ROCKING SPECTRUM INTENSITY MEASURE FOR SEISMIC ASSESSMENT OF ROCKING BLOCKS

Ioannis E. Kavvadias¹, Kosmas E. Bantilas¹, George A. Papachatzakis¹, Lazaros K. Vasilias¹ and Anaxagoras Elenas¹

¹ Democritus University of Thrace, Department of Civil Engineering
Campus of Kimmeria, 67100 Xanthi, Greece
e-mail: {ikavvadi, kbantila, gpapacha, lvasila, elenas}@civil.duth.gr

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Abstract. In this study a spectral ground motion intensity measure (IM) is presented. The proposed IM is calculated throughout the rocking spectrum which is illustrated as a 3D surface diagram, with the slenderness α and the period T, the two horizontal axes. Thus, the IM is defined as the volume under the rocking spectrum, evaluated throughout integration. The suggested IM is intended to have strong correlation with the rocking response of free standing blocks. Several rocking rigid blocks subjected to multiple ground motion records are assumed in order to identify the capability of the IM to predict the rocking response concerning the variability of the block dimensions. The selection of the dimensions was made in order to correspond to slender rocking structures such as electric equipment, ancient monolithic columns and bridges piers. The evaluation of the IM was also made by examining characteristics such as efficiency and proficiency. Further, the performance of the proposed IM is contrasted with the performance of others well-known ground motion parameters, to highlight the adequacy of the former in predicting the rocking seismic response.
1 INTRODUCTION

Probabilistic seismic assessment has emerged in recent years as a useful tool for both the design of new structures and the assessment of existing ones. Vulnerability assessment, are based on the probabilistic seismic demand model (PSDM) that the links the structural response with the intensity of the ground motion. Consequently, these two parameters are described in literature as Engineering Demand Parameter (EDP) and Intensity Measure (IM). The demand uncertainties introduced in the model depends on the IM used. Thus, the selection of the IM that describe with better accuracy the response of the examined structural system is of significant importance. Several criteria has been proposed in the literature in order to suggest an IM as an optimal one [1-4].

Over the years several researchers have developed ground motion parameters to represent the seismic excitation intensity. These IMs can be classified into time history, energy, spectral, and frequency content parameters [5]. Among the others, the spectral parameters are the only that contain structural information, therefore they are aimed to have strong correlation with the structural damage [6]. Nevertheless, response spectra of SDOF oscillators are not adequate to describe the rocking response [7]. Thus, defining spectral intensity measures that could enclose information about the rocking structures, is expected to be a better descriptor of the rocking response.

The seismic response of structural systems that exhibit pure rocking behavior are very sensitive to even minus alteration on the excitation characteristics or on the blocks dimensions [8]. Spanos and Koh [9] taking that fact into account, have been studied the rocking response under a probabilistic framework. Recently, Acikgoz and DeJong [10], examined large flexible rocking structures subjected to pulse type excitations in order to conduct to a predictive framework. Seismic vulnerability assessment of rocking structural systems are presented in the literature. Psycharis et al. [11] have been assessed the seismic reliability of ancient multi-drum columns, while Dimitrakopoulos and Paraskeva [12] have been evaluated the efficiency of dimensionless IMs in predicting rocking response. Kavvadias et al. [13] have been examined several IMs in order to conclude to these that describe with better accuracy the rocking response of rigid blocks.

Taking all the above under consideration, a scalar IM, intended to have strong correlation with the seismic rocking performance is proposed in the present study. This IM, based on rocking rotation spectrum, form an attempt to quantify the information provided by the rocking response of rigid blocks. The ground motion parameter named as Rocking Rotation Spectral Intensity (RRSI), is defined by the volume under the normalized rocking spectrum, as though, is evaluated by integration of the rocking rotation spectrum. The integration limits of the period T and the slenderness α, was adopted in order to construct an IM that could take into account structural response information for the majority of the slender rocking blocks. Using this IM, a reduction of the uncertainty associated with the median demand of rocking response is sought.

To assess the adequacy of the IM, 12 rigid blocks subjected to a set of 35 ground motion record were examined. Correlation coefficients between the proposed ground motion parameter and the developed damage, as well as criteria such as efficiency and proficiency, are adopted in order to evaluate the IM. Moreover, velocity based and frequency content IMs are also examined. These IMs are known that have strong correlation with the rocking response [12, 13], thus the comparison of those performance with the performance of the proposed IM could assure their suitability for usage to vulnerability analysis of rocking structures.
2 SEISMIC ROCKING RESPONSE OF RIGID BLOCK

A rigid block standing free on a rigid base, with slenderness a, semi-diagonal R and frequency parameter p, oscillates about the centers of rotation O and O’ when rocking motion initiates (Figure 1). The minimum acceleration of a ground excitation that enables rocking can be computed from static analysis, and yields to \( \ddot{u} \geq g \tan(\alpha) \). The problem of a rigid rocking block motion under a seismic excitation can be described by the following equation [14]:

\[
\ddot{\theta} = -p^2 \cdot \left\{ \sin[\alpha \cdot \text{sgn}(\theta) - \theta] + \frac{\ddot{u}}{g} \cdot \cos[\alpha \cdot \text{sgn}(\theta) - \theta] \right\}
\]  

(1)

where \( \text{sgn}() \) is the sign function, \( p = \sqrt{3g/4R} \) is the frequency parameter of the rigid block and R is the semi-diagonal.

![Figure 1: Characteristics of a rigid rocking block.](image)

During the rocking motion, energy is lost only during impact (when the rotation changes sign at \( \theta = 0 \)) which causes a reduction of the rotational velocity after it:

\[
\dot{\theta}_{n+1} = r \cdot \dot{\theta}_n
\]

(2)

where \( r \) is the restitution coefficient, \( \dot{\theta}_n \) is the velocity before the impact and \( \dot{\theta}_{n+1} \) is the velocity after the impact.

Considering that the angular momentum remains constant about point O exactly before the impact and right after it, the coefficient of restitution for a rigid rectangular block is given by the following equation [15]:

\[
r = \left[ 1 - \frac{3}{2} (\sin \alpha)^2 \right]^2
\]

(3)

The restitution coefficient depends only on the slenderness and expresses the energy dissipation during rocking motion considering inelastic impact. Since the actual restitution coefficient is material dependent, several experiments were performed in order to determine it accurately [16, 17], and improved formulas has been proposed [18]. In the current study the restitution coefficient of Eq. (3) is used for the analyses. The seismic rocking response is calculated, by solving the equations listed above, in Matlab R2016 [19], using the Runge-Kutta integration technic named ode45.
3 ROCKING SPECTRA

In correspondence with response spectra for single-degree-of-freedom (SDOF) models, rotation and velocity rocking spectra are firstly presented by Makris and Konstantinidis [7]. Despite the fact that the SDOF can be characterized only by the period $T$, the rocking block has two representative parameters, the frequency parameter $p$ and slenderness $\alpha$. Thus, the rocking spectra usually are depicted with diagrams which correspond to blocks of individual slenderness.

In this study rocking spectra are pictured as 3D surface diagrams in which the two horizontal axes are the period $T$ and the slenderness $\alpha$. In such way, the outcome of the ground motions to the rocking response is easily observed. The surface diagram compared with the classic 2D one, provide more clearly the rocking response of blocks with a wide range of geometric characteristics. Moreover, features of the ground motion excitations can be conducted by rocking spectra. The values of the rotational rocking spectra are normalized to the slenderness of the blocks $\alpha$. It is known that in rare cases, a rocking block could develop rotation higher than its slenderness without overturning, while when it overturns the EDP takes infinite value. Nonetheless, the limit of the normalized rotation when collapse occurs is considered $\theta_{\text{max}}/\alpha = 1$. The rocking spectra is created for a range of slenderness $\alpha = 0.1-0.3$ rad and $T = 1-8$ rad/s.

Rocking spectra obtained by two ground motions are presented in Figures 2. In the above figure the surface spectra are depicted among with the acceleration time histories. The excitations have PGA values adequate to uplift the total range of the blocks. However, due to their frequency content, difference at the spectra are observed. As seen in spectrum created from Imperial Valley, rocking is initiated for the most blocks, but due to its low mean period $T_m$ it can barely induce uplift to a block with slenderness higher than $\alpha = 0.2$ rad. On the contrary, the Northridge excitation, which has the same PGA but higher $T_m$, proves to have more detrimental effect on rocking.

![Image of rocking spectra](image_url)

Figure 2: Acceleration time histories and the corresponding normalized rocking spectra.
4 PROPOSED INTENSITY MEASURES

A proper selection of IMs is crucial in order to exhibit high correlation with the structural damage. By representing ground motion intensity through appropriate IMs, it is possible to obtain an optimized response prediction. It is known that rocking response is affected by the frequency content of the excitation [15]. The strong dependence of the velocity characteristics of the ground motion with rocking response is verified in recent studies [12]. Also, Intensity Measures such as Housner’s Spectral Intensity (SI_H), containing information of the elastic response of SDOF, affects significantly the response of free standing rigid blocks [13].

Intensity measures that contain apart from ground motion information, information about the structural response is expected to be a more efficient IM regarding the engineering demand prediction. Spectral parameters constitute IMs that counts information about the structural features. Thought, there are numerous IMs calculated by the SDOF response spectra, none is proposed based on the rocking one. Thus, a rational approach is to select IMs defined by the rocking spectra, which could form a strong correlated parameter with the rocking response.

In the present study, the proposed IM are determined in order to contain information about the earthquake record, as well as information about the structural response of free standing rigid blocks, aiming for a better description of the rocking performance. The proposed parameters named Rocking Rotation Spectral Intensity (RRSI), is defined as the volume under the rocking rotation spectrum calculated via integration of the rocking rotation spectrum. Thus, it forms an attempt to quantify the information provided rocking spectra. The definition of the IMs are the followings:

\[
RRSI = \int_{a_{min}}^{a_{max}} \int_{T_{min}}^{T_{max}} S_r(T,a) dT da
\]  

(3)

where \(S_r\) is the rocking rotation spectrum surface.

The integration limits of period \(T\) and slenderness \(a\) are chosen in order to reflect to a wide variety of slender rocking structures. For the evaluation of the frequency parameter \(p\) examples from previous studies are considered ranged from \(p \approx 3.5\) rad/s for a typical tombstone to \(p < 1\) rad/s for relatively tall building frame structures [10]. As a result, the integration limits for the period \(T\) are adopted from 1-8 s. Concerning slenderness, the adopted limits are \(a = 0.1-0.3\) rad moving from slender structure to firm ones. In these values rocking structures from Electrical transformers (\(a \approx 0.30\) rad) and ancient columns (\(a \approx 0.15\) rad) to bridge piers (\(a \approx 0.10\) rad) are included.

5 OPTIMAL SELECTION OF INTENSITY MEASURE USED FOR VULNERABILITY ANALYSIS

In recent years, probabilistic analysis became widely used for structural engineering purposes and Probabilistic Seismic Demand Models (PSDM) were developed aiming to predict the response of a structure under earthquakes of certain intensities. Specifically, the fragility analysis expresses the probability (P) that the capacity (C) of a specific measured engineer parameter (EDP) of a structure will exceed a certain level of demand (D), for a specific ground motion intensity measure (IM), as defined by the following equation [1]:

\[
P = P[D \geq C | IM]
\]  

(5)
Regarding the rocking response, the results of the time history analyses have to be treated with two different methods. In order to calculate the probability of exceeding a certain capacity value, using only the non-collapse data, the following expression is used [4]:

\[
P[D \geq C \mid IM] = \Phi\left(\frac{\ln(S_d / S_c)}{\sqrt{\beta_{\text{DIIM}}}}\right) \tag{6}
\]

where \(\Phi\) is the standard normal cumulative distribution function, \(S_c\) is the median value of the capacity which is estimated through the adopted limit states and \(\beta_{\text{DIIM}}\) is the dispersion or logarithmic standard deviation for the demand conditioned the IM.

The median seismic demand \(S_\text{D}\), is related with an examined IM with the following expression:

\[
S_d = a \cdot (\text{IM})^b \tag{7}
\]

where \(a\) and \(b\) are the linear regression coefficients for the logarithmic expression of the assumed scale law.

The linear regression analysis is performed between the engineering demand parameter calculated via the time history analyses, and the corresponding IM values. The logarithmic standard deviation (Eq.(8)) of the linear regression analysis indicates the dispersion of the median demand. This parameter consist the demand uncertainty introduced in the probabilistic model.

\[
\beta_{\text{DIIM}} \approx \sqrt{\frac{\sum (\ln(d_j) - \ln(a \cdot (\text{IM})^b))^2}{N - 2}} \tag{8}
\]

The above methodology (Eqs. (6)-(8)) corresponds when rocking occurs excluding the rocking collapse cases. To calculate the fragility of the structural system by taking into account the rocking overturn data the Eq. (6) is altered to:

\[
P[D \geq C \mid IM] = P_o + (1 - P_o) \cdot \Phi\left(\frac{\ln(S_d / S_c)}{\sqrt{\beta_{\text{DIIM}}}}\right) \tag{9}
\]

where \(P_o\) is the probability of rocking collapse.

When rocking overturn occurs the rotation values rich infinite values. Assuming that fact, the problem should be considered as a categorical one, by grouping the data into non-collapse and collapse ones in order to estimate the probability of overturning. So, to estimate the parameters of the fragility function (mean \(\mu\) and standard deviation \(\beta\)) that provides the probability of collapse, the maximum likelihood approach is adopted [20-21]. The maximum likelihood function \(L\) is defined as follows:

\[
L = \prod_{j=1}^{m} \left(\frac{n_j}{z_j}\right) p_j^{z_j} (1 - p_j)^{n_j - z_j} \tag{10}
\]

where the probability of \(z_j\) collapses out of \(n_j\) ground motions of a certain value of IM is given by the binomial distribution, \(p_j\) is the probability that a ground motion of a particular IM value, will cause the collapse of the structure, \(m\) is the number of IM levels and \(\Pi\) denotes a product over all IM levels.
Assuming lognormal cumulative distribution for the overturning probability, Eq. (7) converts to the following:
\[
L = \prod_{j=1}^{m} \left( \frac{n_j}{z_j} \right) \Phi \left( \frac{\ln x_j - \mu}{\beta} \right)^{\gamma_i} \left( 1 - \Phi \left( \frac{\ln x_j - \mu}{\beta} \right) \right)^{n_j - \gamma_i}
\]

The maximization of \( L \) gives the statistical moments \( \hat{\mu}_{\text{MLE}} \) and \( \hat{\beta}_{\text{MLE}} \) via an optimization process.

\[
\left\{ \hat{\mu}_{\text{MLE}}, \hat{\beta}_{\text{MLE}} \right\} = \max_{\mu, \beta} \prod_{j=1}^{m} \left( \frac{n_j}{z_j} \right) \Phi \left( \frac{\ln x_j - \mu}{\beta} \right)^{\gamma_i} \left( 1 - \Phi \left( \frac{\ln x_j - \mu}{\beta} \right) \right)^{n_j - \gamma_i}
\]

The appropriate selection of an IM plays an important role in the accuracy of a probabilistic seismic demand analysis (PSDA) for structures. As though, the choice must be made based on criteria, presented in the literature, that help to distinguish the accuracy of the seismic assessment. An optimal IM is defined by primary factors such as efficiency, practicality, proficiency and sufficiency [1-4] Efficient IMs are the ones that eliminates the dispersion of the results about the median, which means decrease of the uncertainties introduced to the PSDM, resulting in superior fragility curves during the vulnerability assessment. A distinguished IM according to its efficiency is represented as by a lower logarithmic standard deviation \( \beta_{\text{bpm}} \) (Eq. (8)).

Proficiency is a complex measure, assessing the effect of both practicality and efficiency. Practicality refers to whether or not there is any direct correlation between an IM and the demand placed on the structure and is measured by the regression parameter \( b \) in the PSDM. Thus, the proficiency consists of parameters that combine the logarithmic standard deviation and the slope of the regression analysis and defined as:

\[
\zeta = \frac{\beta_{\text{bpm}}}{b}
\]

From the aforementioned characteristics, efficiency and proficiency are most commonly used to distinguish whether an IM is optimal regarding the appropriateness of predicting the structural damage.

Another measure in order to evaluate the grade of interdependency between the examined IMs and the EDP is the correlation coefficient by Pearson’ (Eq. (16)) which shows how well the data fit a linear relationship, and is derived from the regression analysis of each PSDM (Eq. (1)).

\[
r_{\text{Pearson}} = \frac{\sum_{i=1}^{N} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{N} (X_i - \bar{X})^2 \sum_{i=1}^{N} (Y_i - \bar{Y})^2}}
\]

where:

\( X_i \) is the value of the seismic intensity parameter of the i-th accelerogram, \( Y_i \) is the value of the seismic response parameter to the i-th accelerogram and finally \( \bar{X}, \bar{Y} \) are the mean values of \( X_i \) and \( Y_i \).
6 IMS EVALUATION PROCEDURE

The adequacy of the proposed IMs in describing the rocking behavior of slender rigid blocks is examining thoroughly. In order to investigate the robustness of the IMs regarding the structural characteristics, 12 different rigid blocks are considered. The blocks have slenderness of 0.10, 0.15 and 0.20 rad and frequency parameter $p$ equal to 2.73, 2.00, 1.35 and 0.86 (rad/s). The selection of the blocks dimensions and slenderness was made to be in accordance with typical rocking structures such as bridge piers, ancient columns, electrical transformers and laboratory or home equipment. The procedure could be extended also to rocking frames [22], or rocking flexible oscillators [23].

A cloud analysis is conducted using a set of 35 ground motion time histories as input base acceleration for the analyses. Excitations generated from different types of fault types, with earthquake magnitudes (Ms) between 5.3 and 7.6, including both near-fault and far-fault records, are employed in order to present a wide range of intensities and frequency contents. A further aspect which has been taken into consideration is the expected damage potential of the seismic excitation on the rocking structures. Seismic excitations which provide a wide spectrum of structural damage, from negligible to severe, are taken into account. Based on the above assumption, a rigorous selection of ground motions carried out from the Pacific Earthquake Engineering Research Centre (PEER) [24] and the European strong motion [25] databases has been carried out, using non-scaled records. The complete list of the natural ground motions used for the analyses is shown in Table 1.

The engineering demand parameter (EDP) chosen in the present study provides an estimate of the structural damage state and collapse potential. Due to the nature of the rocking behavior, the most representative parameter to assess structural capacity has to be based on the rocking rotation. As a result, the absolute peak rocking rotation $| \theta_{\text{max}} |$ scaled with respect to the slenderness $\alpha$ is adopted:

$$ EDP = \frac{| \theta_{\text{max}} |}{\alpha} $$

The correlation coefficients by Spearman are demonstrated, as the metric of evaluating the interdependency of the IMs with the EDP. Moreover, to highlight the efficiency of the proposed parameters the dispersion estimators $\beta$ and $\zeta$, calculated by Eq. (8) and Eq. (13) respectively are presented. Assuming the maximum likelihood estimator, to calculate the overturning fragility, $\beta_{\text{MLE}}$ values are presented (Eq. (12)), due to the fact that it could considered as an additional efficiency parameter.

Further, the performance of the PGV, $S_{\text{IH}}$, $I_{\text{FVF}}$, $T_m$ and $L_m$ [5, 26] in assessing the vulnerability of the examined structural systems is also presented. These ground motion characteristics are strongly correlated with the rocking response of slender rigid blocks [13]. Thus, the comparison of their results with those obtained by the proposed spectral IMs, will deduce the superiority of the latter.

7 RESULTS

In Figure 3 the linear regression between the developing rocking rotation and the RRSI are depicted. It can be seen that the linear regression is capable to describe the relationship between the IM and the EDP. Moreover, in a first glance the RRSI it seems to be suitable in describing the safe rocking response for the total of the rocking blocks.
In Figure 4 the correlation coefficients between the proposed and the other examined IMs with the EDP are presented. The RRSI displays on average the highest correlation with the rocking response in contrast with the other IMs. In more detail, it affect significantly the blocks with slenderness lesser than 0.15 rad regardless of their size. It performance tend decreasing only according less slender blocks of small size. Regarding the other examined IMs those that constitutes velocity based ones, demonstrate similar performance with the RRSI, while the $T_m$ is the IM that picture the lowest grade of interdependency with the rocking.
The logarithmic standard deviations $\beta$, which are calculated via the linear regression analysis, are depicted in Figure 5. The RRSI present considerable low values of $\beta$ regardless the rigid block examined. Regarding the other IMs examined, the SI$_H$ and the I$_{FVF}$ decreases notably the dispersion of the PSDM. Moreover, T$_m$ and L$_m$ performs well especially for more slender blocks. Between the proposed IM and the other 5 examined, the RRSI present the lowest logarithmic standard deviation on average with value of $\beta=0.62$. 

Figure 3: Linear regression analysis between ln(RRSI) and ln(EDP).

Figure 4: Correlation coefficients between the IMs and the EDP.
Except of the logarithmic standard deviations $\beta$, another more complex dispersion metric, the proficiency $\zeta$, is calculated. Figure 6 displayed the $\zeta$ parameter per block, for each one of the IMs. According to these results, the $I_{FVF}$ displays the better performance in average. The lowest proficiency is presented by $T_m$ and $L_m$, while the other parameters performs as well as regarding the efficiency parameter $\beta$ with the RRSI being one of the best parameters.

The standard deviation $\beta_{MLE}$ obtained by the maximum likelihood estimator approach could also form an efficiency parameter. Therefore these values are presented in Figure 7. The block with slenderness $\alpha=0.20$ rad and frequency parameter $p=0.86$ rad/s did not collapse subjected under any of the 35 ground motions. Thus, at the third subfigure in the row there are...
depicted results from only three blocks. The $T_m$ and the $L_m$ consist the parameters that could predict the overturning of the blocks with the larger values of dispersion compared with the other IMs. Except of the RRSI, the PGV and the $I_{FVF}$ lead to minimum values of dispersion.

8 CONCLUSIONS

A scalar IM named as Rocking Rotation Spectral Intensity (RRSI) is proposed in the present study. The proposed parameter is defined by the volume under the rocking spectrum, which is pictured as a surface diagram. The spectral volumes that consist the RRSI is evaluated by integration of the rocking rotation spectrum. The integration limits of the period $T$ and the slenderness $\alpha$, was adopted in order to construct IM which take into account structural response information for the majority of the slender rocking blocks. The proposed IM are determined with the intention of having strong correlation with the seismic rocking performance.

To assess the adequacy of the IM, 12 rigid blocks subjected to a set of 35 ground motion record were examined. Using only the non-collapse data, linear regression analyses are performed between the proposed IM and the developed rocking rotation, from which the correlation coefficients are extracted. Further, criteria such as efficiency, proficiency and practicality are adopted in order to evaluate the IM. Moreover, the standard deviations calculated by the maximum likelihood method are presented, as an efficiency parameter. In order to highlight the superiority of the IMs, their results are compared with those obtained by 5 well known IMs which affects the rocking response.

- Regarding the correlation coefficients, the RRSI developed the higher values on average ($r=0.86$), while it ranged from 0.6 to 0.94.
- The RRSI proved to be the more efficient IM on average ($\beta=0.62$) having similar performance with the $SI_H$ and the $I_{FVF}$.
- Based on the proficiency criteria, the $I_{FVF}$ presents the better performance. However, RRSI demonstrate low values of the parameters $\zeta$ too.
- Similar outcomes are arise from the standard deviation which is calculated throughout the maximum likelihood estimator process. With the RRSI being the parameter that decrease the variation of predicting the rocking overturn collapse.

REFERENCES


