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RESONANCE PHENOMENA IN SYSTEMS WITH HYSTERETIC NONLINEARITIES

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Abstract. *The work is dedicated to study of the dynamics of model systems that are described by differential equations with additive hysteretic nonlinearities in the case when a hysteretic loop bypass clockwise. In particular, for the model of the harmonic oscillator with hysteretic external force the conditions for existence of unbounded solutions are obtained. For the model of harmonic oscillator with a dry (Coulomb) and viscous friction under external hysteretic affection we obtained a set of conditions that ensure the self-oscillation mode. The numerical examples that illustrate the results of this work are also presented.*

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1 INTRODUCTION

Appearance of mathematical models of hysteretic phenomena is connected with the wide set of applied problems, generally in the theory of automatic control (see, e.g., [1,2,3] and related references). In these systems the sources with hysteretic properties can not be considered separately, but should be considered as a part of some complex system [1,4]. In this way the hysteretic nonlinearity should be taken into account on the step of modeling and should be included to the system of differential equations that describe the system under consideration. Here we should take a following note. In order to make an adequately description of the dynamics of real physical and mechanical systems, it is necessary to take into account the effects of hysteretic nature such as “backlash”, “stops” etc. For example, the backlashes inevitably appear in the mechanical systems during its long operation due to “aging” of the materials. The hysteretic phenomena (especially in the form of control parameters) play an important role in such fields as physics, chemistry, biology, economics etc. It should also be pointed out that the hysteretic phenomena (taking into account its physical nature) are insufficiently known in our days. This fact leads to exciting and interesting problems even in the well-studied classical systems.

Investigation of hysteretic phenomena has a long history. First works on the systems with hysteretic parts were made by A.A. Andronov in 1946. However the rigorous mathematical theory of hysteresis was formulated in the monography of M.A. Krasnoselskii and A.V. Pokrovskii [3] where the hysteretic phenomena are considered in the frame of the system’s theory. Namely, the hysteretic converters are treated as operators (such operators parametrically depend on its initial state) defined on a sufficiently rich functional space (for example, in the space of continuous functions). The dynamics of such converters are described by the relation of “input-state” and “state-output”.

An important problem is the study of resonance in systems with hysteresis [4]. In this way we note a well-known fact: in the presence of viscous friction the harmonic resonance is not realized (for details see, e.g., [6]). In particular, in [6] it is considered the dynamics of the oscillator with strong nonlinearity (authors studied its phase portrait and the trajectory). It is proved that the form of periodic solutions depends on the “origin” of strong nonlinearity. The main result of [6] is the fact that, for a class of equations that describe the harmonic oscillations with resonance external force and hysteretic operator in the right part of equation, the presence or absence of unbounded solutions depend on the amplitude of the external affection.

In this paper we study the resonance properties of systems in which the energy pumping in the system due to the presence of hysteresis level. Examples of such systems are the oscillations of the ferromagnetic ball in a magnetic field, oscillations of the system of coupled oscillators when the “connection force” has a hysteretic nature [7]. It should also be pointed out that these systems can be considered as an effective models in the solid state physics in the modeling of interatomic interactions taking into account various types of dislocations that arise due to material’s aging. Also such a model can be applied for the problem of viscoelasticity.

2 MODEL

Let us consider a system whose dynamics is described by the following equation with the corresponding initial conditions:

$$\begin{aligned} &\ddot{x} + \omega^2 x = R[\omega_0]x, \\ &x(0) = x_0,\ \dot{x}(0) = x_1. \end{aligned} \tag{1}$$

where R is an non-ideal relay operator with the negative spin and ω_0 is an initial state of the operator (see Figure 1).

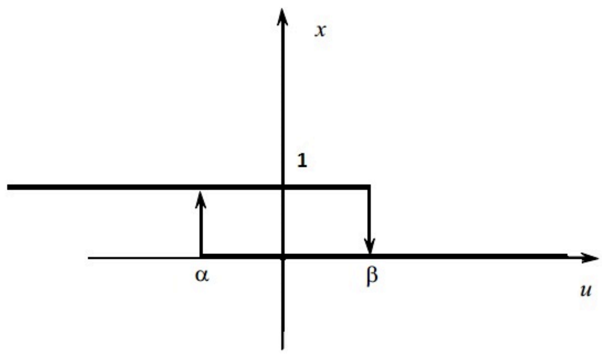

Figure 1: Non-ideal relay with negative spin

Theorem: Let the initial value satisfies the condition $x_0 \notin [\alpha;\beta]$. Then, the corresponding solutions are unbounded.
Proof: Let us assume that the initial conditions obey the following inequality $x_0 > \beta$, then at a certain initial period of time $(t \geq 0)$ the solution of the equation (1) will have the form $x_0(t) = A_1 \cos(t + \varphi_0) + 1, 0 \leq t \leq t_1$, where t_1 is the time at which the equality $x(t_1) = \alpha$ is obeyed. It is clearly that this moment exists. The solution of equation (1) at interval $[t_1 : t_2]$ will be determined by the relation $x_1(t) = A_1 \cos(t + \varphi_1) - 1$. Here t_2 is the moment at which the equality $x_1(t_2) = 1$ will be obeyed. It is also clearly that such a moment exists etc.

Thus, in the absence of switching the solution of the equation (1) is sewn of the functions defined by the following relations: for even *n* we have $x_n(t) = A_n \cos(t + \varphi_n) + 1$, and for odd *n* we have $x_{n+1}(t) = A_{n+1} \cos(t + \varphi_n) - 1$.

Using the continuity conditions for solution and its derivative at the point t_n we obtain the following equality

$$\begin{cases} A_n \cos\varphi_n + 1 = A_{n+1} \cos\varphi_{n+1} - 1, \\ -A_n \sin\varphi_n = -A_{n+1} \sin\varphi_{n+1}, \\ A_n \cos\varphi_n + 1 = 1. \end{cases} \tag{2}$$

Squaring and summing the first two equalities we obtain:

$$\begin{cases} A_n^{\ 2} \cos^2\varphi_n + 4A_n \cos\varphi_n + 4 = A_{n+1}^{\ \ 2} \cos^2\varphi_{n+1}, \\ A_n^{\ 2} \sin^2\varphi_n = A_{n+1}^{\ \ 2} \sin^2\varphi_{n+1}, \\ A_n \cos\varphi_n = 0. \end{cases}$$

Or

$$\begin{cases} A_n^{\ 2} + 4A_n \cos\varphi_n + 4 = A_{n+1}^{\ \ 2}, \\ A_n \cos\varphi_n = 0. \end{cases}$$

Finally we obtain:

$$A_n^{\ 2} + 4 = A_{n+1}^{\ 2}. \tag{3}$$

Similarly, for the next interval, at the point at which the solution has the value -1, we have:

$$\begin{cases} A_{n+1}\cos\varphi_{n+2} = A_{n+2}\cos\varphi_{n+3} + 1, \\ -A_{n+1}\sin\varphi_{n+2} = -A_{n+2}\sin\varphi_{n+3}, \\ A_{n+1}\cos\varphi_{n+2} = -1. \end{cases}$$

Squaring the first two equalities

$$\begin{cases} A_{n+1}^{\ 2}\cos^2\varphi_{n+2} - 2A_{n+1}\cos\varphi_{n+2} + 1 = A_{n+2}^{\ 2}\cos^2\varphi_{n+3}, \\ A_{n+1}^{\ 2}\sin^2\varphi_{n+2} = A_{n+2}^{\ 2}\sin^2\varphi_{n+3}, \\ A_{n+1}\cos\varphi_{n+2} = -1. \end{cases}$$

Then summarize them

$$\begin{cases} A_{n+1}^{\ 2} - 2A_{n+1}\cos\varphi_{n+2} + 1 = A_{n+2}^{\ 2}, \\ A_{n+1}\cos\varphi_{n+2} = -1. \end{cases}$$

Finally we get:

$$A_{n+2}^{\ 2} = A_{n+1}^{\ 2} + 3. \tag{4}$$

Then from (3) and (4) it follows: $A_{n+2}^{\ 2} = A_n^{\ 2} + 4$.

In other words, if the initial value is such that the hysteretic element "works", then the corresponding solution is unbounded.

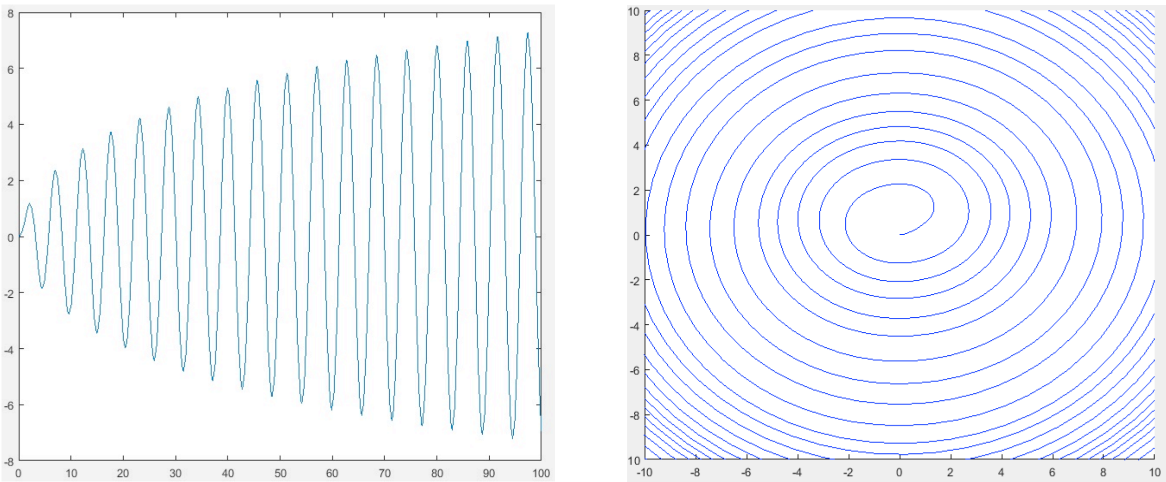

Figure 2: Solution (left panel) and phase portrait (right panel) of equation (1) with the given initial conditions

Note 1. Let us note that the solution oscillates and the rate of growth of amplitude is proportional to $\sqrt{t}$.

Note 2. The theorem remains valid for other types of hysteretic nonlinearities. Main requirement is the positiveness of the loop's area.

If the initial value lies on the segment $[\alpha;\beta]$, then the corresponding solution is limited on the whole positive time interval.

3 VISCOUS AND COULOMB FRICTION

Let us consider the various kinds of friction. The dynamics of oscillator with a viscous friction can be described by the equation:

$$\ddot{x} + 2\xi\dot{x} + \omega^2 x = R[\omega_0]x. \tag{5}$$

In the following consideration we assume that the threshold numbers of non-ideal relay are symmetric relative to origin. Considering the dynamics of the solution it should be noted that once the amplitude of the solution becomes high enough, the work of the friction force balances the energy obtained by the oscillator from the hysteretic element. Let us consider two cases related to the different kind of the roots of characteristic equation of the linear part of the equation (5).

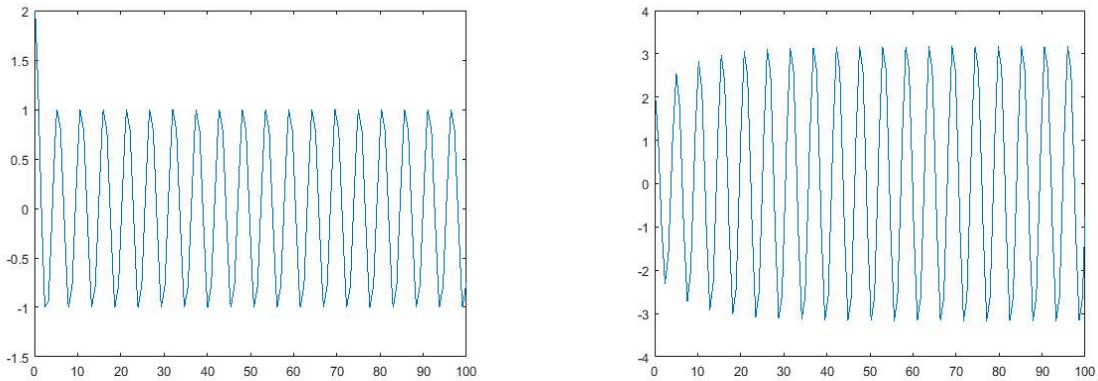

Figure 3: Oscillations in the system (5) at $\xi = 0.1; \xi = 1$ (left and right panels, respectively)

A qualitative change in the nature of the solution (compare the figures 3 and 4) becomes due to the fact that at $\xi = 2$ the roots of the characteristic equation are real. The nature of the oscillations has remained stable with the changed period.

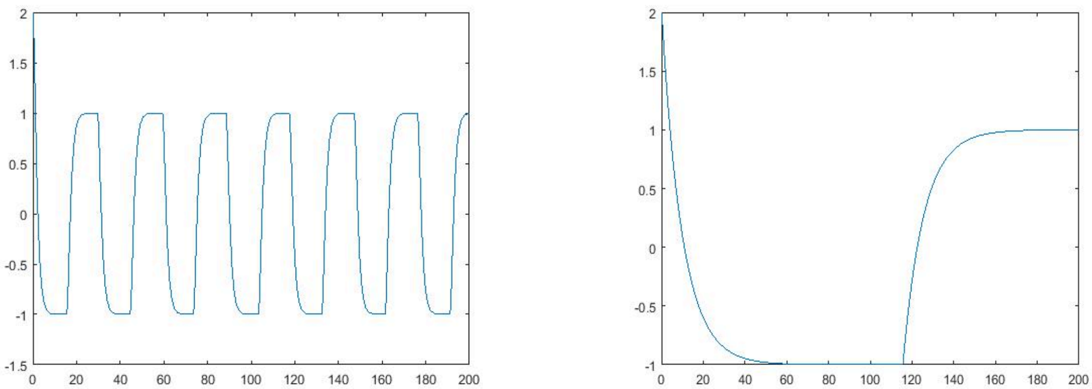

Figure 4: Oscillations in the system (5) at $\xi = 2.1; \xi = 10$ (left and right panels, respectively)

The dynamics of oscillator with the Coulomb friction under external hysteretic affection is described by the equation

$$\ddot{x} + \eta\,\mathrm{sgn}\,\dot{x} + x = R[\omega_0]x. \tag{6}$$

Multiplying both sides of (6) on $\dot{x}$ and integrating over the period T we obtain:

$$\int_0^T \frac{1}{2}\frac{d}{dt}\left(\dot{x}^2 + x^2\right) = -\eta(\operatorname{sgn}\dot{x})\dot{x} + \dot{x}R[\alpha,-\alpha]x, \tag{7}$$

$$\Delta E = -\eta\int_0^T |\dot{x}| dt + S_{loop}. \tag{8}$$

From the equation (5) it follows that the energy gain will be positive if the work of friction forces will be smaller than the loop's area (otherwise it will be negative). In this way the considered system can be treated as a system with the negative feedback. Let us note that at steady-state regime the condition $2(x_{\max} - x_{\min})\eta = S_{loop}$ is satisfied. This means that the amplitude of the oscillation is such that the work of frictional forces on the period is equal to the loop's area. These result illustrates the figure 5.

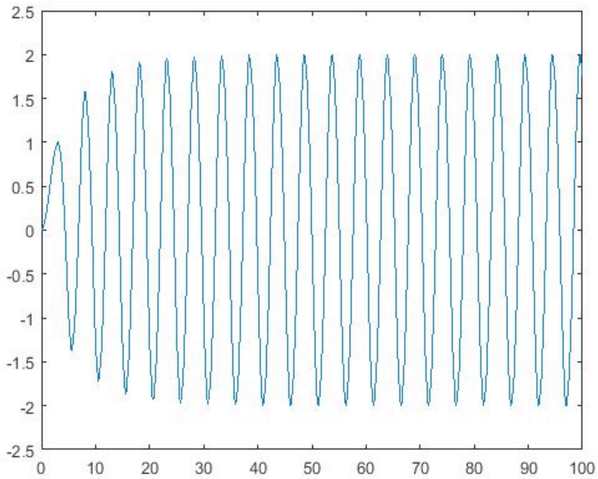

Figure 5: Oscillations in the system (6) at $\eta = 0.5$

4 CONCLUSIONS

In this paper we found that in the dynamical model of harmonic oscillator under external hysteretic affection (force with a hysteretic nature) with a negative spin the unbounded oscillations occur. It is shown also, that in the models of the harmonic oscillator with Coulomb and viscous frictions under hysteretic external force with negative spin the stable self-oscillations are realized.

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