

## PROCESSING OF 3D OPTICAL MOTION DATA OF SHAKING TABLE TESTS: FILTERING OPTIMIZATION AND MODAL ANALYSIS

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**Abstract.** *The use of 3D optical motion data for structural dynamics is both promising and challenging. Further developments for on-the-field applications are still needed, but, at the present stage, such techniques are already feasible for laboratory dynamic tests. A measurement system of this kind was installed for the first time for shaking table testing at the ENEA Casaccia Research Center. This 3D motion capture system is capable of acquiring the positions of more than a hundred passive markers with a constellation of 10 near-infrared cameras. In particular, this paper focuses on filtering and processing the markers displacements. A methodology is proposed for optimizing the displacement data processing to obtain an estimation of markers accelerations. Such methodology implies the optimal choice of the Savitzky-Golay-filter parameters for the implementation of a filtered numerical derivative of displacement data. The optimal parameters are calculated minimizing the error in the estimation of the peak acceleration of a reference marker compared to a conventional accelerometer located at the same measurement point. The accuracy of markers accelerations was estimated in the range of 0.01-0.02 g, which is appropriate for providing interesting indications for studied structures subjected to most shaking table testing. For example, markers displacements data and related estimated accelerations can be combined to obtain the hysteretic behavior of the structure or of portions of it. Markers data were also used for Experimental and Operational Modal Analysis (EMA/OMA) in order to extract the modal parameters and to calibrate/validate the Finite Element Models (FEM) of the structure. In particular, the combined use of OMA by markers data and numerical modal analysis by FEM permits to compare the resulting modal shapes for a more precise dynamic identification. An example of practical application is illustrated on a shaking table experimentation conducted on a two-story tuff-masonry prototype, which reproduces a typical ancient buildings of Central Italy.*

## 1 INTRODUCTION

In shaking table experiments the measurement of motion parameters is typically carried out by means of accelerometers and displacement sensors. This is due to the well-known difficulty of obtaining the complete motion of measurement points through a single device with appropriate accuracy. In the past decades a vast literature has been published over the correction and the processing of accelerometric signals in order to estimate the displacement by numerical algorithms [1]. Nonetheless, results are not always satisfactory, especially when baseline correction is manifestly ineffective [2]. Theoretically, the complete motion of a point can be calculated starting from its trajectory, which means, in practical, to measure its absolute displacement. Subsequently, velocity and acceleration can be estimated through successive operations of numerical derivation. In such a direction, an interesting prospective is provided by the recent developments of displacement sensors based on opto-electronic technologies applied to laser or digital vision systems. Such kind of systems are increasingly utilized as motion measurement devices also for shaking table tests [3, 4].

In the frame of an increasing interest for the measurement of the absolute displacement in seismic tests, in order to provide experimental validation for the application of the increasingly relevant displacement based approach to the seismic design, ENEA is engaged in exploring all its 3D motion capture potentialities. In fact, among the earliest shaking table facilities in Italy to install a 3D motion capture system was the ENEA Casaccia Research Center [5]. This specific system, named 3DVision, offers advantages in terms of instrumentation flexibility, large number of measurement points, easy and low-encumbrance marker installation [6]. Besides, 3D computer graphic potentialities and friendly visualization of real-time data and movies are particularly suitable for experiment remote sharing via the internet, like in the DYSCO virtual lab implemented by ENEA [7].

Particularly interesting is the possibility of measuring even more than a hundred 3D points through passive cheap markers to obtain information also on the acceleration in destructive shaking table tests, as an alternative to installing numerous conventional expensive accelerometers. The accelerations estimated from markers can provide valuable indications for studying structures subjected to shaking table testing, even if accuracy cannot be as accurate as conventional accelerometers. In fact, accuracy in the order of 0.01-0.02 g is still appropriate for providing interesting information for seismic tests with input peak accelerations that typically fall in the range 0.1-1.0 g. The present paper illustrates a methodology for processing 3DVision data in order to obtain an optimal estimation of acceleration for shaking table tests. To such aim, the problems related to error propagation and numerical derivative of data were explored in order to make a proper choice of digital filter and processing algorithm.

The methodology was applied to experimental data acquired in shaking table tests on a two-story tuff-masonry prototype, representing a traditional building typology of Central Italy. In particular, the results in terms of estimation of accelerations and hysteretic behavior of portions of the structure are illustrated.

Another interesting application of markers data is to perform the Experimental and Operational Modal Analysis (EMA/OMA). The great number of 3D measurement points available permits to improve the accuracy in the identification of modal shapes and frequencies [8], as well as to distinguish global and local modes, on the basis of the specific markers selected for the analysis. EMA and OMA techniques, such as FRF-MIMO and FDD, were calculated to extract the modal parameters of the tested tuff-masonry prototype allowing to calibrate and finally validate the Finite Element Model (FEM) of the structure.

## 2 DATA FILTERING

Digital filters are used to attenuate the measurement error from the acquired signals, once that all hardware sources of noise are avoided or limited as much as possible. In the proposed DDP the main objective is to reduce the measurement noise through digital filtering of acquired data in order to optimize the estimation of velocity and, especially, acceleration. In this optic it is relevant to premise how displacement noise propagation to velocity and acceleration was modelled. Moreover, as well known in digital signal processing, even the numerical operation of derivation itself adds another source of noise [9]. Consequently, a proper DDP must include filtering algorithms that should provide good performance also in terms of attenuation the error amplification caused by numerical derivative.

### 2.1 Displacement error propagation to acceleration error

Let us consider a pure sine vibration of amplitude  $A$  and frequency  $f$ . Then the displacement  $s(t)$  can be expressed with the following equation with time  $t$ :

$$s(t) = A \cdot \cos(\omega \cdot t) \quad (1)$$

where  $\lambda = 2ff$ . Besides, be the measurement error dependent only on the vibration amplitude  $A$ , which is true for substantially white noise. According to the general formulation of the error propagation theory, the displacement error can be expressed as follows [10]:

$$\Delta s(t) = \Delta A \cdot \cos(\omega \cdot t) \quad (2)$$

Similarly, for velocity  $v$  and acceleration  $a$  the error can be written:

$$\Delta v(t) = -\Delta A \omega \cdot \sin(\omega \cdot t) \quad (3)$$

$$\Delta a(t) = -\Delta A \omega^2 \cdot \cos(\omega \cdot t) \quad (4)$$

The Eq.s (3) and (4) show that the error in displacement amplitude  $\Delta A$  propagates to  $v$  and  $a$  increasing with  $\lambda$ . In particular, acceleration error is characterized by quadratic growth with the frequency for a white displacement noise (Figure 1).

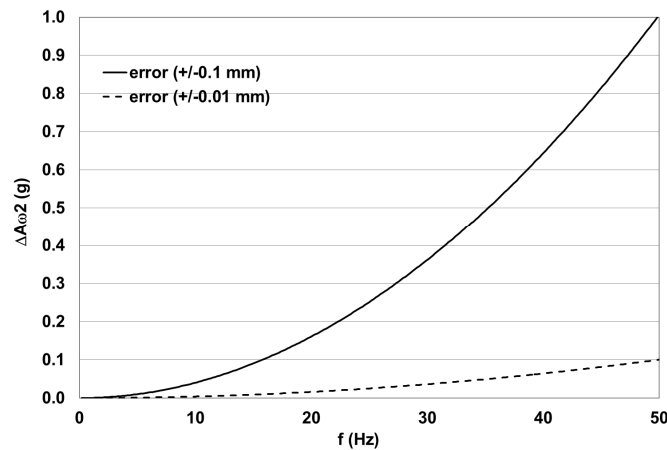


Figure 1. Error propagation effect on the acceleration amplitude ( $\Delta A \omega^2$ ) vs. signal frequency  $f$  (Hz).

## 2.2 Filtering technique

Several filtering approaches can be considered for measurement noise reduction, whose performances mainly depend on the noise and the signal properties in both the time and the frequency domain.

One very common approach is represented by the band-pass family. As a first step, signals are analyzed in the frequency domain. When the frequency content of the signal is clearly different from that of the noise, than band-pass filters performance is optimal. In seismic applications typical cut-off frequencies are between 0.1-0.5 Hz and 25-30 Hz, since natural spectra generally have most of their energetic content within this range [11]. Many band-pass filters are available in literature.

Instead of a band-pass filter, in the present paper the Savitzky-Golay smoothing filter was considered, as it is one of the most effective algorithms in removing random noise from experimental data without excluding completely the contribution of frequencies higher than an upper cut-off limit [12]. It essentially performs a local polynomial regression on a series of equally spaced data points (windowing), to determine smoothed values [13]. This filter is characterized by two parameters: the fitting polynomial degree ( $n$ ) and the windowing width ( $m$  points). Furthermore, the Savitzky-Golay can be implemented to perform differentiation operations, which is particularly useful in case successive derivatives of raw data must be calculated, like in the present case for obtaining velocity and acceleration from displacement data.

In fact, various techniques of numerical differentiation can be implemented. Classical methods based on higher-order central finite differences provide a somewhat low-pass effect [8]. According to such methods a general expression of the first derivative  $f'$  can be written as:

$$f' = \frac{1}{\Delta t} \sum_{k=1}^M c_k (f_k - f_{-k}) \quad (5)$$

where  $M = (N-1)/2$  with  $N$  number of data within the moving window,  $\Delta t$  is the time step and  $c_k$  are called convolution coefficients. Several methods descending from such approach, also known as convolution methods, combine the derivative estimation with the filtering effect by choosing the appropriate  $c_k$  coefficients in Eq. 5. The Savitzky-Golay filter can be considered a specific case of this general method.

## 2.3 Filtering optimization methodology

The used filtering strategy was based on the assumption that filter parameters  $n$  and  $m$  values can be optimized through the minimization of the difference  $\Delta f$  between results by filtered markers and by reference measurements at the same points on the structure (objective function). In the present study, the reference measurements were carried out through a limited number of conventional accelerometers (capacitive devices with sensitivity of 1 V/g and broadband resolution of 0.03 mg in terms of RMS error). The optimization problem can be expressed mathematically as follows [14]:

$$\begin{cases} \min \Delta F(n, m) \\ n, m \in R \end{cases} \quad (6)$$

Generally speaking,  $F$  can be whatever function or physical quantity we are interested in and, more specifically in the present application, whatever motion parameter we want to use to optimize markers data. The simplest choice for  $F$  is the peak acceleration  $a_p$  of the recorded time-history, but also more sophisticated ground motion parameters (Arias Intensity, Housner Intensity, etc.) can be considered.

Assessing the performance of filtering algorithms is a quite elusive issue and depends on the specific result they are design to or intended for. In the proposed methodology, the focus is to optimize noise reduction from displacement data in order to have an estimate of the acceleration good enough for structural analysis. Consequently, the filtering performance was assessed by comparing the final result with signals acquired by control points equipped with accelerometers. In practice, this can be done using only a part of the installed accelerometers as reference for the optimization process and the other ones for checking the filtering performance.

### 3 EXPERIMENTAL APPLICATION TO SHAKING TABLE TESTS

#### 3.1 Tested specimen

A 2/3-scaled two-story tuff-masonry prototype was subjected to shaking table test (Figure 2). The specimen dimensions were  $3.50 \text{ m} \times 3.00 \text{ m}$  and the inter-story height was  $2.30 \text{ m}$ . The walls thickness was  $0.25 \text{ m}$ . The overall weight was  $193.00 \text{ kN}$ .

#### 3.2 3D optical acquisition setup

The equipment installed at ENEA Casaccia Research Center is able to acquire the trajectory of a large number of retro-reflecting markers located within the measurement volume by triangulating data from a constellation of 10 high-frequency Near Infra-Red (NIR) cameras, based on VICON technology. The NIR cameras are equipped with a 4-Mpixel high-frequency CMOS digital sensor. Data were acquired at 100 fps (which as saying the sampling frequency in Hz).

The total number of markers located on the specimen was 141. Two triaxial accelerometers were located at positions depicted in Figure 2.

Measurement error calculated for each marker resulted in the range  $0.014\text{-}0.035 \text{ mm}$  in terms of RMS error of markers displacement and was approximately considered as white noise, as confirmed by its FFT in Figure 3a. FFT of the acceleration error calculated by 2nd numerical derivative of 3D Vision displacement data, depicted in Figure 3b, show a quasi-quadratic growth, in substantial accordance to the proposed model of error propagation.

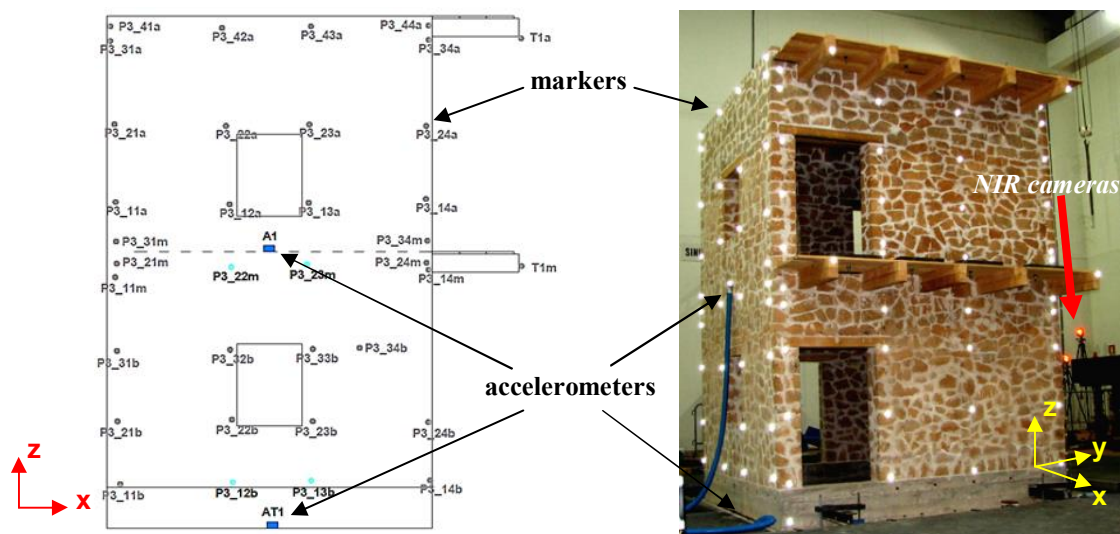


Figure 2. Markers and accelerometers positioned on the specimen. Retro-reflecting markers appear lighted in photos taken by common camera using a flash (right).

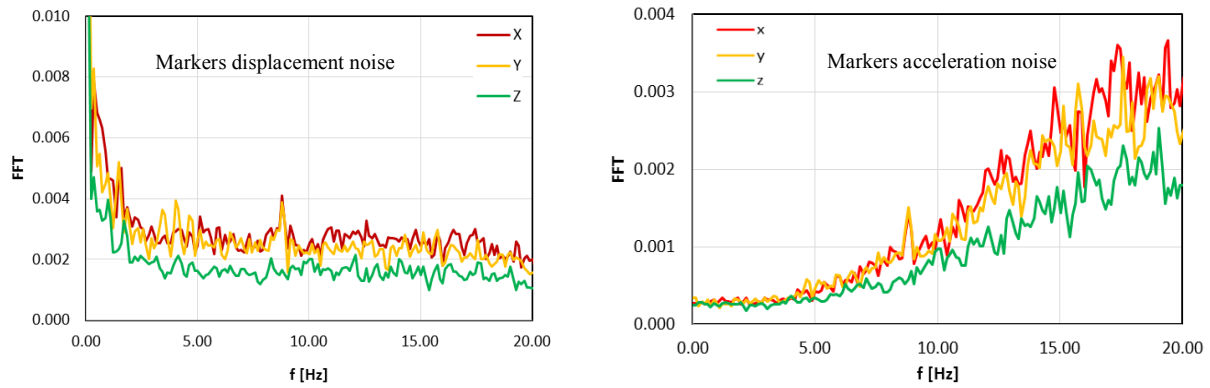


Figure 3. FFT of markers noise in static condition (left) and error propagation to unfiltered acceleration (right).

### 3.3 Experimental campaign

The experimental tests were carried out on the largest of the two shake tables available at ENEA Casaccia laboratory (System 1 in Table 1). A natural input based on the time-history recorded at Colfiorito station during Umbria-Marche earthquake, 1997, was considered for the tests sequence. The above triaxial seismic signal was submitted to the shaking table, increasing gradually the input Peak Ground Acceleration (PGA) at each trial by 0.05 g. Before the test sequence and after each trial a random test (white noise input) at 0.05 g was performed for the dynamic identification of the structure. The specimen showed some initial damage at 0.15 g (nominal PGA) and final collapse occurred at 0.40 g.

	System 1	System 2
Table size	4 x 4 [m]	2 x 2 [m]
Degrees of Freedom	6	6
Frequency range	0-50 [Hz]	0-100 [Hz]
Acceleration	3g peak	5g peak
Velocity	0.5 m/s (0-peak)	1 m/s (0-peak)
Displacement	0.125 m (0-peak)	0.15 m (0-peak)
Specimen Mass	30 [t]	5 [t]

Table 1. ENEA Casaccia shake tables technical specifications.

### 3.4 Filtering optimization application

The filtering optimization methodology was applied considering the accelerometer AT1 as reference point for calculating the objective function  $\hat{a} F$ . Synthetically, the optimization problem was formulated as follows:

$$\begin{cases} \min \Delta a(n, m) \\ \Delta a = |a_{p(AT1)} - a_p^{SG}| \\ n, m \in R \end{cases} \quad (7)$$

where  $a^{SG}$  indicates the acceleration calculated by double differentiation according to Eq. 5 implemented with Savitzky-Golay filtering of the marker located near AT1.

The accelerometer located at first floor (A1) was taken as control point to assess the performance of the filtered double differentiation methodology.

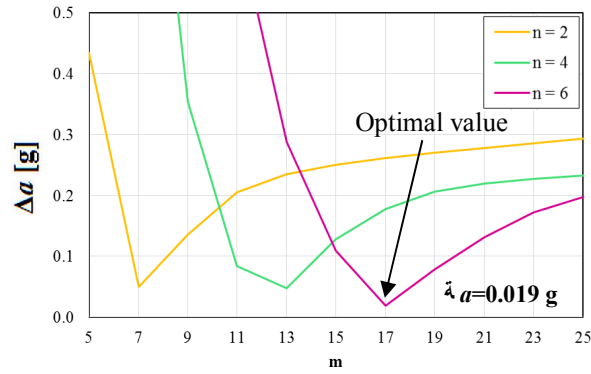


Figure 4. Identification of optimal filter parameters  $n$  and  $m$  by minimizing the acceleration error  $\hat{\Delta} a$  with respect to the reference accelerometer AT1.

## 4 RESULTS

### 4.1 Optimal acceleration estimation

The results of the optimization problem of Eq. 6 are shown in Figure 4. The optimal values of the Savitzky-Golay parameters were  $n = 6$  and  $m = 17$  (corresponding to a windowing half-width of 0.08 s at a sampling frequency of 100 Hz).

The optimal acceleration error at control point A1 obtained in shaking table tests at initial damage and at final collapse of specimen (0.15 g and 0.40 g of PGA, respectively) is depicted in Table 2, showing that in both x and y directions the acceleration error remained in the order of 0.01-0.02 g. The estimated acceleration of markers was utilized to calculate the hysteretic behavior. On the one hand, the hysteretic behavior can be obtained for any portion of the specimen that is described by a given amount of markers. On the other hand, seismic tests are not cyclic, thus hysteretic cycles calculated in shaking table tests are very noisy.

Test input PGA	Acceleration error [g]	
	x	y
0.15 g	0.006	0.024
0.40 g	0.018	0.024

Table 2. Acceleration error at the control point A1 at the first floor in x and y direction in shaking table tests with input Peak Ground Acceleration (PGA) of 0.15 g and 0.40 g.

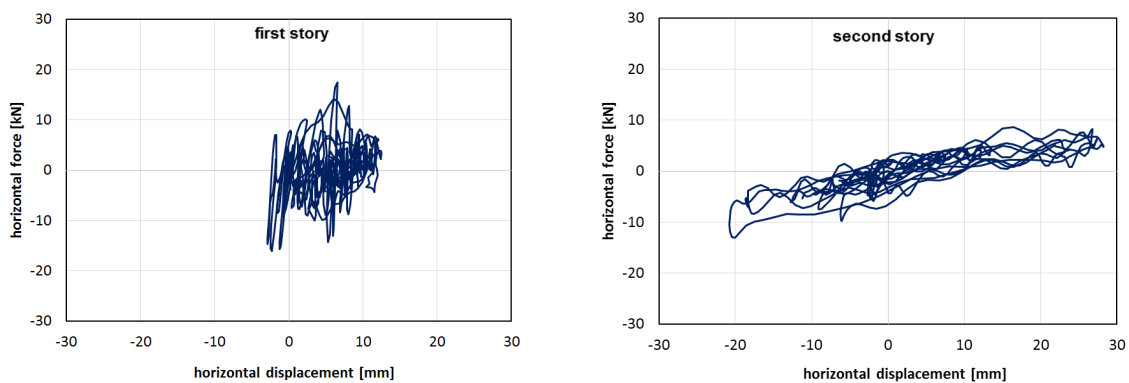


Figure 5. Hysteretic behavior of specimen façade wall of first story (left) and second story (right).



Nonetheless, they can give interesting indications on the walls behavior under dynamic conditions. As an example, in Figure 5 the hysteretic behavior during the trial at 0.40 g was calculated for the façade wall at the first and the second story. It is evident the loss of stiffness at the second story with respect to the first one.

## 4.2 Modal analysis

The availability of a large number of measurement points allowed the application of Frequency Response Function Multi-Input Multi-Output (FRF-MIMO) technique. In particular, the transmissibility function  $H$  was considered to calculate the FRF. The markers at the specimen base were taken as input signals. In Figure 6 the FRF of markers at first story (MIMO 1) and at second story (MIMO 2) are compared with the FRF of the theoretical Single Degree Of Freedom (SDOF). It is evident that the markers located at the second story are characterized by higher amplification, providing a clearer identification of the first mode. This is confirmed by Figure 7 that compares the modal shape and frequency  $f_n$  of the first mode obtained by Frequency Domain Decomposition (FDD) with the same by FEM, indicating clearly a bending mode in x direction.

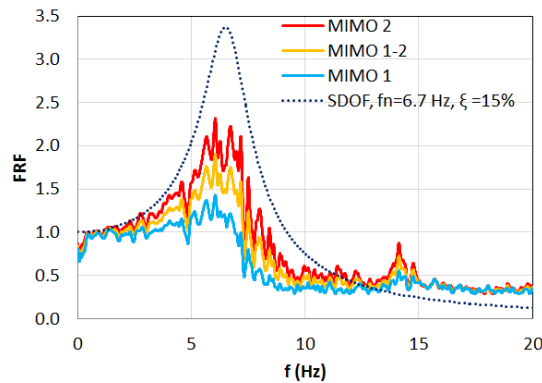


Figure 6. Frequency Response Function (FRF) of markers at first story (MIMO 1), at second story (MIMO 2), and at both stories (MIMO 1-2) with respect to markers at the specimen base compared with FRF of theoretical Single Degree Of Freedom (SDOF).

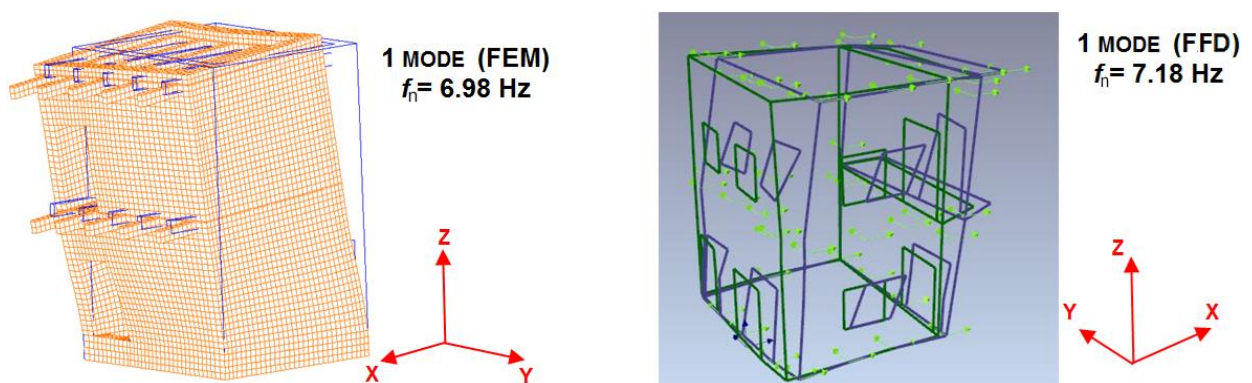


Figure 7. Modal shape and frequency  $f_n$  of the first mode by FEM (left) and by Frequency Domain Decomposition (FDD) with markers data (right).

## 5 CONCLUSIONS

A methodology was proposed for the optimal choice of the filter parameters to process 3D displacement data, such as 3D motion capture markers data, in order to obtain an estimation of the peak acceleration of a large number of measurement points of structures subjected to



laboratory dynamic testing. The above methodology was described and a practical application to shaking table tests on a two-story tuff-masonry specimen was illustrated. The accuracy achieved in the estimation of the peak acceleration resulted in order 0.01-0.02 g. The filtered acceleration was then utilized for calculating the hysteretic behavior of the specimen walls during the seismic tests providing valuable indications on the loss of stiffness of the masonry.

Markers data were also processed for the dynamic identification of the specimen through modal analysis techniques. In particular, FRF-MIMO and FDD were used to extract the modal parameters of the tested prototype. The great number of 3D measurement points available (141) permitted to provide an accurate identification of modal shapes and frequencies. This allowed to calibrate and finally validate a FEM of the specimen for subsequent structural analysis.

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