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NON-SMOOTH CONTACT DYNAMICS OF PLANAR MASONRY STRUCTURES USING MATHEMATICAL PROGRAMMING

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Abstract. *In this paper an incremental formulation for contact dynamic analysis of masonry block structures is presented. The model is composed of rigid bodies interacting at potential contact points located at the vertexes of the block interfaces. A no-tension behavior with finite friction and infinite compressive strength is assumed at contact interfaces. The contact dynamic problem is governed by equilibrium equations, which relate external, inertial and contact forces, and by kinematic equations, which ensure compatibility between contact displacement rates and block degrees of freedom. Quadratic programming is used to solve the optimization problem arising from the formulation of the variational problem associated to dynamics of the block assemblages. To evaluate the accuracy and computational efficiency of the implemented formulation, applications to case studies from the literature are presented.*

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1 INTRODUCTION

In recent years, there has been a growing interest in the use of non-smooth contact analysis to dynamics and rocking behavior of masonry block structures [1-5].

This is mainly because this modelling approach allows to take into account contact interactions directly by setting up a system of proper inequalities and complementarity conditions rather than introducing additional elements in the models such in the case of distinct element models, where penalty methods are generally used.

In this perspective, starting from previous studies in the field of granular materials [6, 7], a non-smooth contact dynamic formulation for the analysis of masonry structures has been recently developed [8, 9] and herein applied to a numerical case study taken from the literature [10] to show potentialities of the adopted modelling approach.

As such, the paper is organized as follows. Sections 2 presents and overview of the adopted rigid block dynamic model. The static and kinematic variables as well as the relationships governing the behavior of the rigid block model are introduced. The quadratic programming formulation which is used to carry out the dynamic analysis of planar masonry assemblages is discussed in Section 3. In Section 4 an application of the proposed formulation to an in-plane wall panel is illustrated to show the ability of the implemented formulation to capture rocking behavior of multi-block assemblages.

2 THE RIGID BLOCK DYNAMIC MODEL

The masonry texture is discretized into a multi-body assemblage made of rectangular rigid blocks i interacting at potential contact points k located at the vertexes of the interface j (Fig. 1).

A no-tension and associative frictional behavior with infinite compressive strength is assumed at contact interfaces.

The dynamic model is formulated following the approach proposed in [6,7] for granular materials, though now detailed for two-dimensional assemblages of rectangular blocks.

The contact variables are the internal forces acting at each contact point k, which are located at a vertex of interface j of block i (Fig. 1a). These variables are collected in vector $\boldsymbol{c}$ and include the shear force component t_k and the normal force n_k along the local coordinate axes.

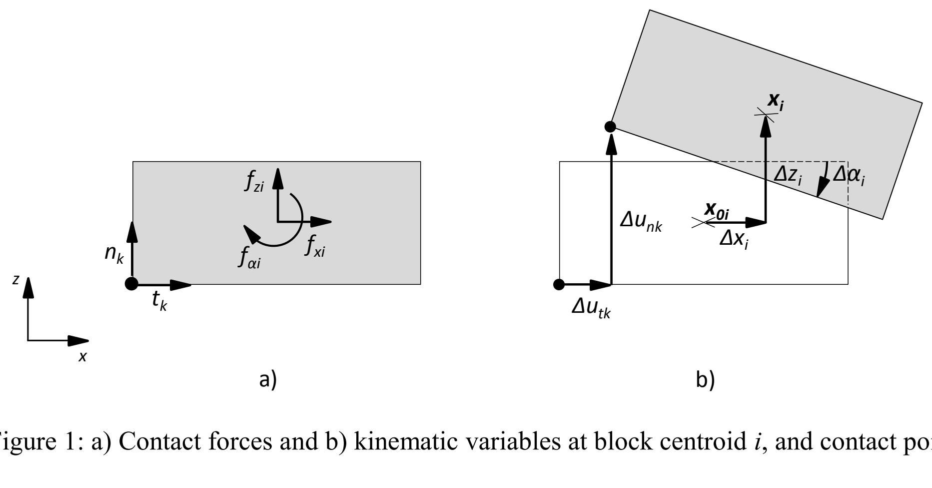

Figure 1: a) Contact forces and b) kinematic variables at block centroid i, and contact point k.

The kinematic variables associated in a virtual work sense to the contact forces are the relative displacement rates at the contact points, namely the tangential and normal displacement rates Δu_{tk} and Δu_{nk} (Fig. 1b), which are collected in the vector $\boldsymbol{\Delta u}$. The positions x_i, y_i and α_i related to the degrees of freedom of each block i are collected in the vector $\boldsymbol{x}$.

External loads applied to the centroid of rigid block i are collected in vector of external forces $\boldsymbol{f_{ext}}$ (Fig. 1a).

The equations of motions are discretized with respect to time using the θ-method and assuming:

$$\dot{\boldsymbol{x}}(t) = \frac{1}{\theta}\left[\frac{\Delta x}{\Delta t} - (1-\theta)\dot{\boldsymbol{x}}_{\boldsymbol{0}}\right] \tag{1}$$

where $\boldsymbol{\Delta x} = \boldsymbol{x} - \boldsymbol{x_0}$ is the displacement vector, $\boldsymbol{x_0}$ and $\dot{\boldsymbol{x}}_{\boldsymbol{0}}$ are the known position and velocity at time t_0 and $\theta \geq 0.5$.

On the basis of the incremental expression (1), the equations of motion of the rigid block assemblage interacting at potential contact points can be posed as follows:

$$\boldsymbol{\bar{M}\Delta x} + \boldsymbol{A_0 c} = \boldsymbol{\bar{f}_0} \tag{2}$$

where:

$\boldsymbol{A_0}$ is the equilibrium matrix corresponding to contact forces; (3)

$\boldsymbol{\bar{M}} = \frac{1}{\theta \Delta t^2}\boldsymbol{M}$, being $\boldsymbol{M}$ the mass matrix collecting the mass m_i and the mass moment of inertia J_i of each block; (4)

$\boldsymbol{\bar{f}_0} = \boldsymbol{f_{ext}} + \boldsymbol{\bar{M}}\dot{\boldsymbol{x}}_0 \Delta t$, (5)

Contact conditions are expressed imposing that the normal component of the relative displacement at contact point k has not to be greater than the initial gap, as follows:

$$\boldsymbol{\hat{n}_{0k}^{\mathrm{T}}}\, \boldsymbol{\Delta u_k} \leq \mathrm{g}_{0k} \tag{6}$$

where $\boldsymbol{\hat{n}_{0k}}$ is the initial normal associated with the surface j and g_{0k} is the initial gap.

For contact interactions, a complementarity condition is included as follows:

$$n_k\left[\boldsymbol{\hat{n}_{0k}^{\mathrm{T}}}\, \boldsymbol{\Delta u_k} - \mathrm{g}_{0k}\right] = 0 \tag{7}$$

Considering that $n_k \geq 0$, this condition ensures that contact forces occur only if the gap is closed otherwise are zero.

Eqs. (6), (7) express in matrix form the so-called Signorini unilateral contact conditions at a contact point k in terms of displacement rates.

The behaviour at contact interfaces undergoing sliding failure is governed by failure conditions which are expressed according to the Coulomb friction law.

In vector notation, the limit conditions for sliding failure can be written as:

$$\pm\boldsymbol{t} \leq \mu\boldsymbol{n} \tag{8}$$

where μ is the friction coefficient.

3 FORMULATION OF THE QP PROBLEM AND IMPLEMENTATION

Under the assumption of associative flow rule for displacement increments, the equilibrium equations (2), kinematic conditions (6-7) and sliding friction conditions (8) are equivalent to the following force–based problem [6-9]:

$$\begin{aligned} \max \quad & -\tfrac{1}{2}\boldsymbol{r}^{\mathrm{T}}\,\overline{\boldsymbol{M}}^{-1}\boldsymbol{r} - \boldsymbol{g}_{\boldsymbol{0}}^{\mathrm{T}}\boldsymbol{c} \\ \text{s.t.} \quad & \boldsymbol{r} + \boldsymbol{A}_{\boldsymbol{0}}\boldsymbol{c} = \overline{\boldsymbol{f}}_{\boldsymbol{0}} \\ & \pm\boldsymbol{t} - \mu\boldsymbol{n} \leq \boldsymbol{0},\;\; \boldsymbol{n} \geq \boldsymbol{0} \end{aligned} \tag{9}$$

where $\boldsymbol{r}$ is the vector of dynamic forces associated with the degrees of freedom of the blocks.

The optimization problem (9) was the problem which was actually implemented and solved in the present formulation, with kinematic variables derived as Lagrange multipliers associated with the various constraints, according to the solution procedure detailed in the following section.

To calculate and update positions of the blocks and contact gaps, a simple incremental procedure was implemented [8, 9].

The procedure was implemented in a computer code, *DynoBlock_2D*, which provides as outputs the time histories of contact forces and kinematic variables as well as the plots of the failure mechanisms at different time steps.

The quadratic programming problem associated to (9) were solved using the primal–dual interior-point solver in MOSEK [11].

The analyses were carried out using a PC containing a 3.3GHz Intel Xeon E3-1245 processor with 8 GB of RAM.

4 DYNAMIC ANALYSIS OF A BLOCK MASONRY PANEL UNDER GROUND ACCELERATION TIME HISTORY

An application of the adopted formulation to the case of a block masonry wall tested by Restrepo-Vélez et al. [10] is reported in this section.

The experimental wall panel comprises a front panel and three side walls. The panel was built adopting a scaling factor of 1:5 and using marble blocks with nominal dimensions of 80×40×30 mm which were dry-jointed. The masonry wall panel was tested on a tilting table to investigate the failure mechanism and the collapse load under constant and uniformly distributed horizontal acceleration.

For numerical analysis, a 2D rigid block model of the side wall was generated (Fig. 2). The average value of the unit weight of the blocks was 26.8 kN/m^3 and the friction coefficient used in the computations was 0.7, in accordance with values measured experimentally. The mass of the half blocks along the sides of the wall was increased to take into account the effects of the orthogonal walls on the failure mechanism considering an effective length of 3.5 bricks. A time increment of 0.001 sec. was used and the θ value was set equal to 0.7.

The numerical model was first validated against experimental test on the tilting table of the small scale specimen using a quasi-static analysis [8].

Afterwards, in order to evaluate the computational cost of the adopted formulation in the case of a real scale structure subjected to earthquake ground motion, a full scale numerical model was implemented to perform dynamic time history analysis. The full scale numerical model was generated from the small-scale model, using a scaling factor of 5 so to comply with geometric dimensions of a real structure.

The failure mechanism predicted by the quasi-static analysis is shown in Figure 2b and the corresponding acceleration magnitude is 0.199g (Table 1). A good agreement with experimental outcomes was observed. The crack pattern in the numerical model mainly develops on the left pier and involves the spandrel as well. The stepped diagonal cracks in the spandrel follow the crack pattern observed after experimental investigation.

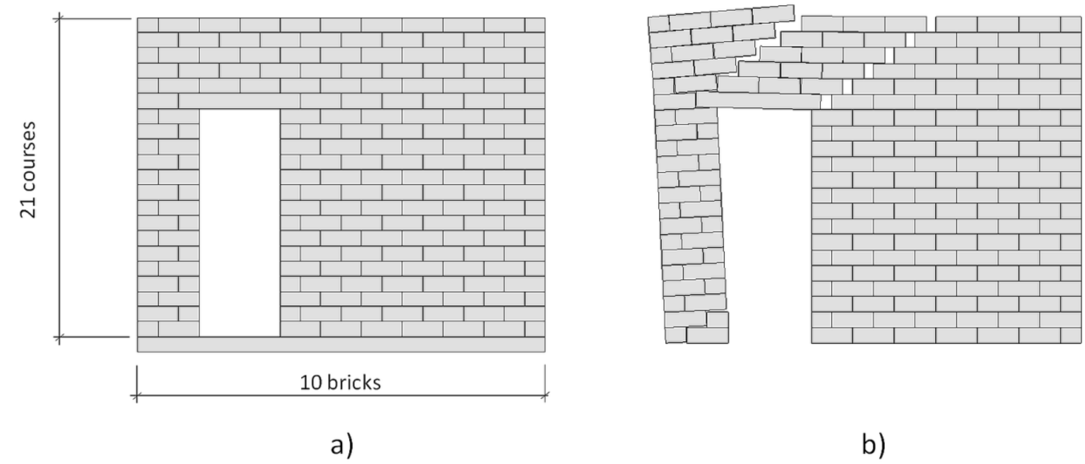

Figure 2: S22 wall panel [Restrepo Vélez et al., 2014]: a) rigid block model used for contact dynamic analysis; b) Failure mode obtained for quasi-static analysis under constant and uniformly distributed horizontal acceleration.

Case study	Model size (*b* × *c*)	Exp. (Restrepo Vélez et al. 2014)	Full scale model			
			Quasi-static analysis		Acceleration time history	
		λ	a_g (g)	CPU Time (s)	a_g (g)	CPU Time (s)
S22	196 × 1072	0.197	0.199	940	1.06	4210

TABLE 1: Block masonry wall S22: ground acceleration at failure and CPU time.

For seismic analysis on the full scale model, the acceleration time series AQA.HNN recorded by the Italian Civil Protection Department seismic network (IT) at L'Aquila, N-S component, occurred on 06.04.2009, was considered [12]. The peak amplitude of the ground acceleration is 0.44g.

A set of dynamic time history analysis was carried out with increasing level of excitation, considering for the record the time interval from 15 to 22 s. For values of the scaling factor greater than 240% AQA, the sliding displacement computed for the lintel was larger than the size of block staggering (half a block), thus involving collapse of the spandrel on the top and overturning of the left pier.

The results of the seismic analysis for AQA earthquake with acceleration amplitudes scaled by the factor of 2.4 (240% AQA) are shown in Figs. 3-4.

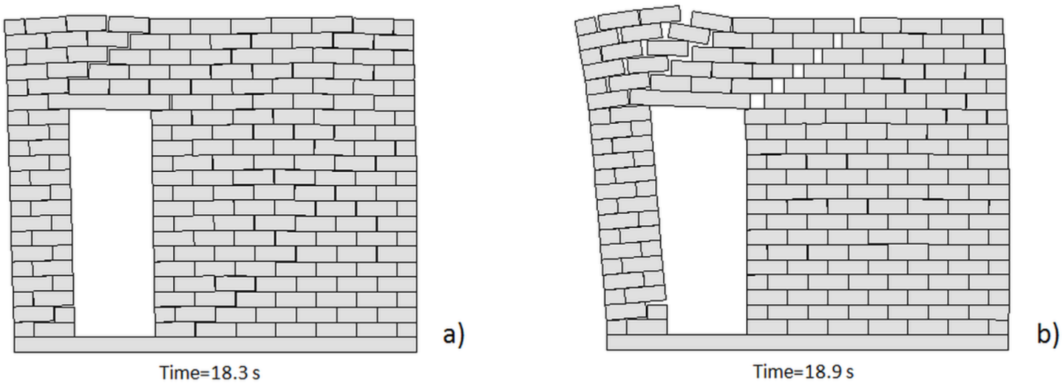

Figure 3: Time history response to the 240% AQA earthquake.

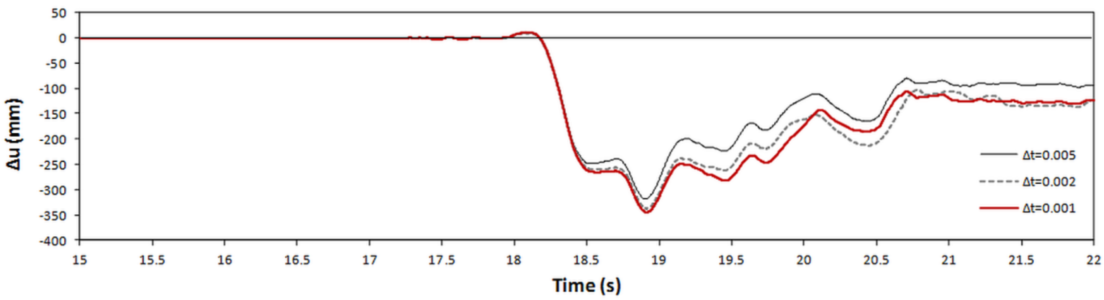

Figure 4: Time history of top relative displacement and sensitivity analysis to time increment.

The results show that in the case of AQA earthquake the seismic resistance of the structure computed from dynamic analysis is up to 5 times the value computed for quasi-static overturning under uniform and constant distribution of acceleration. The difference with quasi-static results should be ascribed to the combined effect of impulsive actions associated to the earthquake motion and to the displacement capacity related to the regular staggering of blocks and their aspect ratio. However, considering that the results of the numerical dynamic analysis are not directly supported by experimental tests and that simple assumptions were adopted for contact point specification, those should be intended as indicative of the differences with quasi-static results.

Sensitivity analyses to time increment were also carried out. Figure 4 shows the time history response computed for the relative horizontal displacement calculated between the top and base blocks. It can be seen that although a remarkable difference of predicted residual displacement is observed using a step of 0.005 s, the peak response is slightly affected by time increment in the considered range.

5 CONCLUSIONS

- A variational formulation for dynamic analysis of masonry block structures was presented in this paper. The formulation assumes a no-tension, frictional behavior at contact joints and uses quadratic programming to solve the contact dynamic problem.
- A quasi-static analysis of a dry-jointed masonry block panel from the literature was carried out using the formulation adopted and a good agreement in terms of failure mechanism and collapse load was observed.
- Dynamic time history analysis under ground acceleration motion was carried out to compare numerical results and to evaluate the computational efficiency and the convergence stability of the implemented procedure. Those were found to be encouraging, also considering the small number of mechanical and algorithm parameters associated to the adopted formulation.

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