

## **LIMIT ANALYSIS APPROACH FOR THE SEISMIC VULNERABILITY REDUCTION OF MASONRY TOWERS THROUGH STRENGTHENING WITH TRADITIONAL AND INNOVATIVE MATERIALS**

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**Abstract.** *The paper investigates the possibility and the effectiveness of reducing the seismic vulnerability of masonry towers by means of composite materials and traditional steel bands. Masonry towers are very widespread in Italy, both as bell towers for churches and defense towers in medieval cities and castles. Masonry material, presenting low mechanical properties, is not suitable to withstand significant tensile and compression stresses induced by earthquake loading. The slenderness of these structures is another factor that can reduce the bearing capacity when significant stresses are present in specific structural parts. The seismic vulnerability of masonry towers is very high, as a consequence of both poor material properties in tension and high compression levels at the base of the structure. Moreover, Italy is characterized by a high potential risk to be stricken by moderate/high seismic events, as experienced in the last decades. In such a situation, the seismic upgrading of masonry towers could appear rather important. Seismic upgrading by introducing both traditional steel bars and composite materials as strips or rebars is analyzed in detail for different towers. Based on some a priori assumed failure modes (one proposed by Heyman), simplified models from a limit analysis approach are here discussed and used to define the most suitable retrofitting solution. The retrofitting possibilities consist of: a) horizontal hooping rings; b) vertical pre-stressed tie rods; c) vertical composite strips. A simplified straightforward relationship is found between the retrofitting requirements and seismic hazard. The procedure is applied for a wide range of geometrical properties and appears to be fast and reliable.*

## 1 INTRODUCTION

Recent seismic events have highlighted that existing masonry towers and churches are particularly vulnerable under earthquake excitations; hence, national and international standards have imposed the evaluation of their structural performance, even in the presence of horizontal loads, [1], [2]. However, seismic vulnerability is not the only threat for masonry towers; for instance, approximately one century ago, the Civic Tower of Pavia collapsed due to self-weight and the degradation of the material, [3]. Other examples of the collapse of masonry towers could be found, e.g. the bell tower in Piazza San Marco in Venice (Italy, 1902) and the bell tower of Saint Magdalena church in Goch (Germany, 1992). As a matter of fact, masonry material has low mechanical properties, exhibits very limited ductility and is characterized by a brittle failure. The safety assessment of masonry towers against earthquakes appears to be of relevant importance in such circumstances, for historical, social and cultural reasons.

The behavior of masonry towers under vertical and seismic loads is widely studied in the literature. Different levels of analysis, such as FE (finite element) analyses [4]–[9], rigid elements [10], Limit Analysis (LA) [11] and simplified models [7], [12], are available in the literature. As regards the towers performance upgrading, very few cases that could be found consist of horizontal and vertical tied rods, which are used to increase the global structural capacity under horizontal loads, [13], [14]. In recent research, different authors have studied the applicability of new composite materials for the retrofitting of existing masonry constructions, [15]–[17], including the possibility to introduce some intrados FRP sheets for the seismic upgrading of bell towers, [18].

The numerical results of a simplified LA procedure are presented in this paper, remarking the collapse mechanism, the corresponding lateral collapse load and the retrofitting design. A subroutine procedure is implemented for different towers, varying the base width from 4 to 12 m and varying the height from 20 to 70 m. A dynamic amplification concept, which correlates the load multiplier of the tower self-weight and the corresponding ground acceleration, is introduced. The results highlight the seismic risk for a limited range of height and width combinations corresponding to a slenderness variation from 4 to 7. For some combinations of the geometrical properties it is quite difficult to reduce the seismic vulnerability with traditional methods; hence, other advanced techniques are recommended. The proposed procedure is quite simple and gives reliable results for a fast seismic retrofitting design of masonry towers.

## 2 FAILURE MECHANISMS

Masonry towers may exhibit different failure modes due to their peculiar properties, mostly related to the level of pre-compression, masonry quality and structural irregularities. Figure 1 illustrates four possible collapse mechanisms of masonry towers due to lateral loads: vertical shear crack (#1), rocking failure (#2), rocking failure with limited tensile strength (#3) and horizontal sliding (#4), [5], [11], [19], [20]. These mechanisms represent simple cases where the stiffness of the tower as a block is relatively high compared to small deformations. Hence, it is possible to make simplified considerations using limit analysis (LA), which results to be a fast and reliable tool, about the capacity and the vulnerability of such constructions, [6], [7], [21], [22]. Such simplified models are developed and recommended for a quantitative evaluation of the seismic risk in Italia Codes, [1], [2]. A FE upper bound limit analysis has been proposed [23], deliberately to evaluate the load multiplier of the most probable failure mechanism, without predefining the failure mechanisms. The first predefined failure mechanism is likely to occur when the wall thickness is relatively small and relatively large openings are present, [3], [6]. The third failure mechanism is very likely to occur if an existing crack pattern, as shown in the image, is prone to be active. Such circumstances are very common for leaning towers, which are very diffused in the Po River valley (Italy), [12], [24]. The failure mechanisms follow the crack pattern propagating through mortar interfaces. A horizontal sliding could occur if low quality deteriorated mortar

constituting the bed joints is combined with low levels of pre-compression [20], e.g. low density or short towers.

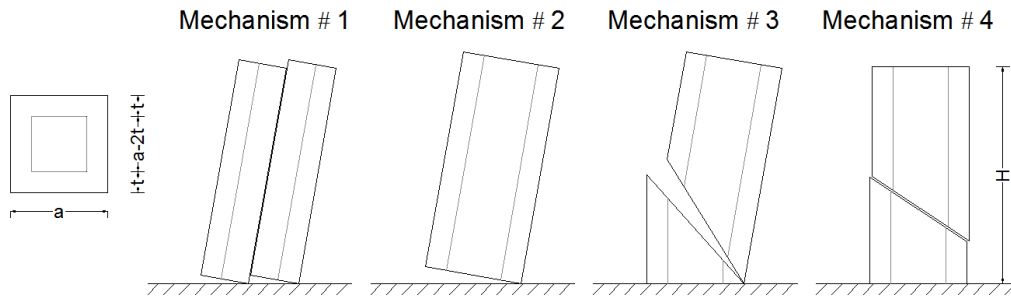


Figure 1. Geometrical properties and possible failure mechanisms of masonry towers.

### 3 RETROFITTING POSSIBILITY

Generally, masonry towers exhibit high vulnerability to lateral loads and consequently their seismic upgrading should be considered. As a matter of fact, their structural behavior is quite complex and their retrofitting is a difficult task, mainly when dealing with cultural heritage preservation. For instance, some proposed retrofitting techniques in the literature consist of: horizontal high-strength bars [25], pre-stressed vertical tie rods [14], viscous dampers at the base [26], intrados FRP sheets [18], masonry material properties upgrading [27]. The efficiency of any retrofitting technique is primarily related to the expected failure mechanism and the strictness of the implementation. Based on the above-mentioned failure mechanisms, some conceptual retrofitting possibilities are proposed in Figure 2.

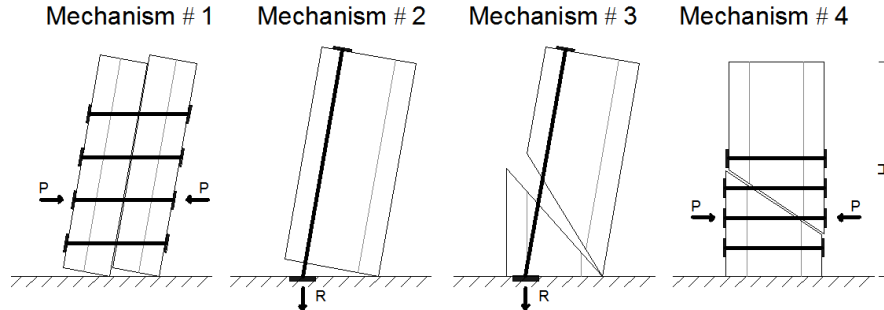


Figure 2. Conceptual retrofitting possibilities for different failure mechanisms of masonry towers.

The possibility of the herein proposed techniques consists of vertical elements, which provide higher capacity against overturning, and horizontal elements, which pre-compress the masonry in order to increase the shear capacity. Vertical elements may be of different typologies, i.e. pre-stressed or not pre-stressed, and different materials, i.e. steel, FRP bars/sheets. Such elements are introduced into the numerical model as applied external forces, limiting their value to the corresponding material strength or related to debonding phenomenon.

### 4 LIMIT ANALYSIS APPROACH

As previously mentioned, a LA approach is developed to investigate the performance of the masonry towers under lateral loads. Taking advantage of the feasibility of the method, a parametric study is performed. The procedure allows for the design the retrofitting devices as a function of the seismic intensity level.

#### 4.1 Load multiplier at collapse

Based on the specified geometry and the virtual works principle, the following collapse load multipliers are extracted for each failure mechanism:

$$\lambda_{\#1} = \frac{\gamma a (a - t) + a \tau_0 + \frac{\sum P}{2 t H} a \tan \varphi}{2 \gamma t (a - t) H} \quad (1)$$

$$\lambda_{\#2} = \frac{2 \gamma a t H + R}{2 \gamma t H^2} \quad (2)$$

$$\lambda_{\#3} = \frac{W_{(a,t,H,\gamma,\theta)} d_{(a,t,H,\gamma,\theta)} + R(a - t)}{W_{(a,t,H,\gamma,\varphi)} h_{(a,t,H,\gamma,\theta)}} \quad (3)$$

$$\lambda_{\#4} = \frac{4 t (a - t) [(\tau_0 + \gamma(H - h_v) \tan \varphi) \sec \varphi - \gamma h_v \sin \varphi] + \frac{\sum P}{2 t H} a \tan \varphi}{\gamma 4 t (a - t) h_v \cos \varphi} \quad (4)$$

where,  $W_{(a,t,H,\gamma,\theta)}$  is the weight of the tower section above the crack,  $d_{(a,t,H,\gamma,\theta)}$  is the distance from the mass center of the tower section above the crack to the pivot point,  $h_{(a,t,H,\gamma,\theta)}$  is the distance from the mass center of the tower section above the crack to the base of the tower,  $\theta$  is the angle between the cracked plane and the horizontal line,  $\varphi$  is the friction angle of the masonry material and  $h_v$  is the height of the tower block above the sliding plane. The last mechanism is considered without pre-stressing, just to prevent any relaxation of the steel due to temperature or creeping effect.

#### 4.2 Retrofitting design

The above equations of the load multiplier have a similar form given by equation (5):

$$\lambda_{\#i} = \frac{S_s + R}{I} \quad (5)$$

where  $S_s$  is the stabilizing contribution of the unreinforced tower,  $R$  of the retrofitting and  $I$  is the horizontal load contribution. The units of  $R$  could be [kN] if the failure mechanism is governed by shear or [kNm] if the failure mechanism is represented by overturning. If a proportional relationship between  $S_s$  and  $R$  is considered, then  $R = K S_s$ . Hence the retrofitting design, related to the lowest allowed load multiplier  $[\lambda]$ , can be performed through the following equation.

$$R = \frac{I [\lambda]}{S_s + 1} S_s \quad (6)$$

#### 4.3 Acceleration factor

The framework of the retrofitting device design for four different failure mechanisms in terms of load multiplier is here defined. For a rigid body approach, this load multiplier can be used for design purposes and corresponds to the design ground acceleration. Despite the rigid block assumption, the masonry towers are flexible and have a dynamic response due to ground shaking. Under a seismic load, one can assume that the tower is safe if the acceleration factor  $f_{acc}$  is greater than 1:

$$f_{a,SLV} = \frac{a_{SLV}}{a_{g,SLV}} \quad (7)$$

where  $a_{SLV}$  is the soil acceleration leading to the SLV ultimate state and  $a_{g,SLV}$  is the acceleration corresponding to the reference return period. The acceleration factor is a purely mechanical parameter, which may be useful for an evaluation of the weakness of the structure in terms of strength. We can assumed in the computations  $a_{g,SLV} = S_d(T_1)$  with a behavior factor  $q$  equal to 2.8, which is on the safe side because it should be 3.6 for regular masonry towers [1].

EC8 spectrum for zone 1 is used for the sake of simplicity.  $T_1$  is the fundamental vibration period, which can be evaluated analytically in case of regular towers by the following equation:

$$T_1 = \frac{H^2}{\alpha_1} \sqrt{\frac{\rho \cdot A}{E \cdot I}} \quad (8)$$

It is worthy to note that when the period of the structure increases, the spectral acceleration decreases and the corresponding peak ground acceleration of the collapse multiplier increases.

## 5 CASE STUDIES

A numerical investigation is performed for a wide range of heights and base widths of different towers by means of a subroutine implemented in MATLAB by the Authors. The geometrical properties and the failure mechanisms introduced into the code are those explained in the previous sections. Two different situations are considered: a) the un-retrofitted case where a retrofitting design is performed in a second step; b) the retrofitted case where the safety checks for the required seismic upgrading are performed.

### 5.1 Seismic vulnerability assessment

Figure 3 depicts different views of four failure mechanisms for masonry towers with different heights and base widths, represented by surfaces. It is notable that different surfaces intersect each other, highlighting that geometrical parameters control the most probable failure mode. The load multiplier is quite proportional with the base width increase and inversely proportional with the tower height increase. Among all the failure mechanisms, the relevant one for a tower is the one activating a failure mechanism with the lowest load multiplier. In this case, a failure surface composed by the lowest load multiplier for a given base and height is defined, Figure 4. For the sake of clarity, the slenderness and the first fundamental period of the idealized towers studies are reported respectively in Figure 5-a,b).

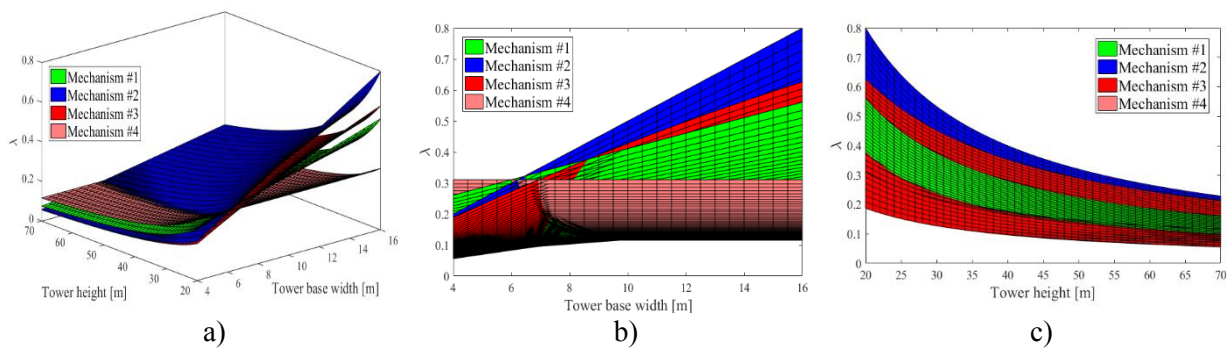


Figure 3. Variation of the load multipliers of the different failure mechanisms for different tower heights and base widths. a) 3D view; b) Variation of the load multipliers for different tower base widths; c) Variation of the load multipliers for different tower heights.

Using equation (7), it is possible to evaluate for each tower an  $a_g/g$  that makes the acceleration factor equal to 1. The acceleration factor is reported in Figure 5-c). It is worthy to note that the acceleration factor gives a straightforward view of the potential vulnerability of the tower, as a function of slenderness, base width and height, see Figure 6. All the cases exhibiting values lower than 1 are unsafe, therefore, they require further structural reinforcement to mitigate the potential risk.

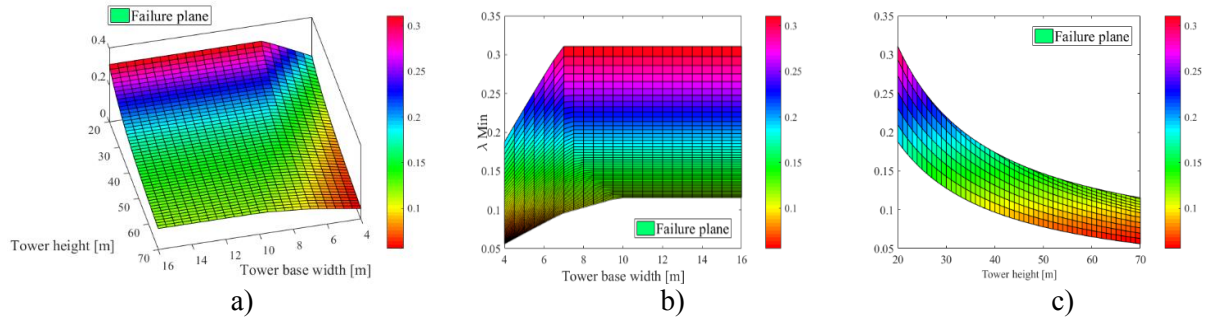


Figure 4. Minimum load multiplier causing the collapse of the towers. a) 3D view; b) Variation of the critical load multiplier for different tower base widths; c) Variation of the critical load multiplier for different tower heights.

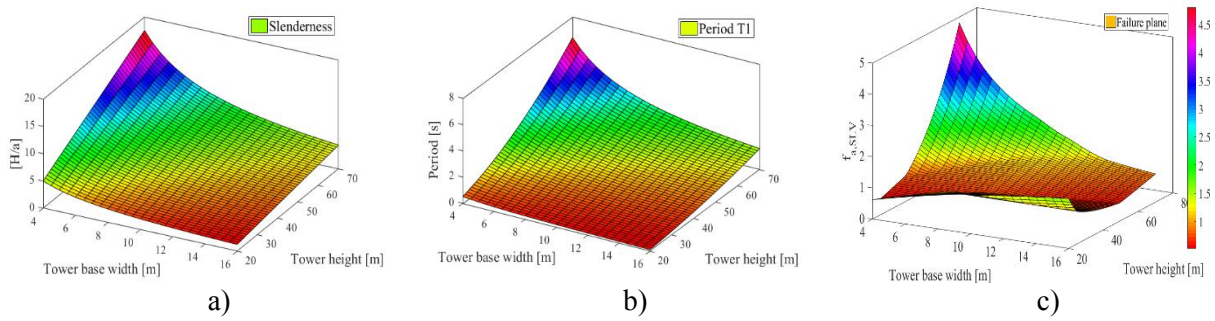


Figure 5. a) Values of slenderness investigated; b) Fundamental vibration period of the towers investigated; c) Corresponding acceleration factor.

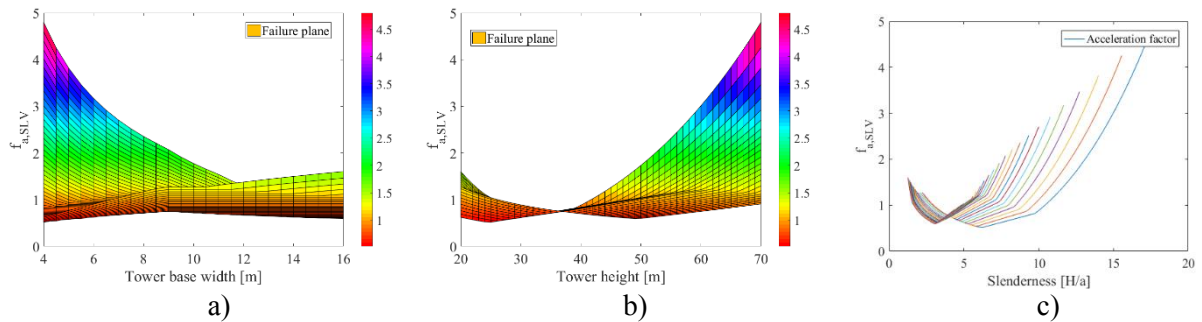


Figure 6. a) Variation of the acceleration factor for different tower widths; b) Variation of the acceleration factor for different tower heights; c) Variation of the acceleration factor for different tower slenderness values.

## 5.2 Results of the retrofitted case

It was highlighted that if the structure exhibits a capacity to withstand an acceleration smaller than that required by the seismic zone, a retrofitting design should be introduced. Figure 7 gives an insight into the geometrical combinations for which the towers require the introduction of retrofitting devices. The information is given by means of  $a_g/g$  increase needed (i.e.  $S_d(T_1) - \lambda$ ) to make the acceleration factor equal to 1 and by means of a non-dimensional retrofitting parameter  $\tilde{r}$ , where  $\tilde{r} = R/W$  if  $R$  is in [kN] (equation (6)) or  $\tilde{r} = R/(W H/2)$  otherwise.

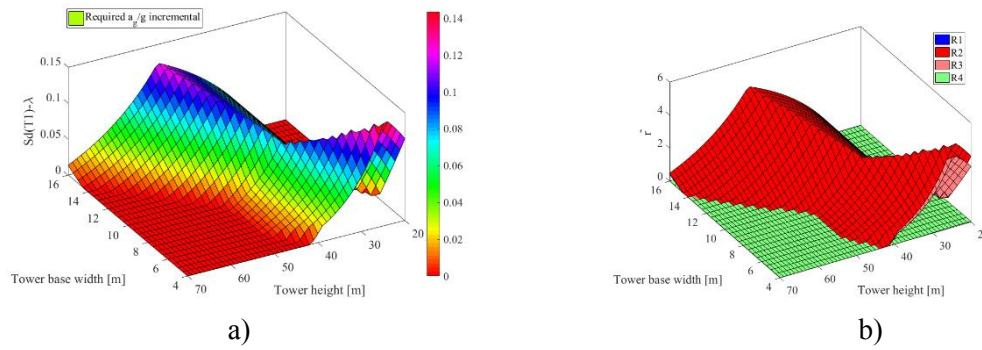


Figure 7. a)  $S_d(T_1) \cdot \lambda$ ; b) parameter  $\tilde{r}$  (amount of retrofitting needed normalized against tower weight), for different failure mechanisms of masonry towers.

The influence of horizontal and vertical bars (i.e. or any strengthening material) is investigated with regard to the vulnerability mitigation. Three different strengthening configurations in terms of vulnerability reduction to zero are reported in **Error! Reference source not found.** and **Error! Reference source not found.**. The unsafe zone is a combination of two quasi triangles, where the right one corresponds to shear failure and the left one corresponds to rocking failure. Increasing the horizontal bars/strips/hoops, the right triangle decreases; the same result is obtained for the left triangle when vertical elements are introduced.

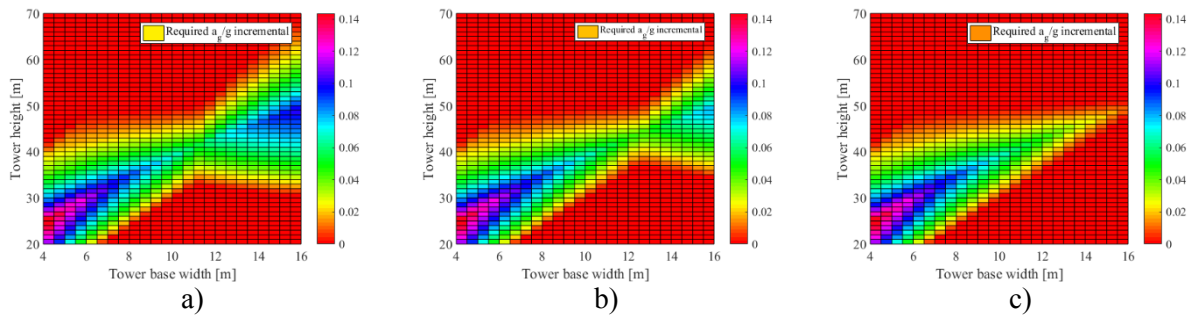


Figure 8. Retrofitting possibilities with horizontal bars. a) 1 m step; b) 0.5 m step; c) 0.1 m step.

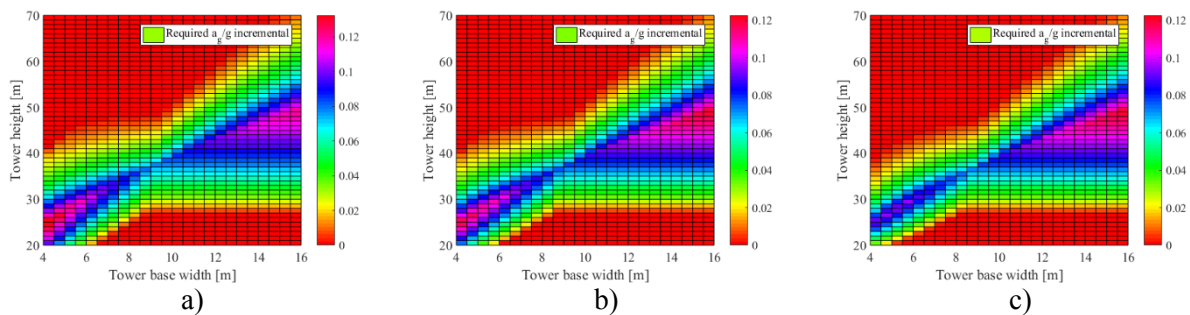


Figure 9. Retrofitting possibilities with vertical bars. a) 20 bars; b) 50 bars; c) 100 bars.

The optimal upgrading performance is achieved by a combination of horizontal and vertical elements, and results are reported in **Error! Reference source not found.**. Table 1 shows different configurations for the required optimal retrofitting. The best options are those with high strength, because they will occupy less space and will be more practical to be implemented in an effective way. As a matter of fact, the delamination phenomenon is present when retrofitting the masonry structures with composites strips, [28], thus the specified strength of the CFRP strips should be reduced as compared with their strength. The reference retrofitting material corresponds to a specimen steel bar with a strength in tension equal to  $260 \text{ MPa}$  and diameter equal to  $25 \text{ mm}$ .

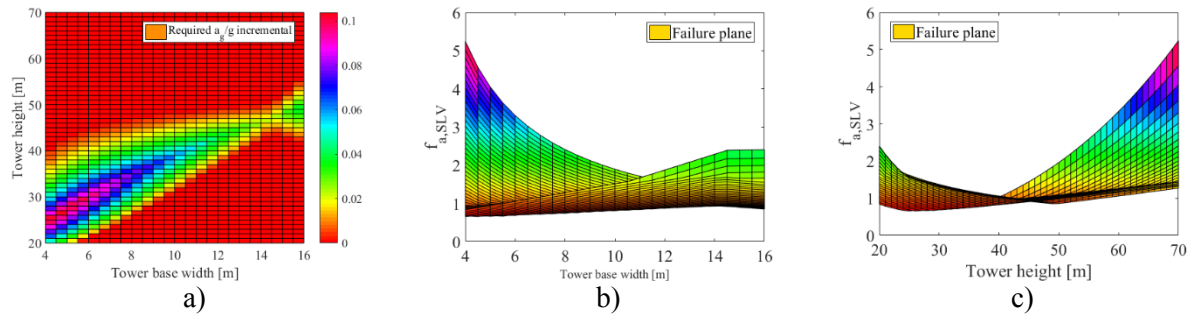


Figure 10. Optimal retrofitting with 80 vertical bars and 0.25 m step for horizontal bars.

Table 1. Different materials configurations for the optimal retrofitting.

Steel bars S 260 NC					Steel bars B450C			
	$f_y$ [MPa]	A [mm <sup>2</sup> ]	d [mm]	nr	$f_y$ [MPa]	A [mm <sup>2</sup> ]	d [mm]	nr
Vertical	260	37500	25	80	450	37500	25	46
Horizontal	260	1875	25	4	450	1875	25	3
CFRP strips					CFRP bars			
	$f_y$ [MPa]	A [mm <sup>2</sup> ]	b [mm]	t [mm]	$f_y$ [MPa]	Area [mm <sup>2</sup> ]	d [mm]	nr
Vertical	100	37500	4000	9.375	1000	37500	25	20
Horizontal	100	1875	1000	1.875	1000	1875	25	1

## 6 CONCLUSIONS

Masonry towers presenting significant slenderness are highly vulnerable under seismic actions. Recent seismic events and numerical investigations have demonstrated their low capacity to resist lateral loads. Their seismic performance upgrading is a very important task required by Technical Codes and Built Heritage Conservation Guidelines.

A simplified LA approach to investigate the vulnerability of masonry towers under lateral loads and further considerations about the retrofitting possibility are presented in this paper. Different geometrical properties are considered for the towers and their behavior is numerically investigated in detail considering four failure mechanisms. For each failure mechanism a retrofitting solution using traditional or innovative materials is then proposed. The procedure is found to be quite suitable to provide fast and reliable results.

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