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DEVELOPMENT OF A CURVILINEAR HYSTERESIS MODEL FOR REINFORCED CONCRETE STRUCTURES

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Abstract. *Multilinear hysteresis models such as the Clough model are generally used for earthquake response analysis of reinforced concrete (RC) structures, while force-deformation relations obtained in experimental studies are curvilinear. Other hysteresis models are developed to consider features such as strength degradation at large deformation regions, stiffness degradation due to cyclic loading or inverted S-shaped slip property. However, these models are generally not widely used because most use many rules to express all the features and require many parameters, which makes the model complicated. In this study, a curvilinear hysteresis model for RC structures is developed with relatively simple rules and a small number of parameters. The model consists of only two functions, the skeleton curve and the inner loop, and nine model parameters. The developed model is applicable to various types of RC beams and columns, including strength deterioration at large deformation regions, stiffness degradation due to cyclic loading or inverted S-shaped slip property. A method to evaluate the capability of hysteresis model is developed using the minimized error index based on genetic algorithm to obtain the best parameter which will reproduce the force-displacement relations. Force-displacement relations of RC specimens are collected from previous experimental studies and the error indices are calculated. The developed model are capable of reproducing force-displacement relations well compared to other hysteresis models for RC structures.*

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1 INTRODUCTION

In general, multi-linear hysteresis models such as the Clough or Takeda model are used to describe the restoring force characteristics of reinforced concrete(RC) structures, although force-displacement relations obtained in experiments of RC members are curvilinear. On the other hand, curvilinear hysteresis models are developed for wooden structures[1][2], which reproduce experiments well using relatively simple rules with a small number of parameters. The same approach may be applicable for RC structures that include slip or stiffness degradation due to cyclic loadings.

In this study, a curvilinear hysteresis model for RC structures is developed. Force-displacement relations obtained in previous experimental studies are reproduced to evaluate the capability of the developed model.

2 HYSTERESIS MODEL

The outline of the developed model is shown in Figure 1, which consists of only two rules, the skeleton curve(I) and the unload-reload curve(II).

2.1 Skeleton curve

The skeleton curve is simply defined by one function

$$f = A_0 \frac{\tanh((\alpha/d_m) \cdot d)}{d^\beta + d_0} \tag{1}$$

where f is the force, d is the displacement, α and β are parameters to control the shape of the curve. The maximum force point (d_m, f_m) are determined by the following equations.

$$A_0 = f_m \cdot \frac{{d_u}^\beta + d_0}{\tanh(\alpha)} \tag{2}$$

$$d_0 = \frac{(0.5\beta \sinh(2\alpha) - \alpha) \cdot d_m}{\alpha} \tag{3}$$

The influences of the shape parameters α and β are shown in Figure 2, in which the maximum force point(d_m, f_m) is fixed at (0.3, 14.0). The parameter α controls the initial stiffness and strength deterioration and β influences the strength deterioration after the maximum force point.

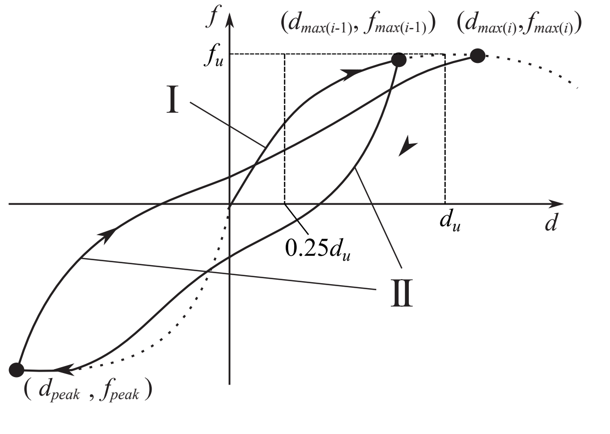
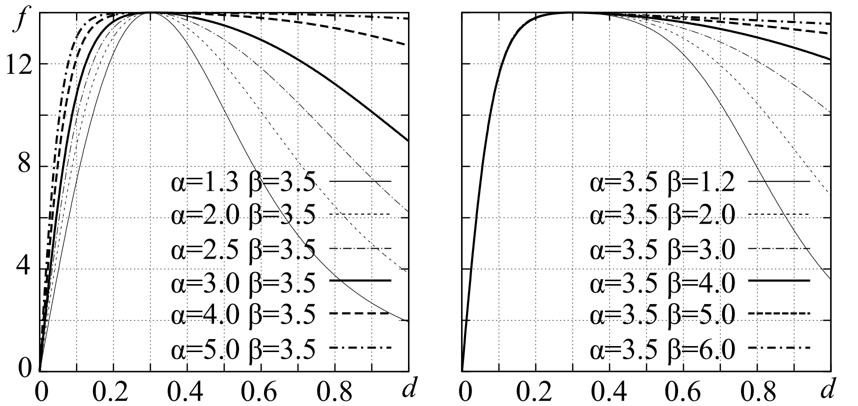


Figure 1: Model outline



(a) Shape factor α (b) Strength deterioration factor β

Figure 2: Influences of the skeleton shape parameters

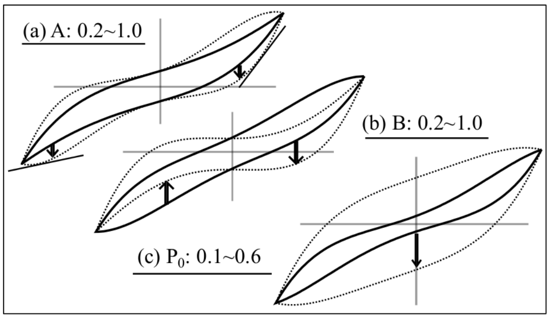


Figure 3: Influences of the inner loop parameters

When the velocity direction is reversed and the absolute maximum displacement exceeds the elastic limit ($0.25d_u$), the hysteresis enters the unload-reload curve (rule II).

2.2 Unload-reload curve

The following function expresses a loop ranging from a spindle-shape loop to a slip-type loop

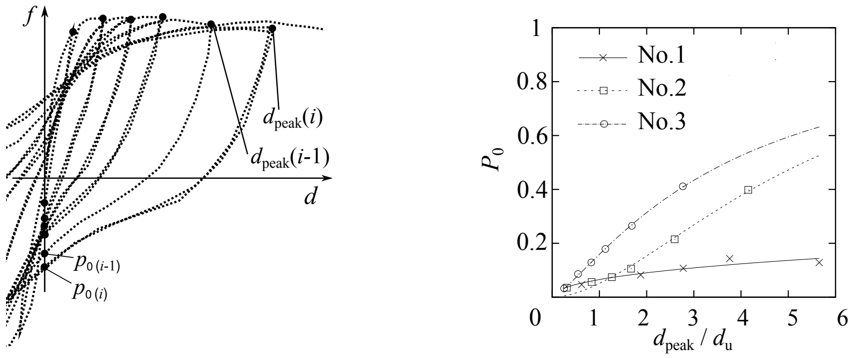
$$F_{bottom}^{top} = \mp AD^4 + BD^3 \mp (P_0 - A)D^2 + (1 - B)D \pm P_0 \tag{4}$$

where A, B and P_0 are parameters to control the shape of the curve. F and D are normalized force and displacement with values of -1 or +1 at the unloading point or the maximum/minimum point on the other side of the unloading point using scale factors.

The influences of the parameters A, B and P_0 on the inner loop are illustrated in Figure 3. The parameter A controls stiffness variation level, B controls slip characteristics and P_0 controls loop area, respectively.

P_0 is the ratio of the force at $D = 0$ to the peak force f_{peak}. According to the load-displacement curves obtained in previous experimental studies, the parameter P_0 cannot be fixed as a constant value, but varies depending on the peak displacement. In many cases, P_0 increases as peak displacement increases as shown in Figure 4(a). Three examples of relations between peek displacements and P_0 are shown by the points in Figure 4(b). Based on the characteristics of these relations, P_0 is formulated as follows

$$P_0 = \frac{-1}{1 + r_{pzc} \cdot \left(\frac{d_{peak}}{d_u}\right)^{\gamma}} + 1 \tag{5}$$



(a) Reading P_0 from experimental data (b) Examples of $P_0 - d_{peak}$ relations [5]

Figure 4: Relations between unloading displacement (d_{peak}) and the force at zero displacement (P_0)

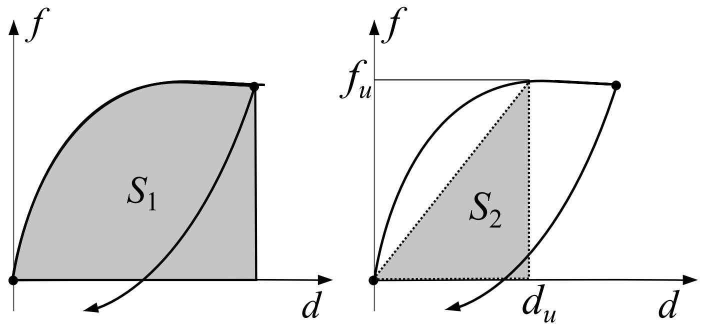


Figure 5: Hysteretic area S_1 and base area S_2

where r_{pzc} and γ are model parameters to determine P_0. The displacement d_{peak} is normalized by the displacement at the maximum force d_u.

For the three examples shown by the points in Figure 4(b), the P_0 values calculated by Equation 5 are illustrated by the lines. The experimental P_0 values are well expressed by the equation with appropriate values for the parameters r_{pzc} and γ.

2.3 Stiffness degradation

Stiffness degradation due to cyclic loading is expressed by increasing the hysteretic maximum displacement, as shown in Figure 1. When the velocity direction is reversed, the unload-reload curve function (Equation 4) scale factors are determined to aim for the previous peak point $(d_{max(i-1)}, f_{max(i-1)})$. The target peak point is moved from $(d_{max(i-1)}, f_{max(i-1)})$ to $(d_{max(i)}, f_{max(i)})$ on the skeleton curve to consider the stiffness degradation. The amount of movement of the maximum experienced displacement is based on the energy dissipation, determined by the following equation with the previous hysteretic area (S1) normalized by the basic area (S2)

$$d_{max(i)} = d_{max(i-1)} \cdot \left(1 + r_s \cdot \frac{S_1}{S_2}\right) \tag{6}$$

where S_1 and S_2 are dissipated energy area and basic energy area in the previous hysteresis loop (Figure 5), respectively, and r_s is a model parameter to determine the amount of stiffness degradation due to cyclic loading.

2.4 Model outline

The required model parameters are listed in Table 1.

Table 1: Model parameters

Model parameter	Variable
Maximum force	f_u
Deformation at the maximum force	d_u
Skeleton curve shape factor	α
Skeleton curve strength deterioration factor	β
Inner loop shape factor	A
Inner loop slip factor	B
Inner loop swelling factor	r_{pzc}
Inner loop swelling index	γ
Stiffness degradation factor	r_s

A summary of the developed hysteresis model is given below:

- Until the absolute maximum displacement exceeds the elastic limit, restoring force is determined by the skeleton curve function (Equation 1).
- After the displacement exceeds the elastic limit and the velocity direction is reversed, restoring force is calculated by the unload-reload curve (Equation 4). The scale factors for both displacement and force are determined so that the normalized displacement and force take a value of 1 or -1 at unloading point (d_{peak}, f_{peak}) and maximum point at the opposite direction (d_{max}, f_{max}).
- To consider stiffness degradation, the maximum experienced point at the opposite direction $({}_{max}, f_{max})$ is increased based on energy consumption in the previous loop.
- If the velocity is reversed in unload-reload state, new scale factors are calculated for the new unloading point and target at the opposite direction.
- The absolute maximum deformations are saved for both directions. When the absolute displacement exceeds the maximum value, restoring force is calculated by the skeleton curve function.

3 REPRODUCING LOAD-DEFORMATION RELATIONS FROM PREVIOUS EXPERIMENTAL STUDIES

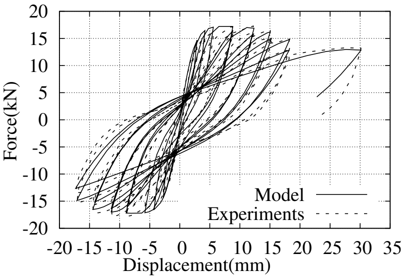
Load-deformation relations are collected from previous experimental studies for reproduction using the developed model.

The normalized error of the reproduction is calculated by the following equation

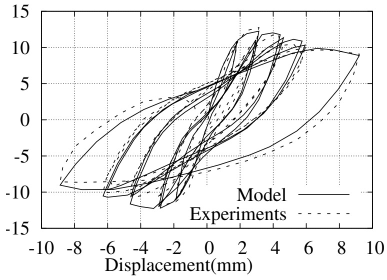
$$E = \frac{1}{f_m}\sqrt{\frac{1}{N}\sum_{i=1}^{N}(f_{exp}(i) - f_{model}(i))^2)} \tag{7}$$

where $f_{exp}(i)$ and $f_{model}(i)$ are the forces at ith step in the experiment and model reproduction and N is the total step number. The error is normalized by the maximum force f_m in the experiment.

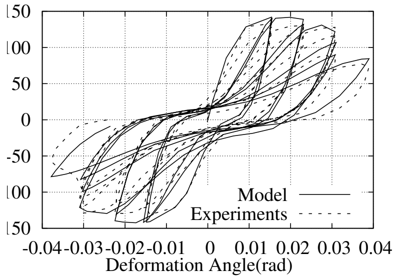
The best values of the parameter sets are determined using genetic algorithm to minimize the normalized error. During optimization, both the skeleton curve strength deterioration factor β



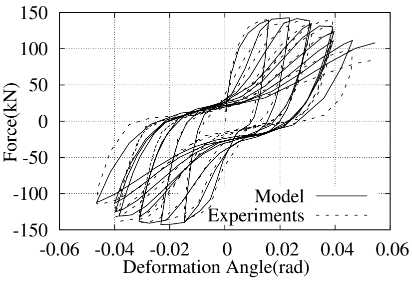
(a) Specimen No.12 [6]



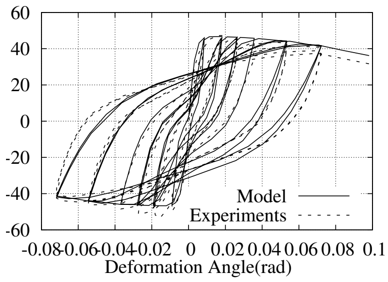
(b) Specimen No.25 [6]



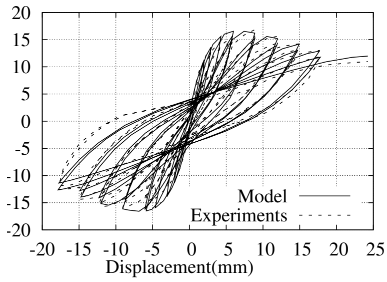
(c) Specimen BA3 [7]



(d) Specimen BA11 [7]

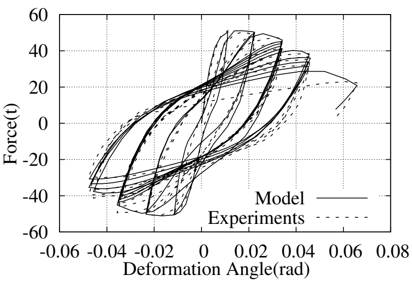


(e) Specimen No.17 [8]

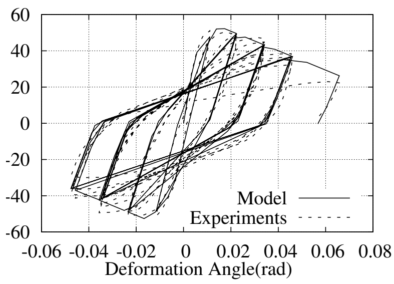


(f) Specimen B04R [9]

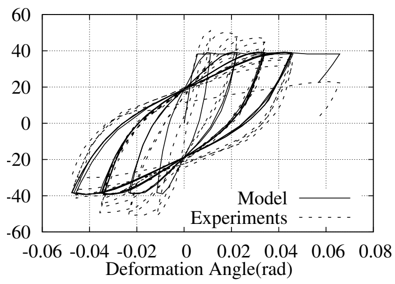
Figure 6: Force-displacement relations in experiments and model reproductions



(a) Developed model



(b) Takeda model



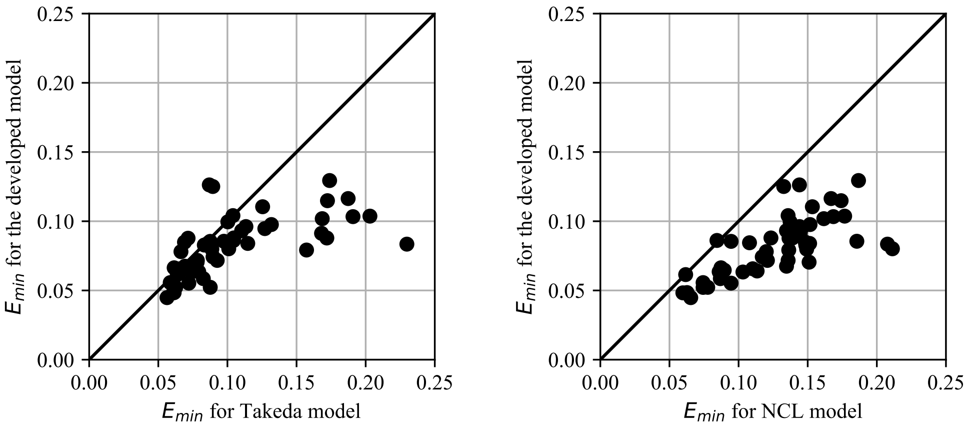
(c) NCL model

Figure 7: Reproduction comparison with other hysteresis models (Specimen[8] No12)

and the stiffness degradation factor r_s influence strength deterioration at the large displacement region, which makes it difficult to get stable values for the parameters. Therefore, the envelope curves are extracted from experimental data and the skeleton parameters, f_u, d_u, α and β, are optimized first before determining the other parameters.

Examples of the reproductions for the best model parameters to fit load-deformation relations in the experiments are shown in Figure 6, where the dotted lines are experiments and solid lines are model reproductions. From spindle-shaped force-displacement relations (ex.(e)) to slip-type ones (ex.(c)), the force-displacement relations are well reproduced including strength deterioration at the large deformation area and stiffness degradation due to cyclic loadings.

The same methodology is applied to other hysteresis models, the Takeda model[3], the most frequently used multi-linear model for RC structures in Japan, and the NCL model[4], a curvilinear model for RC structures, with the results for an experiment shown in Figure 7. In Takeda model, stiffness degradation due to cyclic loading is not considered which may lead to overestimation of the energy dissipating capacity of structures. In NCL model, unload-reload curves are not well reproduced in the first and last several cycles because the force at zero-displacement (P_0 in the developed model) is constant.



(a) vs Takeda model

(b) vs NCL model

Figure 8: Comparison of minimized error with other hysteresis models [6]-[13]

4 MODEL CAPABILITY EVALUATION

To evaluate the capability of the developed model, the error index of the model E_{min} is defined as the minimized normalized error. In other words, the best parameter set is obtained by genetic algorithm for each reproduction and the minimized error is used to assess model capability. Smaller minimized error (E_{min}) means better capability of the hysteresis model to reproduce force-displacement relations.

Fifty-one force-displacement relations of RC beam or column specimens are collected from previous experimental studies with various kinds of failure such as shear failure, flexural failure or shear failure after flexural yielding published in the *Proceedings of the Japan Concrete Institute* or *Journal of Structural and Construction Engineering*. The minimized errors defined above are calculated for each specimen for the developed model, Takeda model and NCL model. The results are compared in Figure 8. In most experiments, the E_{min} for the developed model is smaller than that for Takeda model or NCL model, which shows that the developed model is more capable of reproducing experimental RC force-deformation relations.

5 CONCLUSIONS

A curvilinear hysteresis model for reinforced concrete structures is developed. The developed model is relatively simple and requires a smaller number of parameters even though it considers many features such as strength deterioration at large displacement regions, stiffness degradation due to cyclic loading and inverted S-shaped slip property.

Various force-displacement relations from previous RC member experiments were reproduced by the developed model and compared to other models. The model parameters were determined using genetic algorithm to best reproduce the experimental results. The reproduction can be quite good if the parameters are set appropriately.

The model capability is evaluated using minimum normalized error calculated for 51 force-displacement relations obtained in previous experimental studies and compared to other RC structure hysteresis models. In most cases, the reproduction of the developed model are better than the other models. The developed model showed high capability in reproducing experimental RC force-deformation relations.

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