Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2017) 4381-4392

ECCOMAS Proceedia

COMPDYN 2017
6th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15–17 June 2017

ON THE DESIGN OF PERFORMANCE-BASED PENTAMODE BEARINGS

A. Amendola[1], M. Titirtla[1], R. Penna[1], F. Fabbrocino[2]

[1]Department of Civil Engineering, University of Salerno,
84084 Fisciano (Salerno), Italy
adaamendola1@unisa.it (A. Amendola), titirlamagdalini@gmail.com (M. Tititrla)
rpenna@unisa.it (R. Penna),

[2]Pegaso University, Department of Engineering
80132 Naples, Italy
francesco.fabbrocino@unipegaso.it (F. Fabbrocino)

Keywords: Pentamode Lattices, Seismic Isolation, Hardening, Tension-Capacity

Abstract.

Pentamode lattices are particular metamaterials belonging to the class of extremal materials, which feature a primitive unit cell equipped with four rods meeting at a point. The potential of confined pentamode lattices in different engineering fields, such as, e.g., structural engineering, has not been largely explored yet. In this area, what is particularly interesting is the confinement of pentamode structural "crystals" between stiffening plates for the design of novel impact or seismic protection devices. Such a research line has recently appeared in the literature, with the aim of developing performance-based, vibration-isolation devices. The present study makes use of discrete-to-continuum approaches to the elastic moduli of pentamode lattices, and investigates the feasibility of pentamode structures as innovative anti-seismic devices. Experimental results and analytic formulae are employed to understand the mechanics of pentamode structures equipped with rigid and hinged connections, and the role played by design variables characterizing the aspect ratio of the structure and the response of the junctions. The final part of the work deals with the design of pentamode bearings that feature stiffness and strength properties similar to those of a commercial rubber bearing available on the market for the seismic isolation of buildings and bridges.

Peer-review under responsibility of the organizing committee of COMPDYN 2017.
doi: 10.7712/120117.5731.17751

1. INTRODUCTION

Pentamode lattices are mechanical metamaterials that exhibit the minimal coordination number required to achieve a fully positive definite elasticity tensor in three dimensions [1]-[11]. The peculiar mechanical behavior of pentamode lattices motivates researchers to investigate their use in transformation acoustics and elasto-mechanical cloak [6]. Their potential in different engineering fields, such as, e.g., structural engineering, has not been largely explored yet. In this area, what is particularly interesting is the use of pentamode structural "crystals" for the design of novel impact or seismic protection devices. Such a research line has recently appeared in the literature, with the aim of developing performance-based, vibration-isolation devices [8]-[11]. The results obtained so far in this field have shown several analogies between the mechanics of confined pentamode lattices [8]-[11] and that of elastomeric and triple friction pendulum bearings [12]- [25].

The present study aims at developing discrete-to-continuum approaches to the elastic moduli of pentamode lattices, and investigating the feasibility of pentamode structures as innovative anti-seismic devices. Our approach combines experimental results and analytic formulae to understand the mechanics of pentamode structures equipped with rigid connections, and the role played by design variables characterizing the aspect ratio of the structure and the response of the junctions. We begin with an experimental of the mechanical theory presented in [3] by comparing the theoretical previsions of such a theory with experimental results examining the mechanical response of additively manufactured, physical models of confined pentamode lattices [8]. We then pass to examine the response of computational models of pentamode bearings in the small strain regime, on comparing such a response with that of a commercial rubber bearing. We end by reviewing of the main results of the present study and drawing directions of future research.

2. COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL MODULI OF CONFINED PENTAMODE LATTICES

We present an experimental validation of the theoretical predictions of the pentamode elastic moduli given in Ref. [3] against experimental values obtained through quasi-static laboratory tests on physical samples of pentamode structures. The latter were additively manufactured through Electron Beam Melting (EBM), as described in Ref. [8]. Such samples are made of a Ti-6Al-4V titanium alloy featuring mass density ρ_0 = 4.42 g/cm^3; yield strength σ_{y_0} = 910 MPa; Young's modulus E_0 = 120 GPa; and Poisson's ratio υ_0 = 0.342. Let n_x, n_y and n_z respectively denote the number of unit cells placed along the x, y and z axes in the generic layer, with the z-axis placed in the vertical direction.

The experimental validation presented in the current section relates experimental and numerical values of vertical and horizontal stiffness properties of single-layer pentamode lattices featuring thick and slender macroscopic aspect ratios. The analyzed lattices are composed of two fcc unit cells in the horizontal plane ($n_a = n_x = n_y = 2$) and varying number of unit cells along quasi the z-axis. Following the notation given in Ref. [8], we hereafter name TPM the systems featuring $n_v = n_z = 2$ (*"thick pentamode materials")*, and SPM systems featuring $n_z = 4$ (*"slender pentamode materials"*). For each of such systems, we analyze physical samples featuring different values of d (d_1, d_2, d_3) and fixed values of a

and D, as shown in Table 1. We use the label i to denote the SPM /TMP sample featuring $d = d_i$.

	a [mm]	D [mm]	d_1 [mm]	d_2 [mm]	d_3 [mm]
Built size	30	2.72	0.49	1.04	1.43
(CAD size)	(30)	(2.71)	(0.45)	(0.90)	(1.35)

Table 1. Geometrical properties of physical samples of confined fcc pentamode lattices.

Table 2 shows the mean values and standard deviations of the experimental values of the secant horizontal stiffness $K_{h,eff}$, which were obtained for the above specimens under the analyzed values of the vertical load F_v. Such a quantity was defined as follows

$$K_{h,eff} = \frac{\left|F_h{}^{+}\right| + \left|F_h{}^{-}\right|}{\left|\delta_l^{+}\right| + \left|\delta_l^{-}\right|} \tag{1}$$

where the symbols + and – denote the maximum and minimum values, respectively, of the lateral force and the lateral displacement recorded during the shear tests presented in [8]. The results in Table 3 include the effective shear modulus defined as follows

$$G_{eff} = K_{h,eff} \cdot \frac{H}{A} \tag{2}$$

where H denotes the height of the specimen, and $A = 2a \times 2a$ denotes the lattice covered area of the terminal plates. The effective modulus G_{eff} is compared with the theoretical shear modulus G_{th} obtained through the scale-bridgeing theory presented in Ref. [3]. The results in Figure 2 and Table 2 highlight a marked increase of $K_{h,eff}$ with increasing values of d/a, which follows by the increase in the bending rigidity of the lattice for growing sizes of the nodal junctions. Table 2 also shows a small dispersion of $K_{h,eff}$ with the vertical load F_v in the small displacement regime. We observe that G_{eff} is always larger than G_{th}, which is explained by the stiffening effect of the terminal plates in the physical models of pentamode lattices. Concerning SPM systems, the difference between G_{eff} and G_{th} is equal to 35.62% in SPM1, 26.91% in SPM2 and 27.02% in SPM3. For what instead concerns TPM systems, the difference between G_{eff} and G_{th} is equal to 6.18% in TPM1, 389.7% in TPM2 and 226.67% in TPM3.

The experimental results given in Ref. [8] include vertical force (F_v) vs. vertical displacement (δ_v) tests, on considering one specimen for each examined d/a ratio, and applying incremental weights on top of the specimens in quasi-static conditions (no lateral displacements were applied during such tests). We let $K_{v,eff}$ denote the effective stiffness defined as follows

$$K_{v,eff} = \frac{F_v^{\max}}{\delta_v^{\max}} \tag{3}$$

where $F_v^{\max}$ denotes the maximum applied vertical load and $\delta_v^{\max}$ denotes the corresponding vertical displacement. Table 2 shows the experimental values of the secant stiffness $K_{v,eff}$ obtained for SPM and TPM specimens, and the values of the effective Young's modulus defined as follows

$$E_{eff} = K_{v,eff} \cdot \frac{h}{A} \tag{4}$$

The experimental moduli (4) are compared in Table 2 with the theoretical moduli E_{th} obtained through the mechanical theory presented in in Ref. [3]. Comparing results in Table 2 for SPM systems we observe that E_{th} assumes values very similar to G_{th} and that E_{eff} is markedly greater than E_{th}, especially in presence of low d/a ratios. The difference between E_{eff} and E_{th} is indeed equal to 85.82% in SPM1, 37.67% in SPM2 and 1.72% in SPM3, and is again explained by the presence of the terminal plates (as in the case of the increase of G_{eff}/G_{th}). It is interesting to note that both the G_{eff}/G_{th} and E_{eff}/E_{th} ratios decrease with increasing values of the d/a ratio. Concerning TPM systems, we note that the difference between E_{eff} and E_{th} is indeed equal to 1.75% in TPM1, 1505.93% in TPM2 and 810.18% in TPM3. Overall, we are led to conclude that the elastic response of confined pentamode lattices is rather different from that of the unconfined pentamode lattices analyzed in Ref. [3], due to the marked confinement effect played by the stiffening plates against the deformation of the structure.

		SPM1	SPM2	SPM3	TPM1	TPM2	TPM3
$K_{h,eff}$	mean (std Dev)	2.02 (0.20)	13.23 (0.24)	30.73 (0.60)	13.1 (-0.12)	62.67 (-0.27)	115.54 (-0.66)
G_{eff}		70.43	444.77	1033.01	222.03	1061.89	1957.76
G_{th}		45.34	325.07	972.84	45.34	325.07	972.84
$K_{v,eff}$		15.55	95.22	295.81	43	175.03	470.5
E_{eff}		320,04	522,65	994,25	728.61	2965.27	7963.88
E_{th}		45.37	325.79	977.14	45.37	325.79	977.14

Table 2. Experimental values of the effective values of the horizontal stiffness $K_{h,eff}$ (N/mm); the shear modulus G_{eff} (kPa); the vertical stiffness $K_{v,eff}$ (N/mm), and the Young modulus E_{eff} (kPa) of SPM and TPM specimens. G_{th} and E_{th} denote the theoretical values of the shear modulus and Young modulus obtained through the mechanical model presented in Ref. [3].

3. ENGINEERING APPLICATIONS OF PENTAMODE BEARINGS

The present section aims at investigating the use of pentamode lattices as next-generation seismic isolators. According to the European Standard EN 15129 [21], a seismic isolator is a "*device possessing the characteristics needed for seismic isolation, namely, the ability to support a gravity load of superstructure, and the ability to accommodate lateral displacements*". Our current goal is to compare the mechanical response of computational models of pentamode lattices with that of a commercial rubber bearing with diameter $\varnothing 33.5"\ (0.8509\,m)$ and height $13.85"\ (0.3518\,m)$, which is composed of 29 rubber layers of $7\,mm$ each; 28 steel shims (spacers) of $3.04\,mm$ each; two terminal rubber layers of $31.8\,mm$ each and covers, without lead plug (isolator Type E [RB-800] produced by Dynamic Isolation Systems, Inc, McCarran, NV, USA).

As we have already observed, the multiscale approach to the elastic moduli of pentamode lattices presented in Ref. [3] is not accurate in predicting the elastic response of confined pentamode lattices, since it ignores the confinement effect played by the stiffening plates. In consideration of this, we now make use of the theory presented in Ref. [10] to design pentamode bearings that are equipped with hinged (stretching-dominated regime), and are obtained by repeating in the 3D space a sub-lattice of the fcc unit cell characterizing ordinary pentamode lattices. The latter is formed by two primitive cells, in place of the four primitive unite cells forming the fcc cell (sfcc pentamode lattices, cf. Fig. 1 of Ref. [10]).

We aim at showing that it is possible to design sfcc pentamode bearings equipped with hinged connections that feature the same vertical stiffness K_v of the commercial rubber bearing under consideration. We focus our attention on a square multi-layer pentamode bearing with edge length L, which is formed by $n_a \times n_a$ sfcc unit cells in the horizontal plane and n_z layers, denoting the solid volume fraction of the sfcc unit cell by ϕ. Making use of the design formula given in Sect. 5 of Ref. [10], we obtain

$$\frac{2}{9} E_0 L \frac{n_a}{n_z} \phi = K_v \tag{5}$$

where E_0 denotes the Young modulus of the rods. In order to avoid material yielding under vertical loading, we also make use Eqn. (30) of Ref. [10] to predict the incremental axial stress induced by an incremental vertical displacement $\dot{w}$ of the device from the reference configuration, which corresponds to an incremental vertical displacement of the generic layer equal to $\dot{w}/n_z$. By setting such an incremental stress equal to $f_y \alpha$, where f_y denotes the yielding stress of the material and α denotes a safety factor, we obtain the second design formula that follows

$$\frac{2}{3} E_0 n_a \frac{\dot{w}}{n_z L} = f_y \alpha \tag{6}$$

Upon solving the above design formulae (5) and (6) for n_a and ϕ, we obtain

$$n_a = \frac{3\alpha f_y n_z L}{2E_0 \dot{w}} \quad , \quad \phi = \frac{3K_v \dot{w}}{\alpha f_y L^2} \tag{7}$$

Let us now employ steel bars grade S335JH, with Young modulus $E_o = 210$ GPa and yield strength $f_y = 355$ MPa [27][28], and let us assume $L = 600$ mm, $\dot{w} = 1.00$ mm, $\alpha = 0.5$, and and $K_v = 733$ kN/mm. Table 5 and Figure 5 illustrate different designs of sfcc pentamode bearings that correspond to such a set of design parameters, and exhibit different numbers of layers n_z. It is seen that the bilayer pentamode bearing of design (c) exhibits horizontal and vertical dimensions rather similar to those of the commercial bearing under consideration.

	n_z	n_a	a [mm]	H [mm]
(a)	3	2	300	450
(b)	4	3	200	400
(c)	2	2	300	300

Table 3. Geometrical properties of square sfcc pentamode bearings replicating a commercial rubber bearing.

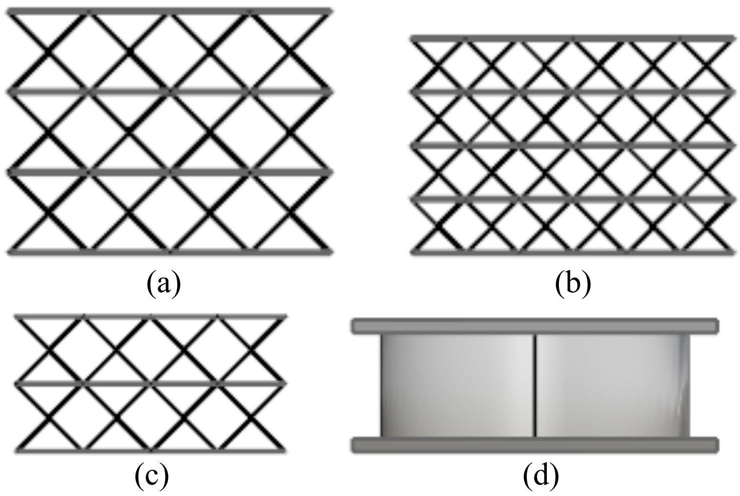

Figure 1. Designs of sfcc pentamode bearings (a, b, c) that exhibit equal vertical stiffness and load-carrying capacity of a commercial rubber-bearing (d).

4. CONCLUDING REMARKS

We have presented novel versions of pentamode materials: artificial structural crystals showing shear moduli markedly smaller than the bulk modulus [3]-[5]. On examining fcc pentamode bearings equipped with rigid connections of variable size and geometry, we have observed that the bulk modulus of such systems is always larger than the shear modulus. The validation of theoretical predictions of the elastic moduli of such systems against the experimental data presented in Sect. 3 highlighted that the elastic response of confined pentamode lattices is rather different from that of the unconfined pentamode lattices analyzed in Ref. [3], due to the marked confinement effect played by the stiffening plates against the deformation of the structure.

As we already noticed, the ability of pentamode lattices to show very soft and very stiff deformation modes, which has been recognized both in the bending- [9] and in the streching-dominated [10] regimes of such structures, suggests they are potentially suitable for use as seismic isolators [8][9][29][30]. The major analogy between the mechanics of confined pentamode lattices and that of seismic isolation devices alternating rubber layers and stiffening plates [8][11][15] consists of the fact that, in both cases, the plates forming such laminated structures stiffen the compressive deformation mode of the system, and, at the same time, keep its compliance against shear actions sufficiently large.

Unlike most other seismic isolators, where the response depends entirely on the properties of the materials used, we wish to remark the response of pentamode lattices depends mostly on their geometry. This is advantageous, as their response can be easily tuned by altering the geometry to control the vertical and horizontal stiffness for each application [8]. As a matter of fact, by simulating the mechanical response of physical models of sfcc systems in the large elastic strain regime, it has been observed a stiffening effect in terms of the lateral force vs. lateral displacement response with increasing amplitude of lateral displacements (cf. Ref. [11]). It is worth noting that a similar hardening response is a desirable performance for seismic isolators potentially experiencing large displacements, like, e.g., triple pendulum bearings [18][19]. In addition, pentamode bearings are tension-capable, i.e., can bear both compression and tension vertical loads during seismic excitations, due to the nonzero tensile strength of the rods forming the pentamode lattices (refer, e.g., to the recent paper [17], and references therein, for the technical relevance of tension-capable bearings).

In Sect. 3 we have designed innovative seismic isolators based on either fcc or sfcc pentamode lattices confined between stiffening plates. The soft modes of such materials have been controlled by tuning of the mechanical properties and the geometry of members and junctions. We have shown that such novel isolators can exhibit mechanical response similar to that of conventional rubber bearings, provided that the material and the geometry of the lattice are suitably designed.

We may conclude that pentamode bearings offer several potential advantages over traditional structural bearings [16]-[18], which essentially follow from the fact that the mechanical properties of such systems are driven by the geometry of the lattice microstructure, more than the chemical composition of the material (i.e, such systems behave as mechanical metamaterials). It is worth noting that it is possible to play with the lattice microstructure in order to achieve the desired combination of shear and compression moduli. Additionally, the choice of the material offers additional design opportunities, both in terms of the elastic response, and for what concerns the energy dissipation properties of the system (cf. Ref.[8]). We have also observed that it is possible to design laminated structures that feature multiple pentamode layers equipped with different materials and properties, while in laminated rubber the only lamination variable relative to the soft layers consists of the use of natural or synthetic rubber (Sect. 3) [31]-[33]. Finally, pentamode bearings can be manufactured on employing rapid prototyping techniques that make use of single or multiple materials (metals, polymers, etc.) [31]-[36].

Future extension of the present research will regard experimental testing of real-scale physical models of pentamode bearings [38]; the modeling of facture damage in the rods of pentamode bearings under stress [39]; discrete-to-continuum approaches to the mechanics of multilayered structures [67]-[69]; and the use of pentamode lattices within innovative materials and structures [70]-[71].

REFERENCES

[1] G.W. Milton, A.V. Cherkaev, "*Which Elasticity Tensors are Realizable?*" J Eng Mater-T 1995; 117(4), 483-493.

[2] G. Gurtner, M. Durand, 2014. Stiffest elastic networks, Proc. R. Soc. A 470, 20130611.

[3] A.N. Norris, 2014. Mechanics of elastic networks. Proc. R. Soc. A 470, 20140522.

[4] M. Schittny, T. Bückmann, M. Kadic, M. Wegener, "*Elastic measurements on macroscopic three-dimensional pentamode metamaterials*". Appl Phys Lett 2013; 103.

[5] M. Kadic, T. Bückmann, N. Stenger, M. Thiel, M. Wegener, "*On the practicability of pentamode mechanical metamaterials*". Appl Phys Lett 2012; 100.

[6] T. Bückmann, M. Thiel, M. Kadic, R. Schittny, M. Wegener, "*An elastomechanical unfeelability cloak made of pentamode metamaterials*". Nat Common 2014; 5, 4130.

[7] M. Kadic, T. Bückmann, R. Schittny, Wegener M., 2013. On anisotropic versions of three-dimensional pentamode metamaterials. New J. Phys. 15, 023029.

[8] A. Amendola, C.J. Smith, R. Goodall, F. Auricchio, L. Feo, G. Benzoni, F. Fraternali, Experimental response of additively manufactured metallic pentamode materials confined between stiffening plates. *Composite Structures*, **142**, 254-262, 2016.

[9] A. Amendola, G. Carpentieri, L. Feo, F. Fraternali, 2016. Bending dominated response of layered mechanical metamaterials alternating pentamode lattices and confinement plates. Compos. Struct. 151,71-77.

[10] F. Fraternali, A. Amendola, 2017. Mechanical modeling of innovative metamaterials alternating pentamode lattices and confinement plates. J. Mech. Phys. Solids, 99, 259-271.

[11] A. Amendola, G. Benzoni, F. Fraternali, 2016. Non-linear elastic response of layered structures, alternating pentamode lattices and confinement plates. Compos Part B-Eng 2016, http://dx.doi.org/10.1016/j.compositesb.2016.10.027.

[12] R.I. Skinner, W.H. Robinson, G.H. Mcverry, 1993. An introduction to seismic isolation. Wiley.

[13] J.M. Kelly, 1993. Earthquake-resistant design with rubber. London: Springer-Verlag.

[14] G. Benzoni, C. Casarotti, 2009. Effects of Vertical Load, strain rate and cycling on the response of lead-rubber seismic isolators. J. Earthquake Eng. 13(3), 293-312.

[15] M. Higashino, H. Hamaguchi, S. Minewaki, S. Aizawa, 2003. Basic characteristics and durability of low-friction sliding bearings for base isolation. Earthquake Eng. Eng. Seismolog. 4(1), 95-105.

[16] G. P. Warn, A. S. Whittaker, M.C. Constantinou, 2007. Vertical stiffness of elastomeric and lead- rubber seismic isolation bearings. J. Struct. Eng.-ASCE 133 (9), 1227-1236.

[17] Z. Zhou, J. Wong, S. Mahin, 2016. Potentiality of using vertical and three-dimensional isolation systems in nuclear structures, Nucl. Eng. Technol, 48 (5), 1237-1251.

[18] T.C. Becker, S.A. Mahin, 2012. Experimental and analytical study of the bi-directional behavior of the triple friction pendulum isolator. Earthquake Eng. Struct. Dyn. 41, 355–373.

[19] D.M. Fenz, M.C. Constantinou, 2008. Modeling triple friction pendulum bearings for response-history analysis. Earthq. Spectra 24, 1011-1028.

[20] D. R. Pant, A. C. Wijeyewickrema, M. A. ElGawady, 2013. Appropriate viscous damping for nonlinear time-history analysis of base-isolated reinforced concrete buildings. Earthquake Engng Struct. Dyn. 42(4), 2321-2339.

[21] European Committee for Standardization. Anti-seismic devices, EN 15129. Brussels, Belgium, 2009.

[22] M. Higashino, H. Hamaguchi, S. Minewaki, S. Aizawa. "*Basic characteristics and durability of low-friction sliding bearings for base isolation*". Earthquake Engineering and Engineering Seismology 2003; 4(1), 95-105.

[23] Toopchi-Nezhad H, Tait MJ, Drysdale RG. Bonded versus unbonded strip fiber reinforced elastomeric isolators: finite element analysis. Compos Struct 2014;93(2):850-9.

[24] Osgooei PM, Tait MJ, Konstantinidis D. Three-dimensional finite element analysis of circular fiber-reinforced elastomeric bearings under compression. Compos Struct 2014;108(1):191-204.

[25] Osgooei PM, Tait MJ, Konstantinidis D. Finite element analysis of unbonded square fiber-reinforced elastomeric isolators (FREIs) under lateral loading in different directions. Compos Struct 2014;113(1):164-73.

[26] J.M. Kelly, J.-W Lai, "*The use of tests on high-shape-factor bearings to estimate the bulk modulus of natural rubber*". Seismic Isolation and Protective Systems 2011; 2(1), 21-33.

[27] EN 10210-2:2006, Hot finished structural hollow sections of non-alloy and fine grain steels - Part 2: Tolerances, dimensions and sectional properties.

[28] European Commission, 2014. HORIZON 2020 Work Programme 2014-2015, PART 5.ii, 98.

[29] Fraternali F., Carpentieri G., Montuori R., Amendola A., Benzoni G., 2015.On the use of mechanical metamaterials for innovative seismic isolation systems, COMPDYN 2015 - 5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, 349-358.

[30] Fabbrocino, F., Amendola, A., Benzoni, G., Fraternali, F., Seismic application of pentamode lattices. Ingegneria Sismica/International Journal of Earthquake Engineering 2016; 1-2, 62-71.

[31] A. Amendola, E.H. Nava, R. Goodall, I. Todd, R.E. Skelton, F. Fraternali, On the additive manufacturing, post-tensioning and testing of bi-material tensegrity structures, *Composite Structures*, **131**, 66-71, 2015.

[32] A. Amendola, G. Carpentieri, L. Feo, F. Fraternali, "*Bending dominated response of layered mechanical metamaterials alternating pentamode lattices and confinement plates*". Compos Struct 2016, 71-77.

[33] A. Amendola, F. Fabbrocino, L. Feo, F. Fraternali, "*Dependence of the mechanical properties of pentamode materials on the lattice microstructure*", (2016) ECCOMAS Congress 2016 - Proceedings of the 7th European Congress on Computational Methods in Applied Sciences and Engineering, 1, pp. 2134-2150.

[34] I. Farina, F. Fabbrocino, G. Carpentieri, M. Modano, A. Amendola, R. Goodall, L. Feo, F. Fraternali, *On the reinforcement of cement mortars through 3D printed polymeric and metallic fibers,* Composites Part B: Engineering, **90**, 76-85, 2016.

[35] R. Singh, S. Singh, F. Fraternali, "*Development of in-house composite wire based feed stock filaments of fused deposition modelling for wear-resistant materials and structures*". Compos Part B-Eng 2016; 98, 244-249.

[36] I. Farina, F. Fabbrocino, F. Colangelo, L. Feo, F. Fraternali, "*Surface roughness effects on the reinforcement of cement mortars through 3D printed metallic fibers*". Compos Part B-Eng 2016; 99, 305-311.

[37] R. Singh, R. Kumar, L. Feo, F. Fraternali; "*Friction welding of dissimilar plastic/polymer materials with metal powder reinforcement*". Compos Part B-Eng, 2016; 101, 77-86.

[38] M. Modano, F. Fabbrocino, A. Gesualdo, G. Matrone, I. Farina, F. Fraternali, "*On the forced vibration test by vibrodyne*" COMPDYN 2015 - 5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, 209-217.

[39] F. Fraternali, Free Discontinuity Finite element models in two-dimensions for in-plane crack problems*, Theoretical and Applied Fracture Mechanics*, **47**, 274-282, 2007.

[40] F. Fraternali, S. Spadea, L. Ascione, Buckling behavior of curved composite beams with different elastic response in tension and compression. *Composite Structures*, **100**, 280-289, 2013.

[41] G. Lomiento , N. Bonessio, G. Benzoni, Friction Model for Sliding Bearings under Seismic Excitation, *Journal of Earthquake Engineering*, **17**(8):1162-1191, 2013.

[42] M. Budek, M. J. N. Priestley, G. Benzoni, The Effect of External Confinement on Flexural Hinging in Drilled Pile Shafts, *Earthquake Spectra*, **20** (1), 1-24, 2004.

[43] Hae-Bum Yun, Sami F. Masri, Raymond W. Wolfe, Gianmario Benzoni , Data-driven methodologies for change detection in large-scale nonlinear dampers with noisy measurements, *Journal of Sound and Vibration*, **322**, Issues 1–2, 24 April 2009, Pages 336–357.

[44] D. D'Ayala, G. Benzoni, Historic and Traditional Structures during the 2010 Chile Earthquake: Observations, Codes, and Conservation Strategies. *Earthquake Spectra*, **28** (S1), S425-S451, 2012.

[45] N. Bonessio, G. Lomiento, G. Benzoni, Damage identification procedure for seismically isolated bridges, *Structural Control Health Monitoring*, **19**(5), 565–578, 2012.

[46] G. Benzoni, C. Casarotti , Effects of Vertical Load, Strain Rate and Cycling on the Response of Lead-Rubber Seismic Isolators, *Journal of Earthquake Engineering* ,**13**, 2009.

[47] A. Budek-Schmeisser, G. Benzoni, Rational seismic design of precast, prestressed concrete piles, *PCI Journal*, **53**(5), 40-53, 2008.

[48] G. Benzoni, M. J. N. Priestley, Seismic response of linked marginal wharf segments, *Journal of Earthquake Engineering* **74**, 513-539, 2003.

[49] F. Fraternali, I. Farina, G. Carpentieri, A discrete-to-continuum approach to the curvatures of membrane networks and parametric surfaces, *Mechanics Research Communications*, **56**, 18-15, 2014.

[50] L. Ascione, F. Fraternali, A penalty model for the analysis of composite curved beams, *Computers & Structures*, **45**, 985-999, 1992.

[51] F. Fraternali, J.N. Reddy, A penalty model for the analysis of laminated composite shells, *International Journal of Solids and Structures*, **30**, 3337-3355, 1993.

[52] A. Amendola, G. Carpentieri, M. De Oliveira, R.E. Skelton, F. Fraternali, Experimental investigation of the softening-stiffening response of tensegrity prisms under compressive loading. *Composite Structures*, **117**, 234-243, 2014.

[53] F. Fraternali, G. Bilotti, Non-Linear Elastic Stress Analysis in Curved Composite Beams. *Computers & Structures*, **62**, 837-869, 1997.

[54] F. Fraternali, G. Carpentieri, A. Amendola, On the mechanical modeling of the extreme softening/stiffening response of axially loaded tensegrity prisms. *Journal of the Mechanics and Physics of Solids*, **74**, 136-157, 2015.

[55] D. Ngo, F. Fraternali, C. Daraio, Highly Nonlinear Solitary Wave Propagation in Y-Shaped Granular Crystals with Variable Branch Angles. Physical Review E, **85**, 036602-1-10, 2012.

[56] R.E. Skelton, F. Fraternali, G. Carpentieri, A. Micheletti, Minimum mass design of tensegrity bridges with parametric architecture and multiscale complexity. *Mechanics Research Communications*, 58, 124-132, 2014.

[57] A. Leonard, F. Fraternali, C. Daraio, Directional wave propagation in a highly nonlinear square packing of spheres. *Experimental Mechanics*, **53**(3), 327-337, 2013.

[58] F. Fraternali, G. Carpentieri, A. Amendola, R.E. Skelton, V. F. Nesterenko, Multiscale tunability of solitary wave dynamics in tensegrity metamaterials. *Applied Physics Letters*, **105**, 201903, 2014.

[59] F. Fraternali, I. Farina, C. Polzone, E. Pagliuca, L. Feo, On the use of R-PET strips for the reinforcement of cement mortars. *Composites. Part B*, **46**, 207-210, 2013.

[60] J.R. Raney, F. Fraternali, A. Amendola, C. Daraio, Modeling and In Situ Identification of Material Parameters for Layered Structures based on Carbon Nanotube Arrays. *Composite Structures*, **93**, 3013–3018, 2011.

[61] C. Daraio, D . Ngo, V. F. Nesterenko, F. Fraternali, Highly Nonlinear Pulse Splitting and Recombination in a Two Dimensional Granular Network. *Physical Review E*, **82**, 036603, 2010.

[62] F. Fraternali, A. Marino, T. Elsayed, A. Della Cioppa, On the structural shape optimization via variational methods and evolutionary algorithms. Mechanics of Advanced Materials and Structures, **18**, 225-243, 2011.

[63] F. Fraternali, S. Spadea, V.P. Berardi, Effects of recycled PET fibers on the mechanical properties and seawater curing of Portland cement-based concretes. *Construction and Building Materials*, **61**, 293-302, 2014.

[64] T. Blesgen, F. Fraternali, J.R. Raney, A. Amendola, C. Daraio, Continuum Limits of Bistable Spring Models of Carbon Nanotube Arrays accounting for Material Damage. *Mechanics Research Communications*, **45**, 58-63, 2012.

[65] F. Fraternali. A Mixed Lumped Stress – Displacement Approach to the Elastic Problem of Masonry Walls. Mechanics Research Communications, **38**, 176-180, 2011.

[66] F. Fraternali, M. Negri, M. Ortiz, On the Convergence of 3D Free Discontinuity Models in Variational Fracture. *International Journal of Fracture*, **166**(1-2), 3-11, 2010.

[67] F. Fraternali, C.D Lorenz, G. Marcelli, On the estimation of the curvatures and bending rigidity of membrane networks via a local maximum-entropy approach, *Journal of Computational Physics*, **231**, 528-540, 2012.

[68] F. Fraternali, G. Marcelli, "*A multiscale approach to the elastic moduli of biomembrane networks*". Biomech Model Mechan 2012: 11(7), 1097-1108.

[69] B. Schmidt, F. Fraternali, Universal formulae for the limiting elastic energy of membrane networks, *Journal of the Mechanics and Physics of Solids*, **60**, 172-180, 2012.

[70] A. Amendola, G. Benzoni, F. Fraternali, Non-linear elastic response of layered structures, alternating pentamode lattices and confinement plates. *Composite Part B, Engineering*, Online first, DOI: 10.1016/j.compositesb.2016.10.027, 2016

[71] F. Fraternali, G. Carpentieri, R. Montuori, A. Amendola, G. Benzoni, On the use of mechanical metamaterials for innovative seismic isolation systems, COMPDYN 2015 - 5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, 349-358.