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WAVE PROPAGATION IN PIPES WITH HELICAL PATTERNS

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Abstract. *Pipe installations are widely used in many industrial applications, particularly in oil, water, natural gas distribution and chemical industry. Although there are many different constructions according to the specific application, these pipes often consists of synthetic rubber which covers reinforced wires wound along helical lines around an inner synthetic rubber tube. Theoretical understanding of wave propagation in these pipes, and in particular dispersion curves and wave–modes, provides information about the dynamic behaviour and stress state of the structure, and it is essential in many applications such as transmission of structure–borne sound, statistical energy analysis, shock response, non-destructive-testing and structural-health-monitoring. In this paper wave characteristics in pipes with helical patterns are studied. In particular wave propagation in cylindrical structural with helical orthotropic characteristics are investigated and some interesting characteristics of wave propagating in these structure are showed and discussed. Two different approaches, the Wave Finite Element method and an analytical model of a thin helically orthotropic cylindrical shell, are used and results are compared. Both methods are very efficient in terms of computational cost, theoretical understanding of the wave characteristics and utilisation of the model for optimisation procedures, where the model is reconfigured for each new set of parameters.*

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1 INTRODUCTION

Pipelines are essential parts in many engineering applications, particularly in oil, water, natural gas distribution and chemistry industry, they are also the main components of the human cardiovascular system. The simulation of their dynamic behaviour and the diagnosis of their structural health conditions is thus of great importance for safety, economic, and environmental reasons.

The prediction and optimisation of the vibro-acoustic behaviour of such structures requires good and low-cost numerical models. However, most of these pipe models shows anisotropic properties and complicated construction, e.g. hard rubber reinforced pipes, and standard approaches are not able to grasp their dynamics. Standard analytical or numerical approaches in fact have limits in the mid and high frequency range, both in terms of computational cost and in terms of utilisation of the model for optimisation procedures, where the model have to be reconfigured for each new set of parameters. Moreover, most of the structural health monitoring techniques requires the knowledge and understanding of wave propagating in such structures up to very high frequency.

Wave propagation in helical waveguides has been a subject of interest since long time ago, e.g. [1], and many original and interesting studies and applications can be found in the literature. Due to difficulties in solving partial differential equations for helical structures, in recent years there has been an increased interest in numerical methods for the analysis of wave propagation in helical waveguides, in particular due to developments in structural health monitoring techniques. Dispersion curves in helical springs were obtained in [2] applying an asymptotic analysis and the dominant balance method. Natural frequencies in helical spring were calculated in [3] using a dynamic stiffness method, while vibrations were studied in [4] applying a pseudo-spectral method and in [5] using the Boundary Integral Method. Several numerical methods based on Finite Element discretization have been also proposed, each one showing some advantages or disadvantages with respect to the other. Vibrations of helical springs with non-uniform ends were studied in [6] using a hybrid Wave Finite Element method. In [7] a Semi Analytical Finite Element method, based on translational invariance of curved waveguides, was presented, while in [8] dispersion curves were obtained based on the Scaled Boundary Finite Element method.

This paper describes the investigation of wave propagation characteristics in pipes with helical patterns. Wave characteristics are obtained by means of an analytical method and a Wave Finite Element (WFE) method. The latter is an extension of the WFE approach for 2D waveguides [9,10] to structures showing a helical pattern. In the first part of the paper the approaches are illustrated. The second part deals with a numerical example and the influence of the helical orthotropic is investigated. Properties of waves propagating in cylindrical helical structures are discussed, in particular concerning spinning waves.

2 THE ANALYTICAL MODEL OF A THIN HELICALLY ORTHOTROPIC ELASTIC CYLINDRICAL SHELL AND THE SPINNING WAVES

Wave propagation in elastic cylindrical shells has been studied in numerous publications. The vast majority of those are concerned with shells made of an isotropic material. Orthotropic shells have been considered in much fewer publications and the principal directions of tensor of elastic constants are customarily taken as coinciding with the cylindrical system of coordinates. It has been shown that all qualitative features of dispersion curves known for isotropic shells are preserved with some quantitative changes in magnitudes of cut-on frequencies, which, obviously, become dependent upon the ratio of elastic moduli in principal directions.

The technology of manufacturing of elastic pipes (cylindrical shells) for some technical applications, however, is such that the principal directions of the tensor of elastic constants are turned at a certain angle ϕ to the cylindrical system of coordinates. This angle is kept constant along the length of a pipe so that fibres in a ply are helically wounded at the cylindrical surface. Often, an orthotropic cylindrical shell is made of many plies and the pitch angle for consecutive plies is switched to opposite. By these means, the principal directions of elastic properties become aliened to the cylindrical coordinate system. In the cases, when an odd number of plies is used and this number is small, the pitch angle affects the waveguide properties of such a shell.

The analytical model of a helically orthotropic cylindrical shell has been obtained in the framework of the general shell theory. It is a trivial task to formulate components of deformation, forces and moments in principal directions of the tensor of elastic constants and transform those to the cylindrical coordinates (x,r,θ). Then the forces and moments are substituted to equations of motion to yield the governing equations of wave motion in an infinite cylindrical shell with helical orthotropy in the cylindrical coordinates (x,r,θ). These equations have been derived by means of the symbolic manipulator Mathematica. They are very cumbersome, and, therefore, not presented here. However, it has been checked analytically that setting $\phi=0$ gives conventional equations for the orthotropic shell, while setting elastic constant to those of an isotropic material gives canonical equations for an isotropic shell for any value of ϕ. The system of differential equations derived as just have been explained allows solution in the form

$$\begin{aligned} u(x,\theta) &= U_x \exp(ikx+im\theta) \\ v(x,\theta) &= U_\theta \exp(ikx+im\theta) \\ w(x,\theta) &= U_r \exp(ikx+im\theta) \end{aligned} \tag{1}$$

Substitution of this equation into the governing equations and equating to zero the determinant of the system of linear algebraic equations with respect to the amplitudes (U_x,U_θ,U_r) yields the dispersion equation in the polynomial form. The polynomial is of the sixth order in the frequency parameter and of the eighth order in wavenumber. As soon as $\phi\neq 0$ it contains both the even and the odd powers of a wavenumber, that suggests the difference between characteristics of waves traveling along the shell in opposite directions. It should also be noted that the conventional solution ansatz, which describes waves travelling without rotation along the axis of an orthotropic shell, i.e.,

$$\begin{aligned} u(x,\theta) &= U_x \exp(ikx)\cos(m\theta) \\ v(x,\theta) &= U_\theta \exp(ikx)\sin(m\theta) \\ w(x,\theta) &= U_r \exp(ikx)\cos(m\theta) \end{aligned} \tag{2}$$

does not allow separation of trigonometric functions in the governing equations. This separation is recovered, when either the elastic parameters describe the isotropic shell, or $\phi=0$. Therefore, we conclude that standing in the circumferential direction waves cannot propagate in an orthotropic cylindrical shell with $\phi\neq 0$. The waveguide properties of the shell are different in positive and negative directions of its axis provided that rotation is the same.

It can also be seen that there is the symmetry with respect to this plane between the dispersion curves for ϕ and $-\phi$. This is readily explained by the simple observation that the waves travelling in the positive direction of the x-axis and rotating clockwise are identical to the waves travelling in the negative direction of the x-axis and rotating anticlockwise.

3 2D WAVE FINITE ELEMENT FORMULATION FOR STRUCTURES WITH HELICAL PATTERNS

The WFE formulation for axisymmetric structures [10,11] can be applied to predict the wave characteristic of structures with helical geometry. As an example a helically reinforced pipe is considered as in Figure 2(a), but the approach presented here can be equally applied to any other curved panels with helical orthotropy. For this structure, an arbitrary periodic segment is a skew segment as shown in Figure 2(b,c), which forms a parallelogram in the (y,x) plane with one side parallel to the y-axis and another parallel to the direction x' at an angle ϕ to the x direction. The segment has arbitrary lengths L_y and L_x and subtends a small angle L_α. Here x corresponds to the circumferential direction while y and z correspond to the axial and radial directions in the local coordinates, ϕ denotes the lay angle of the helix, R is the main radius while h is the pipe thickness. The curved segment in Figure 2(b) can be discretised using either shell or solid elements and then the local coordinates are rotated through a transformation matrix as in standard FE procedures. As an example, Figure 2(d) shows a schematic representation of the FE model of the segment realised using 8-noded elements.

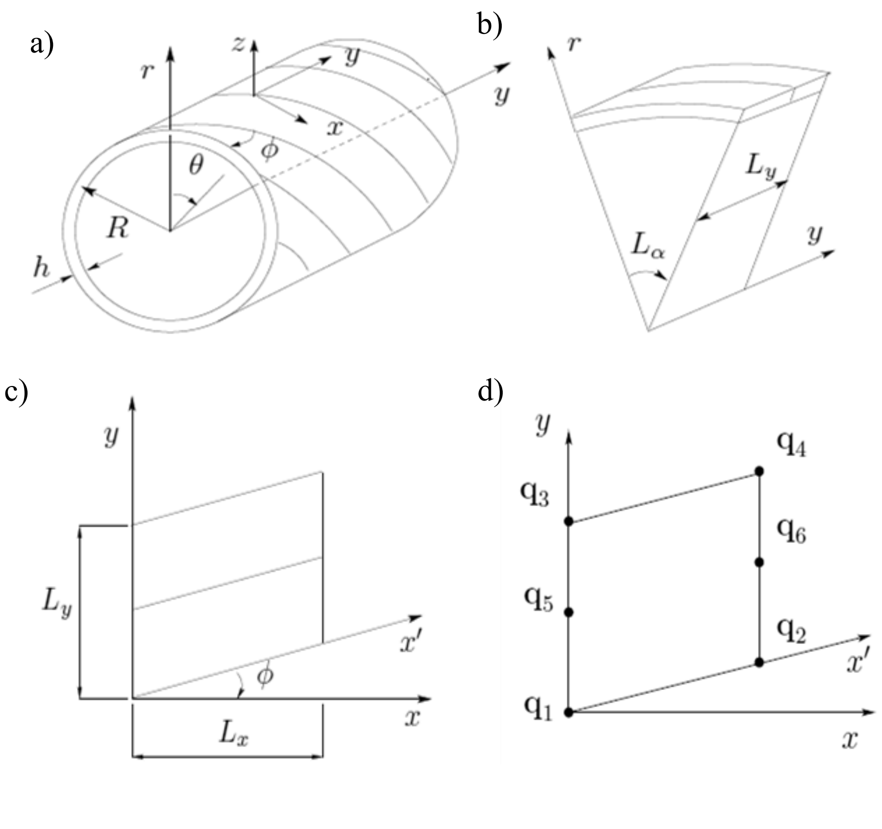

Figure 2: (a) schematic representation of a pipe with a helical pattern; (b) small periodic shew segment of the pipe; (c,d) FE mesh with super-node numbering.

All the degrees of freedom (DOFs) of the FE model are then partitioned in a column vector $\mathbf{q} = \left[\mathbf{q}_1^T\ \mathbf{q}_2^T\ \mathbf{q}_3^T\ \mathbf{q}_4^T\ \mathbf{q}_5^T\ \mathbf{q}_6^T\right]^T$ as in Figure 2(d), where the superscript T denotes the transpose and where $\mathbf{q}_j$ is the vector of the nodal DOFs of all the element nodes through the thickness. A similar expression is used given for the nodal forces $\mathbf{f}$. The FE governing equation of motion the segment of 2(d) is $\left[\mathbf{K} - \omega^2\mathbf{M}\right]\mathbf{q} = \mathbf{f}$. Typically the FE model is realsied using commercial FE packages and therefore the mass and stiffness matrices are obtained from commercial FE software.

The structure is considered arbitrary periodic in the y and x' directions, therefore the propagation of a free wave can be obtained from the modified propagation constants

$$\mu_{x'} = k_x L_x + k_y L_x \tan\phi; \quad \mu_y = k_y L_y; \tag{3}$$

where k_x and k_y are the projections of the wavenumber k in the circumferential and axial directions. Equation (3) refers to the general case of *curved panels* so that k_x can in principle attain any value. However, if the geometry is *cylindrical* the phase change of a wave as it propagates around the circumference must be a multiple of 2π and the propagation constants then become

$$\mu_{x'} = n\,L_x / R + k_y L_x \tan\phi; \quad \mu_y = k_y L_y; \tag{4}$$

where $n = 0,1,2,3...$ is the circumferential mode order [2]. The nodal DOFs are then related by the periodicity conditions in matrix form to give

$$\mathbf{q} = \Lambda_R \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_5 \end{bmatrix}; \quad \Lambda_R = \begin{bmatrix} \mathbf{I} & \lambda_{x'}\mathbf{I} & \lambda_y\mathbf{I} & \lambda_{x'}\lambda_y\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \lambda_{x'}\mathbf{I} \end{bmatrix}^T; \tag{5}$$

where $\lambda_{x'} = \exp\left(-i\mu_{x'}\right)$; $\lambda_y = \exp\left(-i\mu_y\right)$. In the absence of external excitation, equilibrium conditions of the segment can be rearranged in matrix form as

$$\Lambda_L \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_5 \end{bmatrix} = 0; \quad \Lambda_L = \begin{bmatrix} \mathbf{I} & \lambda_{x'}^{-1}\mathbf{I} & \lambda_y^{-1}\mathbf{I} & \lambda_{x'}^{-1}\lambda_y^{-1}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \lambda_{x'}^{-1}\mathbf{I} \end{bmatrix}^T; \tag{6}$$

Substituting Equations (5) and (6) into the equation of motion, the mass and stiffness matrices are then reduced as

$$\overline{\mathbf{M}} = \Lambda_L \mathbf{M} \Lambda_R; \quad \overline{\mathbf{K}} = \Lambda_L \mathbf{K} \Lambda_R; \tag{7}$$

and the equations of free wave motion are therefore reduced to the WFE eigenvalue problem

$$\left[\overline{\mathbf{K}}\left(\mu_{x'}, \mu_y\right) - \omega^2 \overline{\mathbf{M}}\left(\mu_{x'}, \mu_y\right)\right] \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_5 \end{bmatrix} = \mathbf{0} \tag{8}$$

The eigenproblem in Equation (8) involves the three parameters $\mu_{x'}$, μ_y and ω and it takes various forms as discussed in [1]. However, typically these can be recast as a linear eigenvalue problem, which has been found the more robust and computationally convenient formulation. In this case the WFE eigenproblem reduces to a quadratic polynomial eigenvalue problem which is then linearised to a standard linear problem.

4 NUMERCAL RESULTS AND DISCUSSION

In this section complex dispersion curves of a cylindrical structure having helical orthotropic material properties are showed and discussed. A pipe with thickness to mean radius $\frac{h}{R}=0.05$ and $\frac{E_y}{E_x}=3$ is considered. To apply the WFE approach, the FE model of a small periodic segment of the pipe of length $L_x = L_x = 0.002\,\text{mm}$ is discretised using 5 solid elements; this model allows to obtain accurate results up to high frequency. The frequency here is normalised with respect to $\Omega = \frac{1}{R}\sqrt{\frac{E_x}{\left[\rho\left(1-\upsilon_{xy}^{\ 2}\right)\right]}}$ while the axial wavenumber is normalized with respect to the mean radius *R*.

In Figure 3, the 3D dispersion diagrams obtained with the analytical solution as in Equation (1) are plotted, while Figure 4 shows the comparison between the complex dispersion curves of an orthotropic cylinder with $\phi = 0^0$ and $\phi = 30^0$ obtained using the WFE approach. First and second circumferential mode are shown in Figure 4 as an example. Comparison between the analytical and WFE numerical results has shown very good agreement although some discrepancies at high frequency are due to the differences in the models: shell theory used for the analytical model and FE solid elements used for the WFE approach.

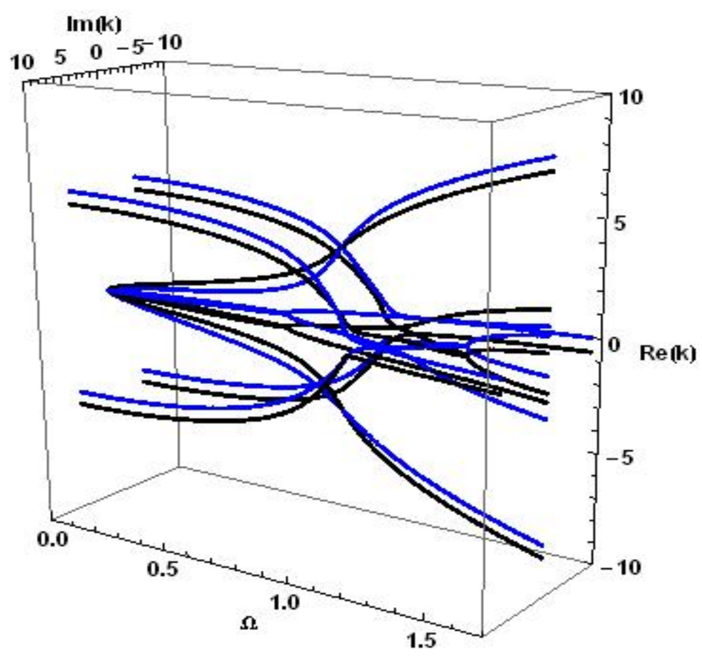

Figure 3: 3D complex dispersion curves of an helical orthotropic cylinder: $\phi = 30^o$ black lines; $\phi = -30^o$ blue lines.

Figures 3 and 4 show clearly that the helical orthotropy introduces non symmetry with respect to the plane $(\text{Im}(k)=0, \Omega = 0)$ and that the waveguide properties of the shell are different in positive and negative directions of its axis, provided that rotation is the same.

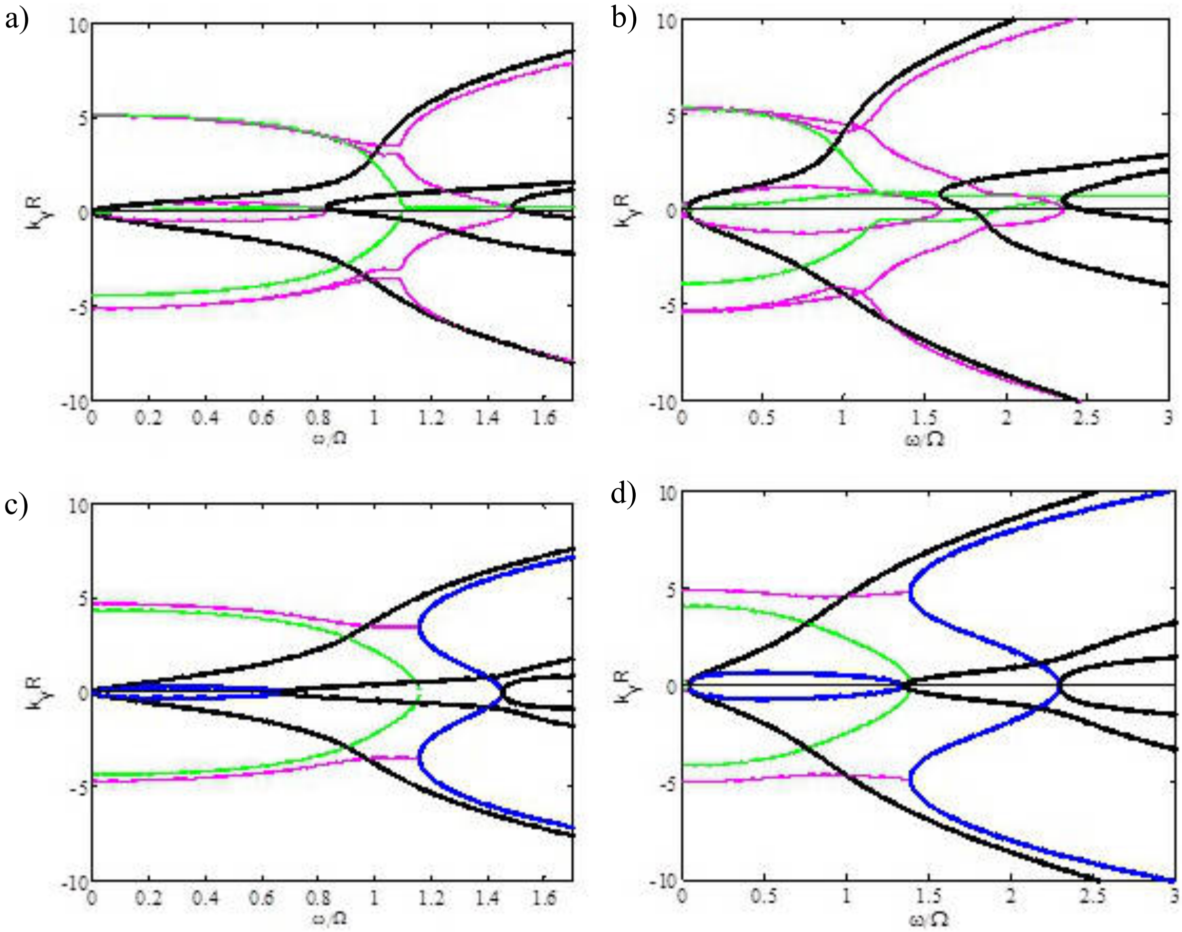

Figure 4: Comparison between the complex dispersion curves of an orthotropic cylinder with helical and axial orthotropy. $\phi = 30^0$: (a) n=1; (b) n=2; $\phi = 0^0$: (c) n=1; (d) n=2. Real wavenumber: black thick line; imaginary wavenumber: blue thick line; complex wavenumbers: green line (real part) and magenta line (imaginary part).

These considerations can be seen if the behavior of the curves in Figure 4(a) and 4(b), with $\phi = 30^0$, are compared with those in Figure 4(c) and 4(d) for $\phi = 0^o$: waves propagating in the positive or negative directions have different wavenumber, phase velocity, and group velocity for the case $\phi = 30^o$, while they show the same characteristics when $\phi = 0^o$ as typically expected. In particular, when $\phi = 30^o$ it can be seen that waves spinning clockwise have different properties from those spinning anti-clockwise. On the other hand, Figure 3 shows also that there is the symmetry with respect to the plane $(\mathrm{Im}(k) = 0, \Omega = 0)$ between the dispersion curves for $\phi = 30^o$ and the dispersion curves for $\phi = -30^o$.

4 CONCLUSIONS

In this work wave characteristics in cylindrical waveguides with helical patterns are studied. Dispersion diagrams for thin cylindrical helically orthotropic shell are obtained using an analytical approach and a WFE method. Comparison between the analytical and WFE numerical results has shown very good agreement although at high frequency some discrepancies are due to shell theory used for the analytical model. Both methods are very efficient in terms of computational cost, theoretical understanding of the wave characteristics and utilisation of the model for optimisation procedures, where the model is reconfigured for each new set of parameters. It is shown that the symmetry in location of dispersion curves for orthotropic shells is broken as soon as the angle between the principal directions of anisotropy and the cylindrical coordinates becomes non-zero. This results is of practical significance in view of possibilities to control power flow by tailoring the pitch angle of helical anisotropy. This issue is related to inspection of the mode shapes and to construction of Green's matrices. These tasks constitute the subjects of on-going work of the authors.

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