A PLASTICITY MODEL FOR 1D SOIL RESPONSE ANALYSIS

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Abstract. A plasticity model is presented for the non-linear ground response analysis of layered sites. The model is the one-dimensional version of that recently proposed by Tasiopoulou and Gerolymos (2016) for sand behavior, designated as TA-GER sand model. Critical state compatibility for monotonic and cyclic loading, anisotropic plastic flow rule and Bouc-Wen motivated hardening law are among the key-features of the developed 1D model, offering considerable flexibility in representing complex patterns of cyclic behavior such as stiffness decay and increase in strength due to build-up of pore-water pressure. Implemented through an explicit finite-difference algorithm into an in-house computer code which performs integration of the wave equations to obtain the nonlinear response of layered soil deposits, the model is first calibrated to match published experimental shear modulus and damping curves and is then validated against results from two wave-propagation codes available in literature.
1 INTRODUCTION

Several constitutive models and numerical codes have been proposed over the last decades for 1D seismic response analysis of horizontally layered soils subjected to vertically-polarized S waves. In general, they can be categorized into three major groups: (a) The equivalent linear viscoelastic models (e.g. [1], [2]), (b) the nonlinear hysteretic (or phenomenological) models (e.g. [3], [4], [5]), and (c) the plasticity-based models (e.g. [6], [7]).

Equivalent linear models are the most popular owing to their computational convenience and simplicity. Their main limitations include their inability to efficiently predict the behavior of a nonlinear system under strong ground motions where large cyclic shear strains dominate as they depend on the distance of the current stress ratio $q/p$ from the phase transformation line. $M_F$ and $M_c$ are the failure stress ratio representing the ultimate strength. Parameter $\zeta_a$ is a hysteretic dimensionless quantity that

Hysteretic models are plausible alternatives to plasticity-based models, but, while capable of overcoming most of the aforementioned limitations, the calibration process is often an arduous task in which the physical meaning of the model parameters is often jeopardized in favour of case-specific accuracy. The absence of a physical law for relating volumetric with shear strains is the main source of this drawback.

This paper presents a downscale version of the recently developed plasticity-based model by Tasiopoulou and Gerolymos for sand behavior [9], [10]. A methodology for the calibration of the model parameters is developed, so that the constitutive stress–strain loops are consistent with experimental shear modulus and damping curves available in the literature. The finite difference wave-propagation code, into which the aforementioned model was implemented, is validated through comparison with results from the equivalent-linear code STRATA [2] and the nonlinear hysteretic code NL-DYAS [4], [5].

2 BRIEF MODEL DESCRIPTION

Tasiopoulou and Gerolymos [9], [10] developed a new plasticity-based model for sand behavior formulated in the 6-dimensional stress-strain space. In this paper, a 2-dimensional (in p-q space) version of the model is presented for the 1D seismic response analysis of layered soils. According to this version, the incremental stress-strain relationship is given in the following matrix form:

$$
\begin{bmatrix}
dp \\
dq
\end{bmatrix} = \eta
\begin{bmatrix}
K - \frac{3Kd}{KM_o^2} \zeta_a^n & \frac{3Kd}{KM_o^2} \zeta_a^n \\
-\frac{3KM_o^2}{KM_i^2} \zeta_a^n & \frac{3KM_o^2}{KM_i^2} \zeta_a^n \\
\end{bmatrix}
\begin{bmatrix}
d\varepsilon_p \\
q
\end{bmatrix}
$$

(1)

in which $K$ and $G$ are the elastic (small strain) bulk modulus and shear modulus respectively, $d$ is the ratio of the plastic volumetric strain increment $d\varepsilon_p^p$, over the plastic deviatoric strain increment $d\varepsilon_q^p$ and is based on Rowe’s dilatancy theory as it depends on the distance of the current stress ratio $q/p$ from the phase transformation line. $M_F$ and $M_c$ is the failure stress ratio representing the ultimate strength. Parameter $\zeta_a$ is a hysteretic dimensionless quantity that
provides the loading and unloading rule and is a function of the Bouc–Wen parameter $\zeta$, while the exponent $n$ controls the rate of transition from the elastic state to the perfectly plastic one. Finally, $\eta$ is inserted as a multiplier of the hardening elastoplastic matrix expressing the dissipated hysteretic energy. It is expressed in a ductility based form as it is a function of $\mu$ which is a reference ductility defined in terms of shear strain, as follows:

$$\eta = \frac{s_1}{s_1 + \mu s_i}$$

(2)

![Figure 1](image_url)
where $s_1$ and $s_2$ are model parameters. Indicative model predictions are shown in Fig. 1 for characteristic values of the aforementioned parameters. The shear stress – strain loops as well as the volumetric strain vs. shear strain are presented for a relative density $D_r = 50\%$ and a mean effective stress $p = 100$ KPa, for two different strain amplitudes $\gamma = 1\%$ and $\gamma = 0.1\%$. The figure also depicts the evolution of the phase transformation and the ultimate strength parameters $M_{pt}$ and $M_s$, from their initial values to their critical state value $M_{sc}$ for the large amplitude excitation.

## 3 PARAMETERS CALIBRATION

To determine the parameters of the model (Eq. 2), $G_{max}$ is first obtained (e.g., from resonant column tests, crosshole / downhole tests, etc.); then, the parameters $n$, $s_1$, and $s_2$ must be assessed. The calibration is then based on matching some established experimental $G : \gamma$ and $\xi : \gamma$ curves from the literature. To this end, the Lavenberg–Marquardt optimization procedure is used, available in mathematical code MATLAB. Two published families of $G : \gamma$, $\xi : \gamma$ curves have been utilized: (a) the Vucetic & Dobry curves for sand [11] and (b) the pressure ($\sigma'_{0}$)-dependent curves of Darendeli et al. [12].

Starting from the Vucetic & Dobry (1991) curves, the results of the calibration are illustrated in Fig. 2. The agreement between computed and experimental curves is quite satisfactory. Small discrepancies are observed for small strain levels.

Darendeli et al. [12] recommended a new family of normalized shear modulus and material damping curves, as functions of plasticity index and mean effective stress. Four confining pressures ($\sigma'_{0} = 25, 100, 400, 1600$ kPa) are examined herein. Comparison of the predicted with the experimental curves is depicted in Fig 3.

![Figure 2: Approximation of the Vucetic and Dobry (1991) shear modulus and damping curve for sand (PI=0) of confinement pressure $p=100$ KPa. Published data is depicted with markers; model results with continuous lines.](image-url)
Figure 3: Approximation of the Darendeli shear modulus and damping curves for sands (PI=0) of various confinement pressure levels. Published data is depicted with markers; model results with continuous lines.

Figure 4: Shear wave velocity distribution of the examined soil profile

4 COMPARISON WITH OTHER METHODS

The 2-dimensional version of the TA-GER sand model [9], [10] is implemented into a computer code which uses the explicit finite-difference technique to integrate the equations of motion for the nonlinear one-dimensional ground response analysis of layered sites.

The effectiveness of the proposed model is checked against the hysteretic model by Gerolymos and Gazetas [4] implemented in the finite difference code NL-DYAS ([4], [5]).

To compare NL-DYAS with TA-GER, a 30-m deep dense sand profile with density \( \rho = 2.1 \) Mg/m\(^3\), constant with depth, and shear wave velocity distribution (Fig 4), is excited at its base and its response is calculated.

A strong motion, the JMA 090 record from the Kobe (1995) earthquake and a moderate one from Kalamata 1986 earthquake are used as excitations at the base of the soil column. We consider the sand to behave according to the Derendeli curves.

To serve as a yardstick, an equivalent linear soil response analysis was also carried out with the use of code STRATA [2] — one of the current state-of-practice soil amplification codes.
The results of the three analyses (TA–GER, NL-DYAS, STRATA) are portrayed in terms of: (a) the acceleration time histories at the ground surface (Figs 5 and 9), (b) the distributions with depth of the peak values of acceleration, displacement, shear strain, and shear stress (Figs 6 and 10), (c) the stress–strain hysteresis loops of the two nonlinear models at the depth of 5m and 15m (Figs 7 and 11), and (d) the corresponding acceleration response spectra (Figs 8 and 12). The following conclusions can be drawn:

Figure 5: Comparison of acceleration time histories at the surface computed with the three models. Shaking with Kalamata 1986 record
Figure 6: Distributions with depth of the peak values of acceleration, displacement, shear strain, and shear stress. Shaking with Kalamata 1986 record.

Figure 7: Comparison of stress–strain loops computed with TA-GER and NL-DYAS at z=15m and z=5m. Shaking with Kalamata 1986 record.
For the moderate excitation, all three codes (and corresponding soil models) predict similar response in terms of distributions with depth and quite similar acceleration time histories, with STRATA exhibiting slightly higher amplitudes.

Regarding the strong seismic excitation, a fairly similar response is predicted by the two non-linear models. On the other hand, STRATA significantly exaggerates the long-period pulses, while it depresses the high-frequency components — a performance within expectations, as such “depression” of high frequencies has been already noted in the literature (e.g. [13], [14], [15], [16]). The response acceleration spectra from the three codes reinforce this conclusion: whereas the two inelastic soil models produce almost identical spectra, the equivalent-linear analysis, having filtered-out the short-period components, underpredicts the spectral values for periods less than 0.45 sec. It is worth mentioning that an improved equivalent-linear method that avoids the overdamping of high frequencies has been developed by Assimaki and Kausel [14]. Such overdamping stems from the facts that damping is a function of strain amplitude and that high frequencies are usually associated with small amplitudes of motion; thus, these components experience substantially less damping than the dominant frequencies and are artificially suppressed when hysteretic damping is taken as constant. The overestimation of the long period spectral accelerations by the equivalent linear method is due to resonance phenomena that take place in a linear analysis. Such phenome-
na cannot be developed when nonlinearity is accounted for, as the shear modulus, therefore the natural periods of soil, are not fixed but change over time.

- The distributions with depth of the peak values of acceleration, shear stress, and horizontal displacement computed with the two nonlinear models for the JMA 090 record are in well agreement, considerably deviating from those of the equivalent linear method. The similarity between the $\tau - \gamma$ diagrams of TA-GER and NL-DYAS analyses is evident for the moderate motion. There are sharp differences for the strong seismic excitation, however, with the TA-GER model predicting broader hysteresis loops that are more regular in shape.

![Comparison of acceleration time histories at the surface computed with the three models. Shaking with Kobe JMA 090 1995 record](image)

Figure 9: Comparison of acceleration time histories at the surface computed with the three models. Shaking with Kobe JMA 090 1995 record
Figure 10 Distributions with depth of the peak values of acceleration, displacement, shear strain, and shear stress. Shaking with Kobe JMA 090 1995 record.

Figure 11 Comparison of stress–strain loops computed with Ta-Ger and NL–DYAS at z=15m and z=5m. Shaking with Kobe JMA 090 1995 record.
5 CONCLUSIONS

A plasticity-based model implemented into a finite differences computer code was presented in this paper and found capable of predicting efficiently the 1D nonlinear site response. The model is a simplified version of that originally proposed by Tasiopoulou and Gerolymos [9], [10]. The few model parameters were calibrated against experimental results in terms of the shear modulus reduction and damping ratio increase curves available in the literature. The capability of the model in simulating the nonlinear response of horizontally layered deposits was checked through comparison with two codes available in the literature: NL-DYAS and STRATA. While the three codes exhibited similar results for the moderate seismic excitation case, validating the proposed plasticity-based model, the equivalent linear method fails to yield satisfactory results for the strong motion case, significantly underestimating the high-frequency components of the ground response and overestimating the low-frequency ones.

REFERENCES


