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TORSIONAL BALANCE OF TWO-WAY ASYMMETRIC PLANS WITH OPTIMAL DISTRIBUTION OF VISCOUS DAMPERS

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Abstract. *Lateral-torsional coupling in asymmetric-plan buildings results in an increase of lateral displacement in the end-points of the building plan and therefore the production of disorderly deformation demand in seismically resistant frames. The demand for deformation in resistant frames depends on the relative magnitude of the plan translation and rotation and also the correlation between these two signals. Therefore, great correlation of small rotations with lateral displacement may result in considerably different deformations in the resistant frames of the structure's two-edges. With regard to the inability to eliminate the asymmetric state due to various reasons such as architectural issues, in this study an attempt has been made to use supplemental dampers to decrease the lateral-torsional correlation of the plan displacement. This can result in a nearly uniform demand of deformation in seismically resistant frames. On this basis, using the concept of "torsional balance", the optimized distribution of viscous dampers is determined for the decrease of this correlation by moving the "Empirical Center of Balance" (ECB) to the geometrical center of the structure. Which results in an equal mean-square-values of displacement in the edges of the plan. One-way and two-way mass asymmetry has been considered for torsional stiff and flexible structures and the optimal layout of dampers has been determined using the particle swarm optimization algorithm. The results show that the optimum center of viscous dampers depends on size of the mass eccentricity, uncoupled torsional-to-lateral frequency ratio and the amount of supplemental damping considered.*

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1 INTRODUCTION

The previous studies on recent earthquakes showed that sometimes, asymmetry of structures was one of the reasons for building's collapse in earthquake [1]. The main factors of structure asymmetry are including mass asymmetry, stiffness asymmetry, and strength asymmetry in plan and structure height which plays an important role in increasing the structural response in earthquake. For this reason, many studies have been carried out in order to control the seismic response in asymmetric structures. In line with such investigations, one of the proposed methods for controlling the structure's torsion is using energy dissipation devices such as fluid viscous dampers (FVD) regarding to its advantages including structure's high level of performance.

Fluid viscous dampers are effective specifically in controlling structure's seismic response by increasing the ratio of damping of total system. To this end, it is needed to establish appropriate distribution of dampers in plan and height of the structure to decrease structural response in earthquake and minimize the response raised by structure asymmetry.

Goel (1998) has analyzed the effect of supplemental fluid viscous damping devices on the response of asymmetric structures. The results showed that supplemental fluid viscous damping devices can decrease the structure drift in comparison with the systems without damping system. Moreover, when the center of damping is on the opposite side of stiffness center, respect to mass center and in a distance equal to stiffness eccentricity, the displacement of stiff and flexible edges decreases in an optimized manner. On the other hand, if damping radius of gyration is increased more, the response in each edges will decrease [2].

During investigating the plan asymmetric structures in earthquake, Goel (2000) referred to this subject that the amount of damping of first mode increases with the increase of damping eccentricity in opposite side of stiffness center and whereas the flexible edge displacement is controlled by first mode, such a distribution of dampers causes to high decrease in deformation of flexible edge [3].

De La Llera et al (2004) submitted an investigation into the concept of "torsional balance". Torsional balance is a status of dampers distribution in plan (including fluid viscous, viscoelastic, and frictional dampers) which causes to equality of mean square value (MSV) of building response (such as acceleration, velocity and displacement) during earthquake in diaphragm points which have the same distance from the center of diaphragm[4].

Figure 1 shows a typical plan of a one-way asymmetric, single-story structural model with its mass center (CM), stiffness center (CS), strength center (CR) and the center of supplemental damping (CSD) as well as its flexible and stiff edges considered by De La Llera et al. (2005). The Parameters of C_{xi} and C_{yi} is related to damping constants which is the *i*th supplemental dampers along the X and Y directions, respectively. Moreover, S_{xi}, R_{xi}, S_{yi} & R_{yi} are stiff nesses and strengths of the resistant frames elements in the X and Y directions, respectively. The Stiffness, strength and damping eccentricities are defined as $e_s=\frac{E_s}{L_x}$, $e_R=\frac{E_R}{L_x}$ and $e_d=\frac{E_d}{L_x}$ which each one of E_s, E_R & E_d are the distance of CS, CR & CD from CM, respectively.

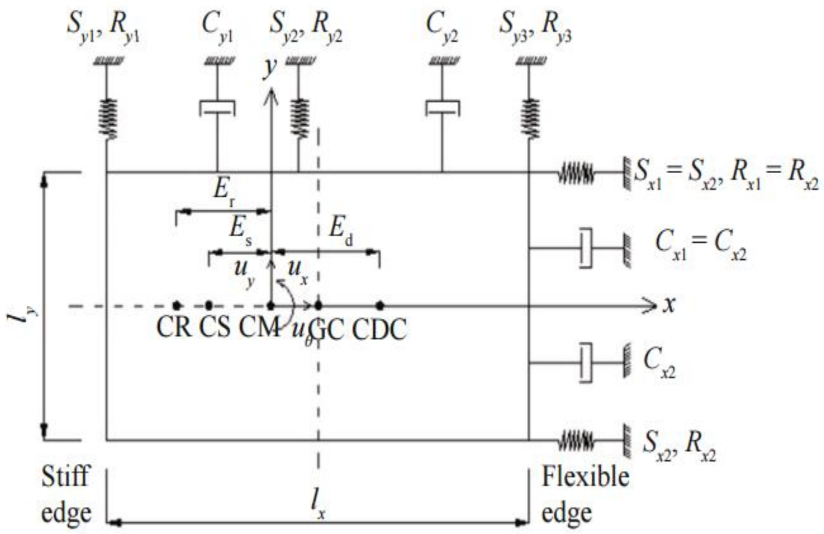

Fig. 1: Single-story structural model considered with one-way stiffness, strength and damping constant eccentricities [5]

In order to investigate the torsional balance, diaphragm of a single story structure, in which CM is placed in a desired point and is only asymmetric in X direction, is considered. Degrees of freedom in CM include uncoupled degree of freedom in in X direction (u_x), coupled degree of freedom of lateral displacement in Y direction (u_y) and rotation (u_θ). Omitting uncoupled displacement of u_x:

$$u(t)=[u_y(t,e_d)\ ,\ L_x u_\theta(t,e_d)] \tag{1}$$

Consequently by considering a rigid diaphragm, displacement vector in a point which have a distance of P from Cm is equal to:

$$u_y^{(p)}(t,e_d)=u_y(t,e_d)+Pu_\theta(t,e_d) \tag{2}$$

The MSV of displacement at a distance of P from CM is equal to:

$$E[u_y^{(p)}(t,e_d)^2] = E[\ u_y(t,e_d)^2] + 2\ PE[u_y(t,e_d)u_\theta(t,e_d)] + P^2\ E[u_\theta(t,e_d)^2] \tag{3}$$

In a point, at a distance of P^*, which placed from CM, the minimum square of expected response that is similar to a point which the coefficient correlation between two lateral and rotational movement is zero, is happened.

The amount of P^* is equal to:

$$P^* = -\frac{E[u_y\, u_\theta]}{E[u_\theta^2]} = -\rho_{y\theta}\frac{\sigma_y}{\sigma_\theta} \tag{4}$$

Where $\rho_{y\theta}$ is linear correlation coefficient between lateral and rotational motions at the CM and σ_y & σ_θ are lateral and rotational displacement standard deviation, respectively. The P^* point is known as “Empirical Center of Balance” or ECB. The MSV of the expected lateral displacement in a point with distance of d from ECB is in a form of parabola which its value is minimum in ECB.

$$E[u_y^{\sim(d)}(t,e_d)^2]= E[u_y^{\sim*2}] + d^2\ E[u_\theta^{*2}] \tag{5}$$

Therefore, if ECB is located in the diaphragm middle point (surface geometrical center in the form of rectangular diaphragm), for resistant elements with the same distance form surface center, the amount of lateral displacement MSV, became equal. Even though, this is not meant to zero rotation of asymmetric structure, but due to the existence of equal expected maximum displacement of two edges of structure, it shows the optimized mode of performance of resistant elements [6].

Mansoori and Sarvghad Moghadam (2009) assessed the possibility of simultaneous control of irregular structure's acceleration and displacement by using viscous damper. They used single-story systems with moment-resistant steel frames and one-way strength and stiffness eccentricity. They came to the conclusion that damping distribution can put a significant effect on modal properties and structure seismic response [7].

De La Llera and Almazan (2009) presented more complete concept of torsional balance in one-way single-story asymmetric structure considering the elastic behavior of structure. The effects of structure period, uncoupled torsional-to-lateral frequency ratio, static eccentricity and damping radius of gyration are studied too. Moreover, they submitted some criteria for torsional balance [8].

Almazan et al (2013) submitted an optimization method for control of seismic response of structures with one-way plan asymmetry considering linear and non-linear behavior relying on the concept of torsional balance. They decreased not only the two-edged displacement around the structure, but also they leveled the amount of displacements. They came to the conclusion that the location of damping eccentricity was not considerably affected by structure non-linear behavior and the results of two linear and non-linear models were the same [9].

Rahimzadeh Rofoie et al (2016) investigated the improvement of torsional responses of asymmetric structures with moment-resisting concrete frames and equipped with viscous dampers. They assessed a single-story model with one-way stiffness eccentricity with various arrangements of dampers and came to conclusion that dampers' appropriate distribution has a considerable effect on the torsional response of asymmetric concrete structures [5].

2 DAMPING PARAMETERS

The inherent damping matrix of a structure is defined as follows:

$$C_o = \alpha M + \beta K \tag{6}$$

Where M & K are mass and stiffness matrix and the amounts of α & β is calculated based on dominant vibrating modes. Total damping matrix of structure including inherent damping and supplemental damping is measured as below:

$$C_t = C_o + C_{sd} \tag{7}$$

C_{sd} is damping matrix resulted from FVDs. According to single-story structure by stiffness eccentricity, the strength and damping of figure 1, C_{sd} can be calculated as below:

$$C_x = \sum_i C_{xi} \quad , \quad C_y = \sum_i C_{yi} \quad , \quad C_\theta = \sum_i C_{xi}\, y_i^2 + \sum_i C_{yi}\, x_i^2 \tag{8}$$

Where C_{xi} and C_{yi} are damping factor of *i*th damper in X,Y direction and x_i,y_i, are distance of *i*th damper from CM in X and Y direction, respectively. Also, damping eccentricities in X, Y directions and normalized damping eccentricity is expressed as follows:

$$E_{sdx} = \frac{1}{C_y} \sum x_i\ C_{yi} \quad , \quad e_{sdx} = \frac{E_{sdx}}{L_x} = \frac{1}{L_x C_y} \sum x_i\ C_{yi} \tag{9}$$

$$E_{sdy} = \frac{1}{C_x} \sum y_i \, C_{xi} \quad , \quad e_{sdy} = \frac{E_{sdy}}{L_y} = \frac{1}{L_y \, C_x} \sum y_i \, C_{xi} \tag{10}$$

Supplemental damping matrix $[C_{sd}]$ is:

$$[C_{sd}] = \begin{bmatrix} C_x & 0 & -E_{sdy} \, C_x \\ 0 & C_y & E_{sdx} \, C_y \\ -E_{sdy} \, C_x & E_{sdx} \, C_y & C_\theta \end{bmatrix} \tag{11}$$

One of the most important parameters in the response for systems containing torsional radius of gyration supplemental viscous damper. For calculating the amount of this parameter, it is needed that the amount of torsional damping ratio is calculated in proportion to center of damping.

$$C_{\theta csd} = C_\theta - E_{sdx}^2 \, C_y - E_{sdy}^2 \, C_x \tag{12}$$

By such a definition, the amount of damping radius of gyration in X and Y directions is equal to:

$$\rho_{sd,x} = \sqrt{\frac{C_{\theta csd}}{C_x}} \tag{13}$$

$$\rho_{sd,y} = \sqrt{\frac{C_{\theta csd}}{C_y}} \tag{14}$$

3 MODELING

The considered model of this study is including a symmetric single-story steel structure of 3×3 bays. The bay dimensions are 6 and 5 meters in X and Y direction respectively and the height of the structure is 3.3 meters. The mass, stiffness and strength properties are assumed in a symmetric form in X and Y directions. Therefore, centers of mass, stiffness and strength are coincident with the geometrical center of the rigid diaphragm. Figure 2 shows a three-dimensional view of the basic model. Two models of torsional stiff and torsional flexible structure is extracted from the basic model. The parameters of these two models are shown in Table 1. Furthermore, for accounting the effect of uncoupled torsional-to-lateral frequency ratio (Ω_s), two flexible and stiff torsional structures with Ω_s=1.2 and Ω_s=0.89 are modeled according to table 1.

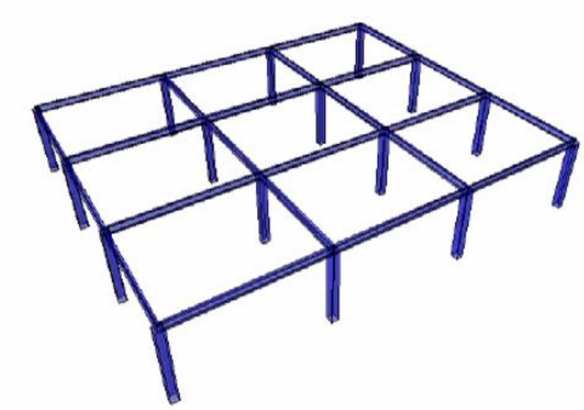

Fig.2: 3D view of the basic model [7]

Model No.	%e_s	%e_R	%e_m	T_x(sec) (Uncoupled)	T_y(sec) (Uncoupled)	T_θ(sec) (Uncoupled)	Ω_{sx}	Ω_{sy}
1	0.0	0.0	0.0	0.3633	0.3529	0.3052	1.16	1.19
2	0.0	0.0	0.0	0.3365	0.3277	0.3728	0.88	0.90

Table 1: Static and dynamic parameters of models 1&2

Asymmetry in structure is considered as a mass eccentricity. In order to create the mass eccentricity it is needed the mass center to move to a distance of e_m from the geometric center of the plan. It is worth mentioning to the change in structure mass moment of inertia which should be considered in calculations In order to create one-way mass eccentricity, Method applied by Mansoori et.al (2014) [10] is considered in this study.

3.1 Creating two-way mass eccentricity

In order to create two-way mass eccentricity, CM has been moved in plan diameter length. To this end, the plan has been divided into four sections and it is supposed firstly that, structure mass is uniformly distributed with surface density σ and then the surface density is increased in a tape with determined width in right side above the structure plan (location 1 in Fig.3) and the counterpart density of tape (Locations 2, 3&4 Fig 3) is decrease the same in a way that the total mass of plan remains constant.

Structure plan with the dimension of a & b is considered in X & Y directions according to figure 3 having mass density of σ where $\sigma=\frac{m}{a\,b}$ .The center of the surface is considered as the origin of coordinate and it is supposed that in a tape with the width of λa (λ=0.5) in right side of structure, the amount of surface density is equal to σ(1+η) and in counterpart tape, surface density is equal to σ(1-η). Moreover, in a tape with the width of λb at the top of plan, the amount of surface density is equal to σ(1+η) and in counterpart tape, surface density is equal to σ(1-η). In this equation, *η* (*-1* < *η* < *1*), indicates density variation ratio. Here, the amount of normalized eccentricity is as follows:

$$e_{mx}=\frac{1}{a}\left(\frac{\sigma(1+\eta)\lambda ab\left(\frac{a}{2}-\frac{\lambda a}{2}\right)+\sigma(1-\eta)\lambda ab\left(-\frac{a}{2}+\frac{\lambda a}{2}\right)}{\sigma ab}\right)=\eta\lambda(1-\lambda) \quad (15)$$

Also, with refer to diagonal form of mass eccentricity, it can be observed that:

$$e_{mx}=e_{my} \quad (16)$$

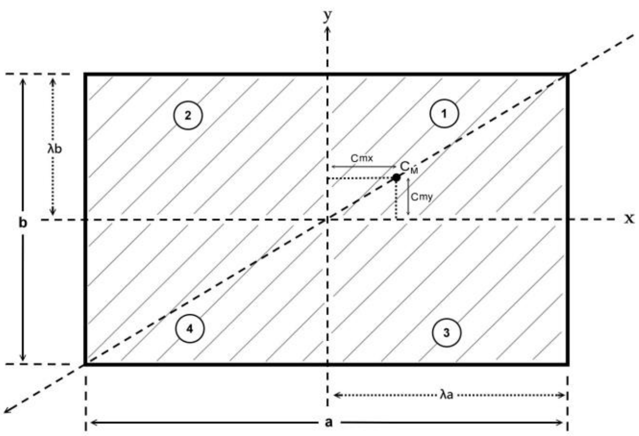

Fig. 3: Method of creating two-way mass eccentricity

In the case of uniform mass distribution, mass moment of Inertia is measured to $I_o=\frac{m(a^2+b^2)}{12}$. In order to calculate plan mass moment of inertia around second mass center, it is needed that each tape inertia calculated separately and added together. Therefore, in can be observed that:

$$\begin{aligned} m_1 &= m\,(1+\eta)\,(1+\eta)\,/4 \\ m_2 = m_3 &= m\,(1+\eta)\,(1-\eta)\,/4 \\ m_4 &= m\,(1-\eta)\,(1-\eta)\,/4 \end{aligned} \tag{17}$$

The amount of plan mass moment of inertia around second mass center is as follows:

$$I_{CM}=I_{CM1}+I_{CM2}+I_{CM3}+I_{CM4} \tag{18}$$

I_{CM1}, I_{CM2}, I_{CM3} and I_{CM4} are mass inertia moment for section 1, 2, 3 and 4 (Fig.3) and is determined as follows:

$$I_{CM1} = m_1\left[\frac{(\lambda^2a^2+\lambda^2b^2)}{12} + \left(\frac{a}{2}-\frac{\lambda a}{2}-ae_{mx}\right)^2 + \left(\frac{b}{2}-\frac{\lambda b}{2}-be_{my}\right)^2\right] \tag{19}$$

$$I_{CM2} = m_2\left[\frac{(\lambda^2a^2+\lambda^2b^2)}{12} + \left(\frac{a}{2}-\frac{\lambda a}{2}+ae_{mx}\right)^2 + \left(\frac{b}{2}-\frac{\lambda b}{2}-be_{my}\right)^2\right] \tag{20}$$

$$I_{CM3} = m_3\left[\frac{(\lambda^2a^2+\lambda^2b^2)}{12} + \left(\frac{a}{2}-\frac{\lambda a}{2}-ae_{mx}\right)^2 + \left(\frac{b}{2}-\frac{\lambda b}{2}+be_{my}\right)^2\right] \tag{21}$$

$$I_{CM4} = m_4\left[\frac{(\lambda^2a^2+\lambda^2b^2)}{12} + \left(\frac{a}{2}-\frac{\lambda a}{2}+ae_{mx}\right)^2 + \left(\frac{b}{2}-\frac{\lambda b}{2}+be_{my}\right)^2\right] \tag{22}$$

The numerical models are created in OpenSees software [11] and several non-linear analysis are performed. In order to consider the non-linear effects for beams and columns, fiber elements with strength hardening behavior is used for steel. The dampers are modeled as linear viscous zero length elements in both structure's edges. The models are analyzed under two synthetic records compatible with design spectrum of European code (EC8) with soil type B.

According to figure 4, one component record (Y direction, figure 4-b) is used for structure with one-way eccentricities and two component record (X and Y direction, figure 4-a and 4-b) is employed for structures with two-way eccentricities.

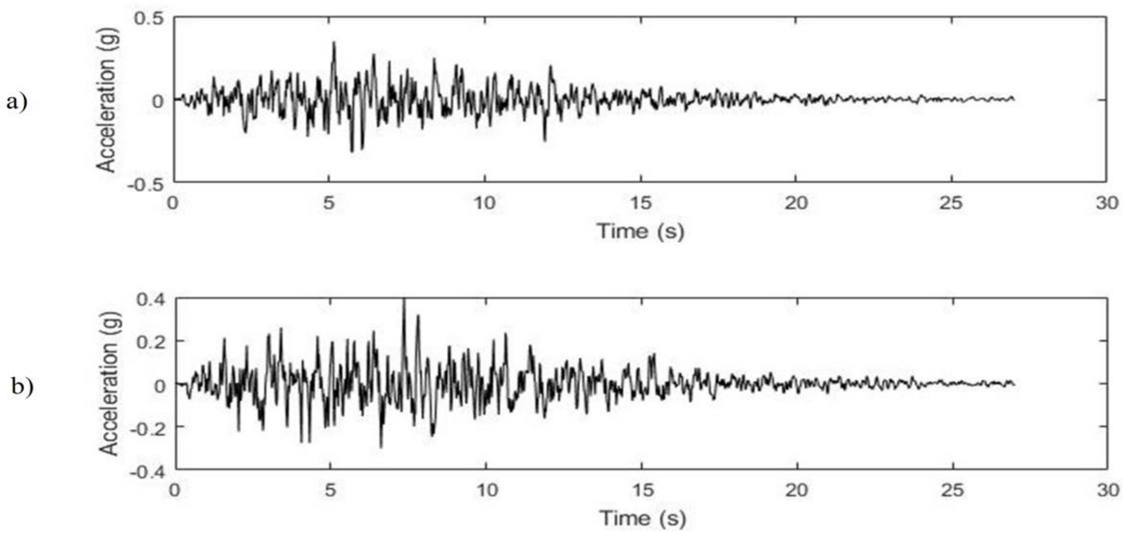

Fig.4: Synthetic records. a) Ground motion in X direction b) Ground motion in Y direction

Also, particle swarm optimization algorithm is used to minimize torsional response and reaching to torsional balance conditions. Objective function of PSO algorithm is defined as below:

$$\text{Objective Function} = (E[u_{yfe}^2] - E[u_{yse}^2]) + (E[u_{xfe}^2] - E[u_{xse}^2]) \quad (23)$$

u_{yfe} and u_{yse} are structure's flexible and stiff displacement in Y direction and u_{xfe} and u_{xse} are structure's flexible and stiff displacement in X direction Linking of Matlab and OpenSees softwares are used for implementation of the PSO algorithm.

4 ANALYSIS RESULTS:

The results of time history analyses for the lateral displacement by using torsional balance concept are shown in figures to. For simplicity the expected values are only shown in flexible and stiff edges of diaphragm.

For example, figure 5 shows the displacement of flexible and stiff edges of structure with normalized one-way eccentricity equal to 0.1, with uncoupled torsional-to-lateral frequency ratio of 1.2 in two modes without damping (figure 5-a) and with optimum damper distribution using torsional balance conditions (figure 5-b).

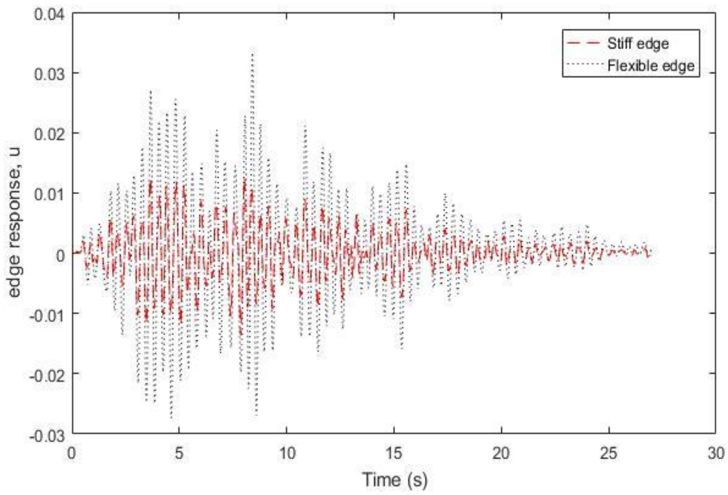

Fig.5-a: Typical response of a torsionally stiff asymmetric structure without FVDs

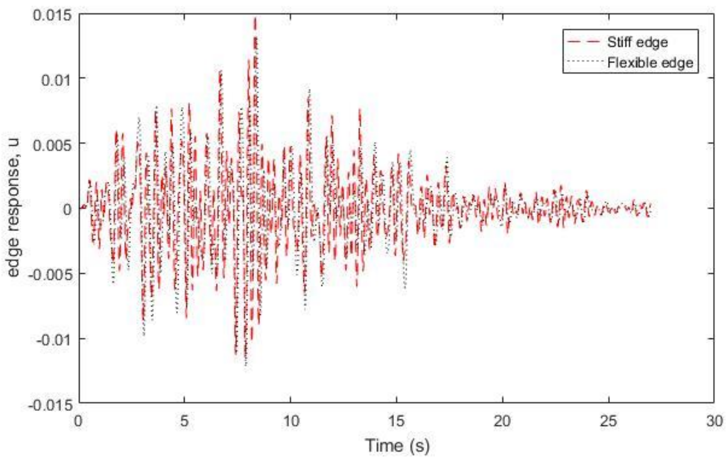

Fig.5-b: Typical response of a torsionally stiff asymmetric structure with optimum distribution of FVDs

It can be observed that in torsional balance condition, not only responses of two edges are decreased significantly, but also the responses are almost equal and improved in comparison with the case that no damper is used in the structures.

Figure 6 indicates the standard deviation of normalized displacement of structure plan having normalized one-way eccentricity 0.1 and Ω_s=1.2 in two modes without supplemental damping and with supplemental damping considering torsional balance concept.

It can be seen that in a torsional condition, the standard deviation of plan displacement is parabola and two points with the same distance from geometrical center of structure plan have the same standard deviation of displacement according to equation 5.

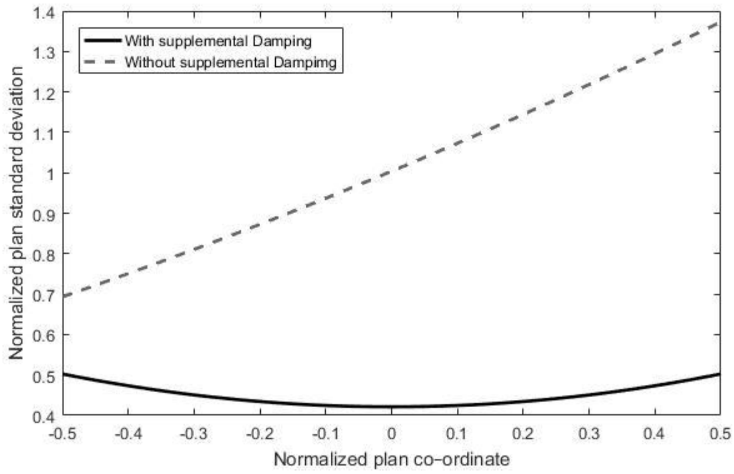

Fig.6: Standard deviation profiles of plan displacement for asymmetric-plan structures

figure 7 shows the optimized values of normalized damping eccentricity versus normalized one-way mass eccentricity for two torsional stiff and flexible structures with Ω_s=1.2 (Fig. 7-a) and Ω_s=0.89 (Fig.7-b) and supplemental damping of 15 percent by using torsional balance concept. In figure 7-a, it can be seen that in order to achieving torsional balance condition, damping center should be in same side with CM and bigger than CM distance to geometric center. Moreover, by increasing the structure asymmetry and consequently, increase of mass eccentricity, it is needed to have more damping eccentricities to achieve torsional balance.

It can be seen that for a normalized mass eccentricity bigger than 0.125 (e_m>0.125), and damping ratio of 15% is not be a response to torsional balance conditions. Figure 7-b show the optimized normalized mass damping eccentricity versus normalized eccentricity for torsional flexible structure.

The results gained from two torsional flexible and stiff models are the same, but with a difference that in torsional flexible structure it is needed to have less mass damping eccentricity in order to achieve optimized distribution of viscous dampers in comparison with torsional stiff structure. As can be seen, for mass eccentricity bigger than 0.175 (e_m>0.175) supplemental damping ratio of 15% is not be a respond to achieve the optimized condition.

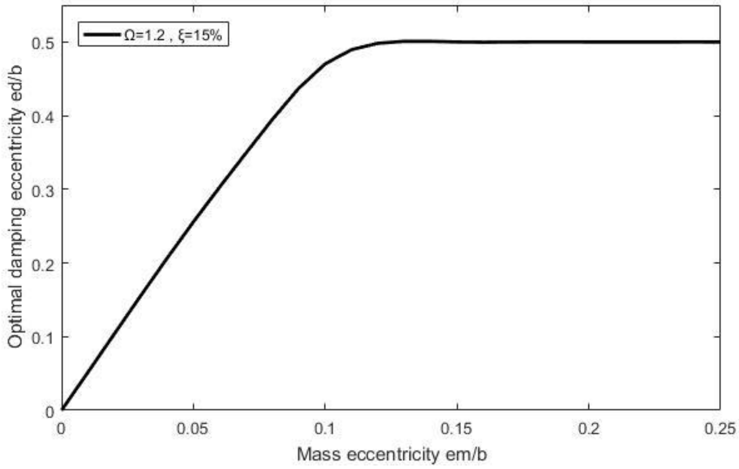

Fig.7-a: Optimal eccentricity curve e_d for asymmetric-plan structure. (Ω_s=1.2, ξ=15%)

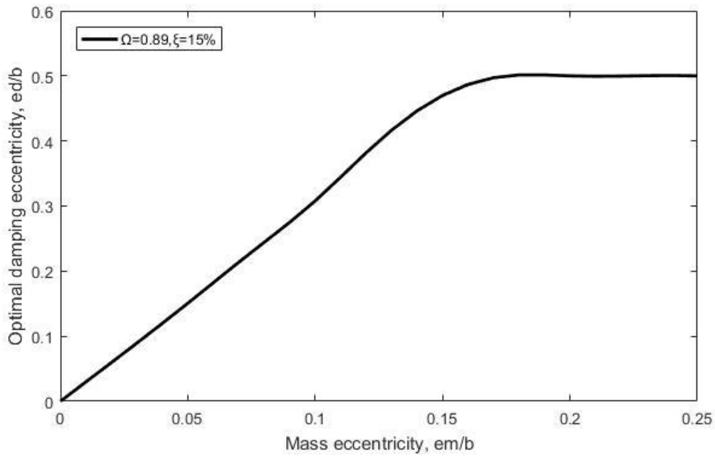

Fig.7-b: Optimal eccentricity curve e_d for asymmetric-plan structure. (Ω_s=0.89, ξ15%)

Figure 8 shows the changes of normalized damping center in terms of normalized one way mass eccentricity of torsional stiff structure for 15, 20 & 25 percent of supplemental damping. It can be concluded that it is needed less damping eccentricity in order to achieving torsional balance condition by increasing the damping percent.

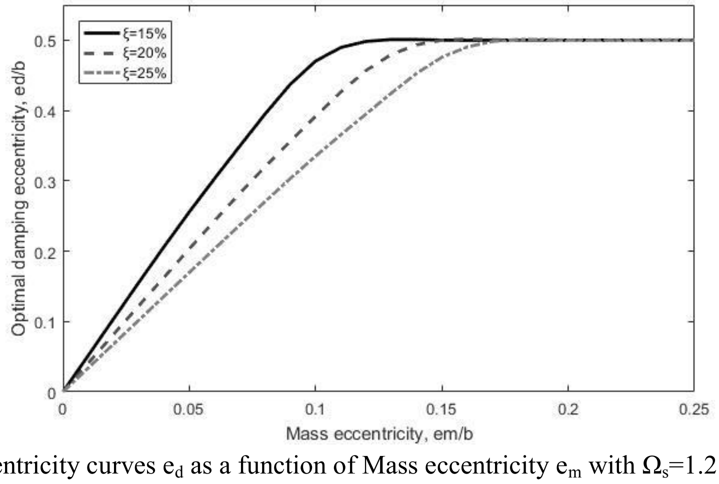

Fig.8: Optimal eccentricity curves e_d as a function of Mass eccentricity e_m with Ω_s=1.2 and ξ=15, 20&25%

Figure 9 presents normalized optimized changes of damping center in X and Y directions versus of normalized two-ways mass eccentricity for two torsional stiff structures of 9-a and torsional flexible structure of 9-b and ξ=15%. The diaphragm center has been changed in order to considering two-ways asymmetry in structure. The results show that in order to achieve optimized design, dampers in two directions of X and Y should be distributed in a way that damping eccentricity be in a same side with CM and more than the distance of CM to geometric center.

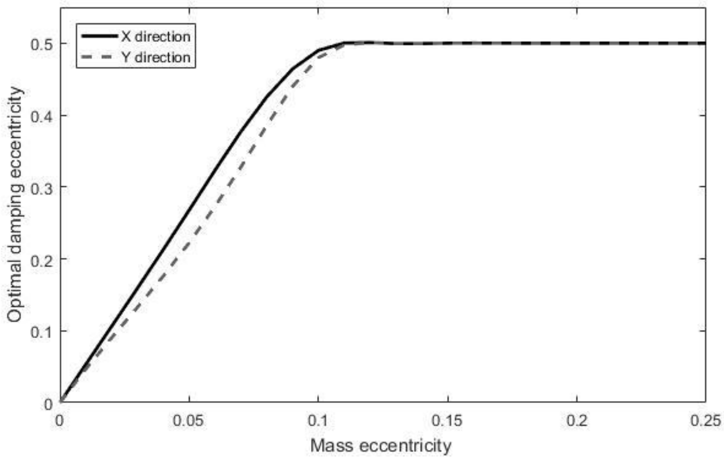

Fig.9-a: Optimal eccentricity curves e_d as a function of Mass eccentricity e_m with Ω_s=1.2 and ξ=15%

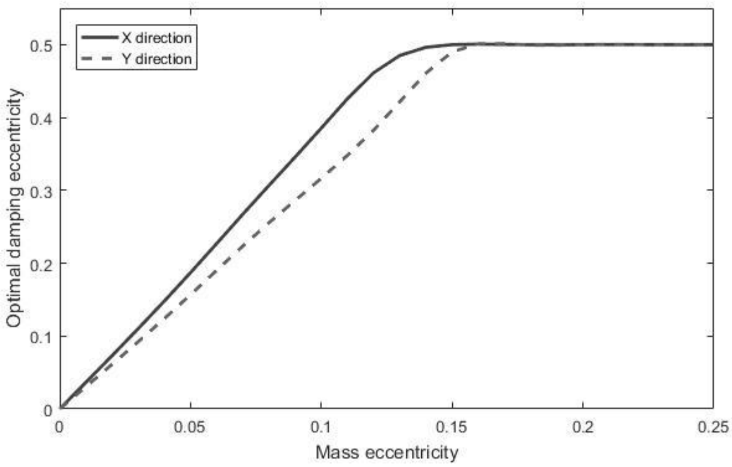

Fig.9-b: Optimal eccentricity curves e_d as a function of Mass eccentricity e_m with Ω_s=0.89 and ξ=15%

Moreover, similar to one-way mass asymmetric model, with increasing of mass eccentricity, higher values of damping eccentricities is needed to achieve torsional balance condition. It can be seen that; in comparison with torsional stiff structures, it is needed to have less damping eccentricity in torsional flexible structure for achieving torsional balance in two directions.

Besides, by comparing figure 9-a and 10 , it can be observed that less damping eccentricity is needed in order to achieve optimized design by increasing damping ratio in torsional stiff structure and the structure can reach to torsional balance against more two-ways asymmetry by increasing damping ratio.

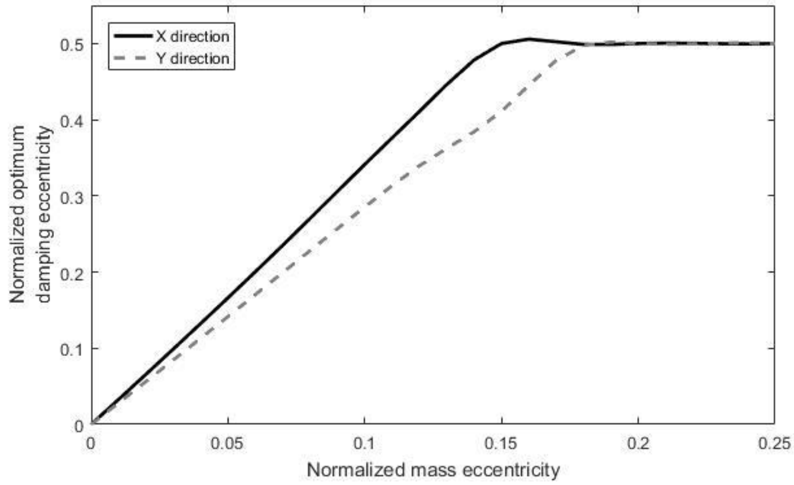

Fig.10: Optimal eccentricity curves e_d as a function of Mass eccentricity e_m with Ω_s=1.20 and ξ=25%

5 CONCLUSIONS:

In this research several parametric analysis has been carried out on a single-story torsional stiff and flexible structure with one-way and two-ways mass asymmetry .The lateral strength and stiffness in all models are constant for comparing the results in different items.

By using the theory of torsional balance, the MSV of lateral displacement of two edges of diaphragm is measured and the amount of optimized damping eccentricity is determined using PSO optimizing algorithm in terms of different values of mass eccentricity, damping ratio and uncoupled torsional-to-lateral frequency ratio.

The summaries of results are as follows:

1- It can be seen that using of viscous damper not only caused to decrease responses of structural lateral displacement, but also it makes displacement of two asymmetric edges of structure close together by appropriate distribution of dampers in structure plan.
2- Damping optimized eccentricity for controlling the displacement, mostly have the same direction of mass eccentricity and bigger than the distance of stiff center to geometric center.
3- By increasing the asymmetry of structure resulted from increasing mass eccentricity, it is needed to have more damping eccentricity or more damping ratio for controlling the displacement of two asymmetric edges of structure.
4- It seems that it is needed less damping eccentricity in order to achieving torsional balance by increasing damping ratio.
5- Torsional stiff structures need more damping eccentricity in order to achieving the control of displacement of two stiff and flexible edges of structure in comparison with torsional flexible structures.
6- The results gained from two-ways mass eccentricity with the results gained from one-way mass eccentricity are conformed and it is seen that the result of these two comparisons is almost the same.

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