & -K_{13} \\ & \text{symm.} & & & K_{22} & K_{23} \\ & & & & & K_{33} \end{bmatrix} \cdot \begin{Bmatrix} w_1 \\ u_1 \\ \varphi_1 \\ w_2 \\ u_2 \\ \varphi_2 \end{Bmatrix} \tag{2}$$

Therefore, the calculation of the complete impedance matrix can be achieved by imposing the appropriate loading (vertical force, horizontal force, moment) in only one of the two foundations. In particular, the following displacements are considered to be necessary: (i) vertical loading – complete displacement field of the two foundations, (ii) horizontal loading – horizontal displacements and rocking angles of both foundations and (iii) moment loading - rocking angles of both foundations.

The soil is assumed to be an elastic half-space. The shear modulus (G) equals to 132MPa, the Poisson's ratio (v) equals to 0.25 and the density (ρ) equals to 1.75t/m^3. In the case of a strip foundation, referring to a track, dam or building foundation with a high ratio of length to width, plane strain conditions is reasonable to be assumed [1]. Therefore, the finite element model is created in 2D space by using 9-node quadrilateral 2-D solid elements [27].

The wave propagation problem that takes place in a half-space cannot be simulated perfectly by the finite element method due to the restriction imposed by the model length and the finite elements size [28]. In particular, the model must be large enough in order to reduce any reflected waves at its boundaries [29] and the finite elements size must be as small as possible in order to accurately describe a wavelength. Therefore, in order to avoid any wave reflections, quiet boundaries suggested by Lysmer and Kuhlemeyer (1969) were used. Furthermore, the minimum finite element size for each frequency was equal to $\lambda_R/10$, where λ_R is the wavelength of Rayleigh waves.

3 FINITE ELEMENT RESULTS

3.1 Validation study

In order to validate the finite element model, the impedance functions of an isolated, rigid, massless strip foundation bonded to an elastic half-space were compared with the results of the studies by Luco and Westmann (1972), Hryniewicz (1981) and Wang et al. (2015). The calculated impedance matrix was transformed into the flexibility matrix by $[K]^{-1}=[F]$. In Fig. 2 the comparison charts are depicted.

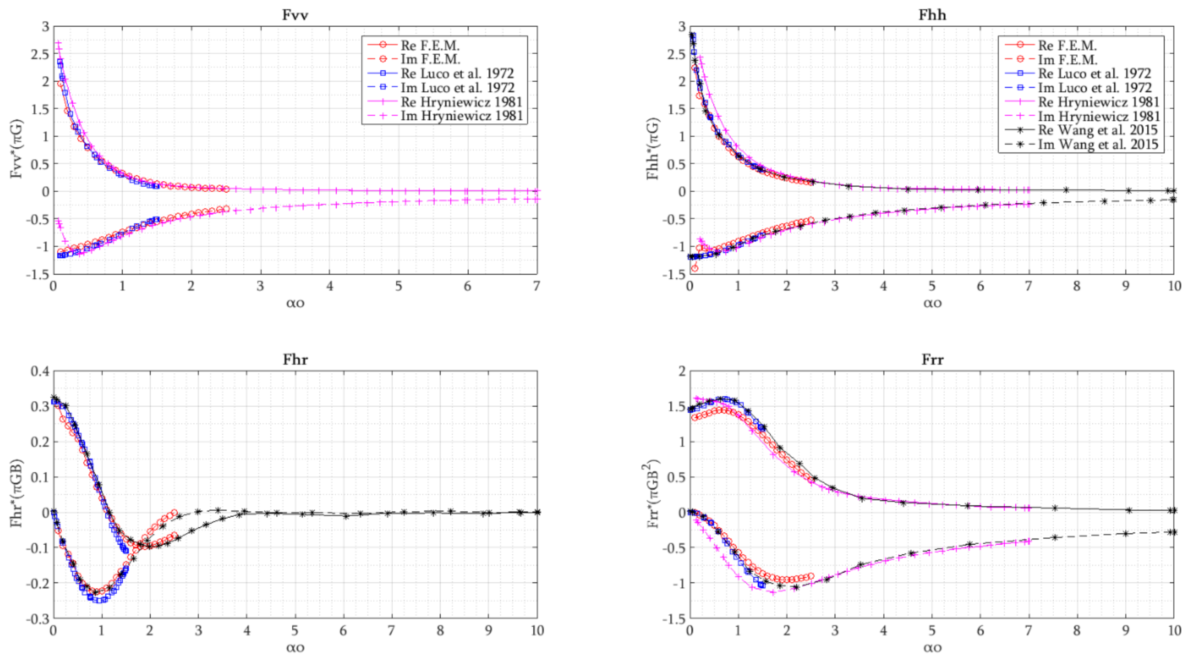

Figure 2. Validation study results for the case of an isolated, rigid, massless strip foundation bonded to an elastic half-space.

The charts refer to the vertical, the horizontal, the rocking and the coupled horizontal-rocking degree of freedom. The vertical loading of the strip foundation does not produce any horizontal or rocking movement. Therefore, there are no coupling terms for the vertical degree of freedom. It can be seen that the finite element results are in good agreement with the results of the other researchers. It must be taken into account that the comparison results were produced by the use of the boundary element method, which treats accurately the wave propagation problem in half-space.

3.2 Coupling of two foundations study

The main part of this study refers to the estimation of the dynamic cross interaction between two rigid, massless, strip foundations bonded to an elastic half-space. Therefore, the question that must be answered is the following: How much an external loading imposed on a foundation affects the dynamic behavior of its adjacent one? In order to answer the above question the necessary terms of the impedance matrix, based on equation (2), were estimated for the following ratios of S/L: 0.125, 0.5, 3 and 5.

In order to establish the accuracy of the finite element analyses, a comparative study was performed with the results available from Wang et al. (2015). Therefore, the real and imaginary parts of K22 and K33 are depicted for the various S/L ratios in Fig.3. The term K22 is related to the horizontal and the term K33 to the rocking movement of foundation 1. The vertical axis of the charts is normalized by the parameters π, G and B, which corresponds to foundation's half-width and the horizontal axis represents the normalized frequency αo, $\alpha_o = (\omega \cdot B)/V_s$, where V_s is the shear wave velocity. It can be seen that the F.E.M. overestimates the real parts of the impedances for low values S/L (0.125, 0.5). On the contrary, the agreement between the real parts of the impedances for high values of S/L (3, 5) and the imaginary parts for all the values of S/L, is considered to be very good. In general, taking into account the limitations of the F.E.M. on wave propagation in half-space, the finite element model can be considered to be accurate in a satisfactory manner.

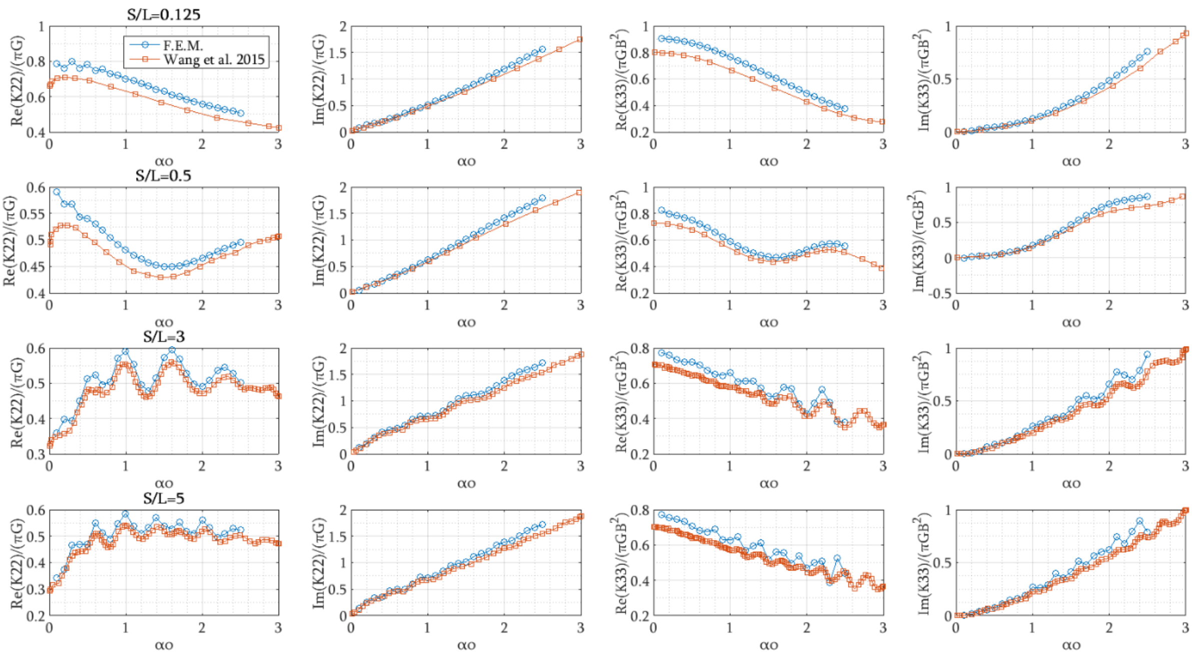

Figure 3: Validation study results for the case of two rigid, massless strip foundations bonded to an elastic half-space.

The following part of the finite element analyses refers to the investigation of the through-soil-coupling of two strip foundations. Based on the analyses results, it was found that the displacement field of each foundation is attributed not only on its external loading but on the adjacent foundation's loading as well. The smaller the ratio S/L is the bigger the effects of the foundation-soil-foundation interaction phenomenon are. Therefore, not only there are coupling terms between the horizontal and rocking movement of the foundation itself but coupling terms also appear between the two foundations even for the vertical excitation. In particular, Fig. 4 depicts the foundations movement at a random time frame for the case of S/L=0.5. It can be observed that even the vertical excitation causes a rocking movement not only on the foundation itself but on the adjacent foundation as well. The blue color depicts the initial position of the foundations while the cyan the deformed one.

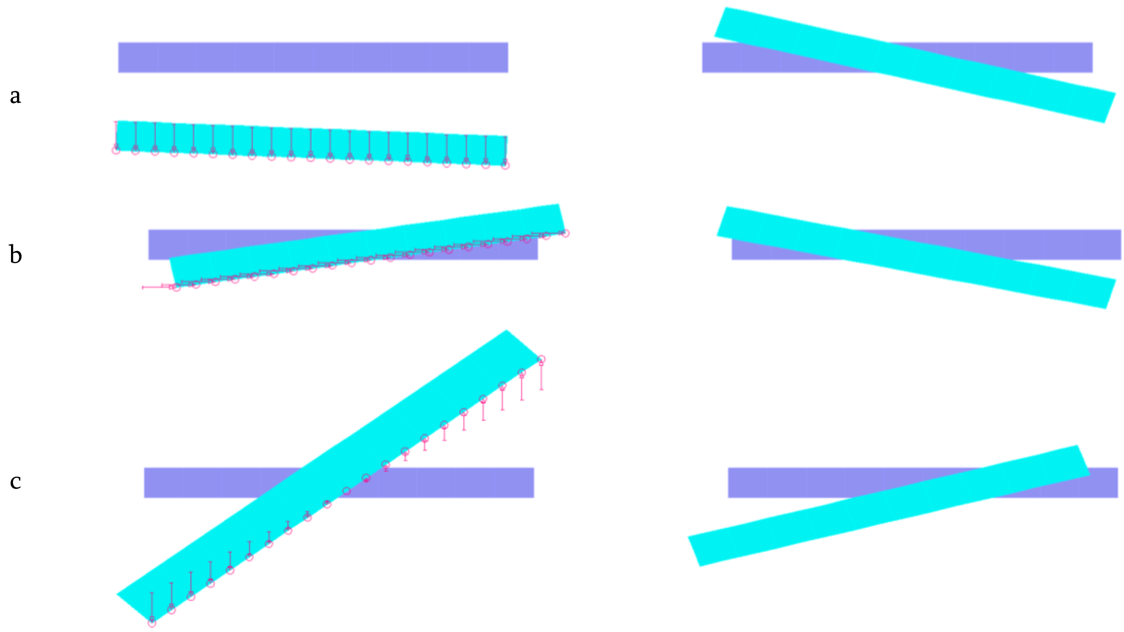

Figure 4. Foundations response at a random time frame for the case of S/L=0.5: a) vertical loading, b) horizontal loading, c) moment loading.

The foundation-soil-foundation interaction is expressed by the impedance matrix which connects the external loads imposed on the foundations to foundations displacements. The following figure, Fig.5, depicts the real and parts of the necessary terms for the definition of the impedance matrix, based on equation (2) while the next one, Fig. 6, depicts the imaginary parts.

Some of the most important observations regarding the real parts are the following:

- At low values of ratio S/L the real parts can be represented by a smooth line, while at high values of ratio S/L they present spikes at various frequencies.
- The higher the value of S/L, e.g. S/L=0.5, the smaller the differences with an isolated strip foundation.
- Vertical excitation. Regarding the loaded foundation (terms K11, K12 and K13): the coupling vertical-rocking (K13) is stronger than vertical-horizontal (K12) for low values of S/L while it fades for high values of S/L.
- Vertical excitation. Regarding the adjacent foundation (terms K14, K15 and K16): a) the vertical-vertical coupling (K14) cannot be ignored for low values of S/L; b) the vertical-vertical coupling (K14) receives negative values at the low range of ao for low

values S/L, while for high values of S/L, negative spikes are noticed through the whole range of ao; c) the coupling between the two foundations cannot be ignored at high frequencies, αo, even for high values of S/L; d) the vertical-horizontal (K15) and vertical-rocking (K16) coupling is strong for all values of S/L.

- Horizontal excitation. Regarding the loaded foundation (terms K22, K23): the horizontal-rocking coupling (K23) receives extremely small values in comparison to term K22 for all values of S/L.
- Horizontal excitation. Regarding the adjacent foundation (terms K25, K26): a) the horizontal-horizontal coupling (K25) cannot be ignored for low values of S/L; b) the horizontal-horizontal coupling (K25) receives negative values at the low range of ao for low values S/L; c) the horizontal-rocking coupling (K26) receives extremely small values in comparison to term K22 but equal range values in comparison to term K25 for all values of S/L.
- Rocking excitation. The foundation coupling cannot be ignored at high frequencies, αo, for all values of S/L.

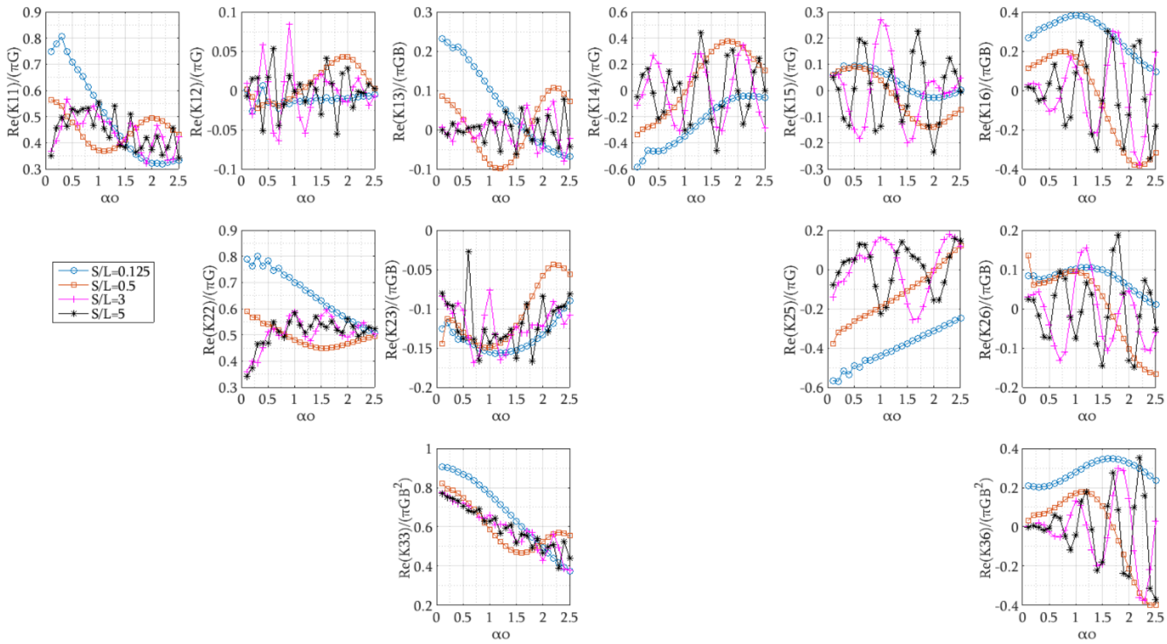

Figure 5. Real parts of the necessary terms for the definition of the impedance matrix.

Some of the most important observations regarding the imaginary parts are the following:

- At low values of ratio S/L the real parts can be represented by a smooth line, while at high values of ratio S/L they present spikes at various frequencies.
- Vertical excitation. The values of all coupling terms (K12-K16) are much smaller in comparison to K11 for all values of S/L.
- Horizontal excitation. The values of all coupling terms (K23-K26) are much smaller in comparison to K22 for all values of S/L.
- Rocking excitation. The values of K36 cannot be ignored for high values of frequency, αo, for all values of S/L.

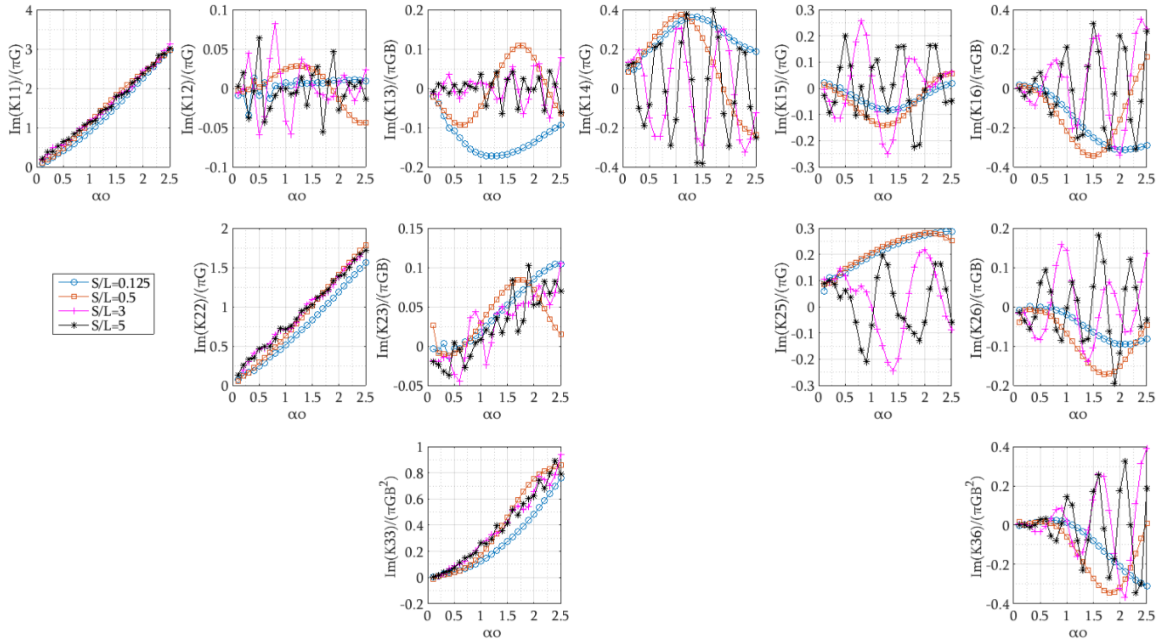

Figure 6. Imaginary parts of the necessary terms for the definition of the impedance matrix.

In the following figure, the impedance matrix terms are depicted in the complex number space. In particular, the horizontal axis represents the real part while the vertical axis represents the imaginary part of each impedance term. Each point in every chart follows a frequency, αo, step of 0.1, starting from 0.1.

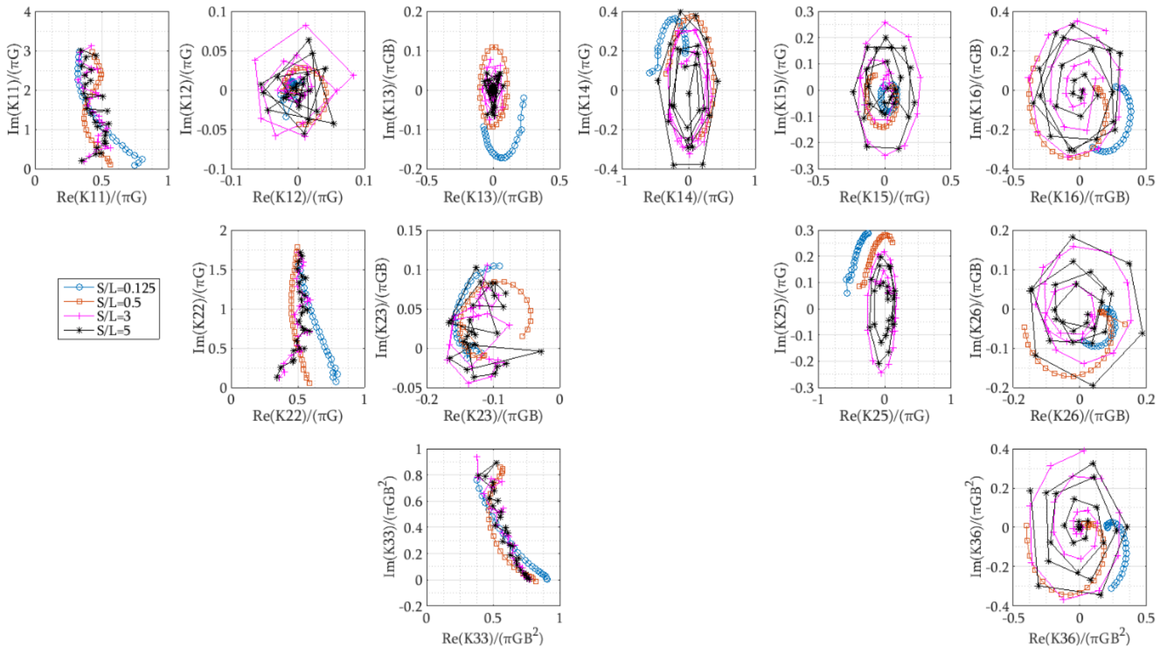

Figure 7. Representation of impedance terms on complex number space.

It can be seen that the representation of the diagonal terms (K11, K22, K33) corresponds to a line, almost smooth for low values of S/L and with spikes for high values of S/L. Whereas, the representation of the off diagonal terms corresponds to a spiral line for low values of S/L and a polygon for high values of S/L.

4 CONCLUSIONS

The through-the soil coupling of two rigid, massless strip foundations bonded to an elastic half-space has been studied by the use of the finite element method. The accuracy of the model has been verified by the comparison studies referring to an isolated strip foundation and a system of two. The foundation-soil-foundation is expressed by the impedance matrix. For low values of S/L, the coupled response cannot be ignored even for the vertical excitation. The charts of real and imaginary parts can be used in a simplified model of springs and dashpots.

RFERENCES

[1] J. Wang, D. Zhou and W. Liu, Simplified model for coupled vibration of adjacent strip footings. M. Papadrakakis, V. Papadopoulos, V. Plevris eds. *5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamic and Earthquake Engineering (COMPDYN 2015)*, Crete Island, Greece, May 25-27, 2015.

[2] L.G. Tham, J. Qian and Y.K. Cheung, Dynamic response of a group of flexible foundations to incident seismic waves. *Soil Dynamic and Earthquake Engineering*, 17, 127-137, 1998.

[3] A. Bharadwaj and S. Ahmad, Rocking impedance of embedded strip foundations in layered soil. *Journal of Geotechnical Engineering*, 118(5), 796-813, 1992.

[4] D.L. Karabalsi and M. Mohammadi, 3-D dynamic foundation-soil-foundation interaction on layered soil. *Soil Dynamics and Earthquake Engineering*, 17, 139-152, 1998.

[5] J.E. Luco and R.A. Westmann, Dynamic response of a rigid footing bonded to an elastic half-space. *Journal of Engineering Mechanics Division ASME*, 39(2), 527-534, 1972.

[6] B. Suhol and L.C. Hong, Coupled horizontal-rocking impedance functions for embedded square foundations at high frequency factors. *Journal of Earthquake Engineering*, 3(4), 561-587, 1999.

[7] J. Wang, S.H. Lo, D. Zhou and B. O. Xu, Frequency-dependent impedance of a strip foundation group and its representation in time domain. *Applied Mathematical Modelling*, 39, 2861-2881, 2015.

[8] P. Karasudhi, L.M. Keer and S.L. Lee, Vibratory motion of a body on an elastic half plane, *Journal of Applied Mechanics ASME*, 35(4), 697-705, 1968.

[9] Z. Hryniewicz, Dynamic response of a rigid strip on an elastic half-space. *Computer methods in applied and mechanical engineering*, 25, 355-364, 1981.

[10] G.M. Wickham, The forced two dimensional oscillations of a rigid strip in smooth contact with a semi-infinite elastic solid. Mathematical Proceedings of Cambridge Philosophical Society, 81 (2), 291-311, 1977.

[11] G. Gazetas and J.M. Roesset, Forced vibrations of strip footing on layered soils. *Proceedings of Methods in Structural Analysis ASCE*, 1, 115-131, 1976.

[12] O. Von Estorff and G. Schmid, Application of the boundary element method to the analysis of the vibration behavior of strip foundations on a soil layer. *Proceedings of the International Symposium on Dynamic Soil-structure interaction*, A.A. Balkema, Minneapolis, Minn., 11-17, 1984.

[13] J. Dominguez and R. Abascal, Seismic response of strip footings on zoned viscoelastic soils. *Journal of Engineering Mechanics*, 115, 913-934, 1989.

[14] S. Ahmad and A. Bharadwaj, Horizontal impedance of embedded strip foundations in layered soil. *Journal of Geotechnical Engineering*, 117(7), 1021-1041, 1991.

[15] A. Jendoubi, F. Legeron and M. Karray, Numerical simulation of displacement functions of strip footings. *Geotechnical Earthquake Engineering and Soil Dynamics IV Congress*, 1-10, 2008.

[16] T.A. Cruze and F.J. Rizzo, A direct formulation and numerical solution of the general transient elastodynamic problem. *International Journal of Mathematical Analysis and Applications*, 22, 244-259, 1968.

[17] J.E. Luco and L. Contesse, Dynamic structure-soil-structure interaction. *Bulletin of the Seismological Society of America*, 63(4), 1289-1303, 1973.

[18] H.L. Wong, Dynamic soil-structure interaction. *Report EERL-75-01*, Earthquake Engineering Research Lab., California Institute of Technology, Pasadena, California, 1975.

[19] H.L. Wong and M.D. Trifunac, Two dimensional, antiplane, building-soil-building interaction for two or more buildings and for incident plane SH-waves. *Bulleting of the Seismological Society of America*, 65(6), 1863-1885, 1975.

[20] M. Murakami and J.E. Luco, Seismic response of a periodic array of structures, *Journal of the Engineering Mechanics Division ASCE*, 103 (EM5), 967-977, 1977.

[21] J.T. Lysmer, T. Udaka, H.B. Seed and R. Hwang, A computer program for complex response analysis of soil-structure interaction. *Report EERC 74-4*, Earthquake Engineering Research Center, University of California, Berkeley, CA, 1975.

[22] V.C. Liang, Dynamic response of structures in layered soils. *Report 1274-10*, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, 1974.

[23] M.N. Aydinoglu and A. Cakiroglu, Dynamic interaction between soil and a group of buildings. *Proceeding of the 6th World Conference on Earthquake Engineering*, New Delhi, India, 2, 1596-1601, 1977.

[24] S. Nakai and N. Fukuwa, Boundary element analysis of approximate three-dimensional soil-structure interaction. *Proceedings of the 6th Japan Earthquake Engineering Symposium*, Tokyo, Japan, 1633-1640, 1982.

[25] H. Antes and V. O. Estorff, Dynamic soil-structure interaction by BEM in the time and frequency domain. *8th European Conference on Earthquake Engineering*, Lisbon, Portugal, 1986.

[26] F.T. Kokkinos and C.C. Spyrakos, Dynamic analysis of flexible strip-foundations in the frequency domain. *Computers and Structures*, 39 (5), 473-482, 1991.

[27] ADINA, Theory and Modeling Guide, Volume I: ADINA. *Report ARD 11-8*, ADINA R & D, Inc., 2011.

[28] W. Smith, A non-reflecting plane boundary for wave propagation problem. *Journal of Computer Physics*, 15, 492-503, 1974.

[29] J.M. Roesset and M.M. Ettouney, Transmitting boundaries: comparison. *Journal of numerical and analytical methods in geomechanics*, 1(2), 151-176, 1977.

[30] J. Lysmer and R.L. Kuhlemeyr, Finite dynamic model for infinite media. *Journal of the Engineering Mechanics Division ASCE*, 95, 859-878, 1969.