THE INFLUENCE OF GEOMETRY, LOADS AND STEEL GRADE FOR THE DEVELOPMENT OF A SPECIFIC COLLAPSE TYPE OF MR-FRAMES

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Keywords: TPMC, Kinematic theorem of plastic collapse, Earthquake Engineering, seismic design, mechanisms of collapse

Abstract. The work is devoted to the analysis of the influence of geometry, loads and steel grade for the development of a specific collapse type of steel Moment Resisting Frames (MRFs). In order to provide the results, the Theory of Plastic Mechanism Control (TPMC) has been applied to a large number of steel frames with different geometrical configurations and different loads and steel grades. TPMC is based on the extension of kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve that allows accounting for second order effects that are very relevant especially in MRFs. This wide parametric analysis has been carried out on more than 100000 cases pointing out that for ordinary and regular MRFs type-1 mechanism is the one controlling the failure mode. In these last case, a simplified application of the Theory of Plastic Mechanism Control can be used.
1 INTRODUCTION

The fundamental principle of capacity design of seismic-resistant structures states that plastic hinge formation in columns during an earthquake should be avoided, in order to make sure that the seismic energy is dissipated in predefined zones only. In the case of Moment Resisting Frames (MRFs) this preselected zones are the beam ends whose rotational capacity is higher than columns one [1]. In facts, it is not desirable to develop plastic hinges in columns, because columns have to support upper storeys and because columns are hard to develop large inelastic deformations. However, in order to develop a complete mechanism of collapse, plastic hinges should form also at the base of first storey columns by determining a collapse mechanism of global type [2], [3], [4].

With reference to the most common structural typologies, design rules suggested by actual seismic codes [5], [6], [7], among which also the Italian seismic code [8], and also codified in Eurocode 8 [9] and ANSI/AISC 341-10 [10] are not able to assure the development of a collapse mechanism of global type. In fact, the hierarchy criteria provided by such codes constitute, dealing with column design, only a simplified application of the well-known “capacity design principles”. For this reason, with reference to MRFs, a rigorous design procedure based on the kinematic theorem of plastic collapse was presented in 1997 [11] aiming to assure a collapse mechanism of global type. Starting from this first work, the “Theory of Plastic Mechanism Control” (TPMC) has been progressively outlined as a powerful tool for the seismic design of steel structures. TPMC consists in the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve and on its application devoted to column design. In fact, for any given structural typology, the design conditions to be applied in order to prevent undesired collapse mechanisms can be derived by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to be located below those corresponding to all the other undesired mechanisms up to a top sway displacement level compatible with the local ductility supply of dissipative zones.

Starting from the first work devoted to MRFs, TPMC was successively extended to MRFs with semi-rigid connections [12], [13], EB-Frames [14], [15], [16], [17], [18], [19], [20], braced frames equipped with friction dampers [21], [22], [23] knee-braced frames [24], dissipative truss-moment frames DTMFs [25] and MRF-CBF dual systems [26], [27], [28], [29], [30], [31], MRF-EBF dual systems [32] and reinforced concrete frames [33] [34] [35].

Starting from the above background, in this paper the state of the art regarding the “Theory of Plastic Mechanism Control” is reported. In particular, by means of new considerations regarding collapse mechanism typologies, new advances have been developed leading to a closed form solution [36].

Moment Resisting Frames (MRFs) are the most common seismic-resistant structures. They are characterized by high dissipation capacity, because of the large number of dissipative zones under cyclic bending represented by the beam end sections. Nevertheless, such structural system could be not able to provide sufficient lateral stiffness, as required to fulfill serviceability limit states and also adequate robustness [37], [38], [39], [40], [41]. Influence is also given for isolated systems by soil category [42]. Finally, a wide parametric analysis to investigate the influence of geometry, loads and steel grade for the development of a specific collapse mechanism is reported.

2 STATE OF THE ART OF TPMC

The “Theory of Plastic Mechanism Control” has the aim to provide structures able to fail with a collapse mechanism of global type (Figure 1). Global mechanism represents the optimum in term of structural dissipation capacity being all the dissipative zones involved in the pattern
of yielding while the non dissipative ones remain in elastic range. Dissipative zones of MRFs are the beam ends and the bases of the column at the first storey.

In particular, TPMC allows the theoretical solution of the problem of designing structures failing in global mode, i.e. assuring that yielding develops only in the dissipative zones while all the columns remain in elastic range with the only exception of base sections at first storey columns. The structural members containing the dissipative zones are assumed to be known quantities, because they are preliminarily designed, according to the first principle of capacity design, to withstand the internal actions due to the load combinations given by code provisions. Therefore, according to the second principle of capacity design, the unknowns of the design problem are constituted by the non-dissipative members, i.e. the column sections, which are designed to assure the desired collapse mechanism, i.e. the global mechanism. Compared to hierarchy criteria given in code provisions, TPMC allows a rigorous application of the second principle of capacity design.

TPMC is based on the kinematic or upper bound theorem of plastic collapse within the framework of limit analysis. According to the theory of limit analysis, the assumption of a rigid-plastic behaviour of the structure until the complete development of a collapse mechanism is usually made. It means that the attention is focused on the condition the structure exhibits in the collapse state by neglecting the lateral displacements corresponding to each intermediate condition. However, the simple application of the kinematic theorem of plastic collapse is not sufficient to assure the desired collapse mechanism, because high horizontal displacements occur before the complete development of the kinematic mechanism. These displacements give rise to significant second order effects which cannot be neglected in the seismic design of structures, particularly in case of moment-resisting steel frames. Therefore, the basic principle of
TPMC is essentially constituted by the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. The mechanism equilibrium curve is represented by a straight line whose intercept with the vertical axis in a Cartesian diagram is the first order collapse mechanism multiplier \( \alpha_0 \), while its slope is represented by the parameter \( \gamma \). The expression of the linearized collapse mechanism equilibrium curve is given by:

\[
\alpha = \alpha_0 - \gamma \delta
\]  

(1)

where \( \alpha \) is the multiplier of horizontal forces and \( \delta \) is the top sway displacement of the structure. Within the framework of a kinematic approach, for any given collapse mechanism (Fig. 3), the mechanism equilibrium curve can be easily derived requiring that external work is equal to the internal work due to the plastic hinges involved in the collapse mechanism, provided that the external second-order work due to vertical loads is also evaluated [11].

In fact, with reference to the global mechanism, it is easy to recognize that, for a virtual rotation \( d\theta \) of the plastic hinges of the uprights involved in the mechanism, the internal work can be expressed, as:

\[
W_i = \left[ \sum_{i=1}^{n_s} M_{c,i1} + 2 \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} (M_{b,\text{Rd},jk}) \right] d\theta
\]  

(2)

where \( n_s \) is the number of storeys, \( n_b \) is the number of bays, \( M_{c,i1} \) is the first storey column base plastic moment of \( i \)-th column while \( M_{b,\text{Rd},jk} \) is the plastic moment of beams. The external work due to the horizontal forces and to the uniform load acting on the beams is:

\[
W_e = \alpha \sum_{k=1}^{n_s} F_k h_k d\theta + \delta \sum_{k=1}^{n_s} V_k h_k d\theta
\]  

(3)

where \( F_k \) and \( h_k \) are, respectively, the seismic force applied at \( k \)-th storey and the \( k \)-th storey height with respect to the foundation level, \( h_{n_s} \) is the value of \( h_k \) at the top storey, \( \delta \) is the top sway displacement and \( V_k \) is the total vertical load acting at \( k \)-th storey.

Figure 2: Second order vertical displacements.

The first term of Eq. (3) represents the external work due to seismic horizontal forces, while the second term is the second order work due to vertical loads. In order to compute the slope of the mechanism equilibrium curve, it is necessary to evaluate the second-order work due to vertical loads. With reference to Figure 2, it can be observed that the horizontal displacement of
the $k$-th storey involved in the generic mechanism is given by $u_k = r_k \sin \theta$, where $r_k$ is the distance of the $k$-th storey from the centre of rotation $C$ and $\theta$ the angle of rotation. The top sway displacement is given by $\delta = h_{ns} \sin \theta$.

The relationship between vertical and horizontal virtual displacements is given by $dv_k \tan \theta \approx du_k \sin \theta$. It shows that, as the ratio $dv_k / du_k$ is independent of the storey, vertical and horizontal virtual displacement vectors have the same shape. In fact, the virtual horizontal displacements are given by $du_k = r_k \cos \theta d\theta \approx r_k d\theta$, where $r_k$ defines the shape of the virtual horizontal displacement vector, while the virtual vertical displacements are given by $dv_k = \delta / h_{ns} d\theta$ and, therefore, they have the same shape $r_k$ of the horizontal ones. It can be concluded that:

$$dv_k = \frac{\delta}{h_{ns}} h_k d\theta$$  \hspace{1cm} (4)

where $dv_k$ is the virtual displacement occurring at $k$-th storey.

By equating the internal work (Eq. (2)) to the external one (Eq. (3)), the following relationship is obtained for the global mechanism:

$$\alpha = \frac{\sum_{i=1}^{n_c} M_{c,i1} + 2 \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_{b,Rd,jk} + \sum_{i=1}^{n_c} M_{c,ii_m}}{\sum_{k=1}^{n_s} F_k h_k} - \frac{1}{h_{ns}} \frac{\sum_{k=1}^{n_s} V_k h_k}{\sum_{k=1}^{n_s} F_k h_k} \delta$$  \hspace{1cm} (5)

that can be arranged in the following form:

$$\alpha^{(g)} = \alpha^{(g)}_0 - \gamma^{(g)}_{i_m} \delta \text{ with } t = 1, 2, 3 \quad i_m = 1, ..., n_s$$  \hspace{1cm} (6)

for the global mechanism and in the following form:

$$\alpha_{i_m} = \alpha^{(g)}_{0,i_m} - \gamma^{(g)}_{i_m} \delta \text{ with } t = 1, 2, 3 \quad i_m = 1, ..., n_s$$  \hspace{1cm} (7)

for each undesired mechanism where:

$$\alpha^{(1)}_{0,i_m} = \frac{\sum_{i=1}^{n_c} M_{c,i1} + 2 \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b,Rd,jk} + \sum_{i=1}^{n_c} M_{c,ii_m}}{\sum_{k=i_m}^{n_s} F_k h_k}$$  \hspace{1cm} (8)

for type-1 mechanism;

$$\alpha^{(2)}_{0,i_m} = \frac{\sum_{i=1}^{n_c} M_{c,ii_m} + 2 \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b,Rd,jk}}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})}$$  \hspace{1cm} (9)

for type-2 mechanism;

$$\alpha^{(3)}_1 = \frac{2 \sum_{i=1}^{n_c} M_{c,i1}}{h_1 \sum_{k=1}^{n_s} F_k}$$  \hspace{1cm} (10)

and for type-3 mechanism, where $F_k$ and $h_k$ are, respectively, the seismic force applied at $k$-th storey and the $k$-th storey height with respect to the foundation level; $M_{c,ii_m}$ is the plastic moment of $i$-th column of $k$-th storey reduced due to the contemporary action of the axial force; $n_c$, $n_b$ and $n_s$ are the number of columns, bays and storeys, respectively.

Regarding the slope $\gamma$ of the mechanism equilibrium curve, they are given by:

$$\gamma^{(1)}_{i_m} = \frac{1}{h_{i_m}} \frac{\sum_{k=1}^{n_s} V_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} V_k}{\sum_{k=1}^{n_s} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k}$$  \hspace{1cm} (11)

for type-1 mechanism;

$$\gamma^{(2)}_{i_m} = \frac{1}{h_{ns} - h_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} V_k (h_k - h_{i_m-1})}{\sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1})}$$  \hspace{1cm} (12)

for type-2 mechanism;
where $V_k$ is the total vertical load acting at $k$-th storey. The collapse mechanism equilibrium curves can be easily represented in a Cartesian diagram ($\alpha$-$\delta$). The curve that is located below all the other ones is the one governing the collapse mechanism. For this reason, in order to assure that the collapse mechanism of global type develops, the collapse mechanism equilibrium curve of global mechanism (Eq. (6)) has to be located below those corresponding to each undesired mechanism (Eq. (7)) until a design displacement $\delta_u$, compatible with the ductility supply of dissipative zones (Figure 3) and usually assumed equal to 0.04.

\[
\gamma_{i_m}^{(3)} = \frac{1}{h_{i_m} - h_{i_m-1}} \sum_{k=i_m}^{n_s} V_k
\]

(13)

This sentence can be mathematically translated in the following inequality:

\[
\alpha_0^{(g)} - \gamma_{i_m}^{(g)} \delta_u \leq \alpha_0^{(t)} - \gamma_{i_m}^{(t)} \delta_u \quad t = 1, 2, 3 \quad i_m = 1, \ldots, n_s
\]

(14)

from which, by substituting in Eq. (14) the corresponding quantities provided by Eqs. (8) to (13) it is possible to provide the conditions to avoid partial mechanism at each storey and, as a consequence, the sum of reduced plastic moment at each storey needed to design column sections. In particular, this quantity for the first storey column sections is given by one only condition:

\[
\sum_{i=1}^{n_c} M_{c,i1}^{(1)} \geq \frac{2 \sum_{k=1}^{n_s} \sum_{j=1}^{n_b} M_{b,Rd,jk} + (4^{(3)} - \gamma^{(g)}) \delta_u \sum_{k=1}^{n_s} F_k h_k}{2 \frac{\sum_{k=1}^{n_s} F_k h_k}{h_1 \sum_{k=1}^{n_s} F_k} - 1}
\]

(15)

while for the other storey is the maximum among the following three conditions:

\[
\sum_{i=1}^{n_c} M_{c,iim}^{(1)} = (\alpha^{(g)} + \gamma_{i_m}^{(1)} \delta_u) \left( \sum_{k=1}^{i_m} F_k h_k + \sum_{k=i_m+1}^{n_s} F_k \right) - \sum_{i=1}^{n_c} M_{c,i1}^{(1)} - 2 \sum_{k=1}^{i_{m-1}} \sum_{j=1}^{n_b} M_{b,Rd,jk}
\]

(16)

needed to avoid type-1 mechanism,

\[
\sum_{i=1}^{n_c} M_{c,iim}^{(2)} = (\alpha^{(g)} + \gamma_{i_m}^{(2)} \delta_u) \sum_{k=i_m}^{n_s} F_k (h_k - h_{i_m-1}) - 2 \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b,Rd,jk}
\]

(17)

needed to avoid type-2 mechanism,

\[
\sum_{i=1}^{n_c} M_{c,iim}^{(3)} = (\alpha^{(g)} + \gamma_{i_m}^{(3)} \delta_u) \left( \frac{h_{i_m} - h_{i_m-1}}{2} \right) \sum_{k=i_m}^{n_s} F_k
\]

(18)
needed to avoid type-3 mechanisms, where $\sum_{i=1}^{n_c} M_{c,i1}^*$ sum of plastic moments of columns at first storey reduced for the contemporary action of axial load after selecting the column sections from the standard shapes.

As preliminarily stated, the maxim among the quantities given by Eqs. (16), (17) and (18) is given by:

$$\sum_{i=1}^{n_c} M_{c,i1} = \max \left\{ \sum_{i=1}^{n_c} M_{c,i1}^{(1)}, \sum_{i=1}^{n_c} M_{c,i1}^{(2)}, \sum_{i=1}^{n_c} M_{c,i1}^{(3)} \right\}$$

(19)

The design algorithm that has to be followed to achieve the design purpose, i.e. the design of MRFs collapsing with a global failure mode is reported in Figure 4.

**Figure 4: Design algorithm**
3 PARAMETRIC ANALYSIS

As observed in numerical examples reported in previous works [36], [33] type-1 mechanism usually governs the design of structures, i.e. the term $\sum_{i=1}^{n_c} M_{c,i,m}^{(1)}$ is greater than $\sum_{i=1}^{n_c} M_{c,i,m}^{(2)}$ and $\sum_{i=1}^{n_c} M_{c,i,m}^{(3)}$. Now it is important to point out what are the parameters deciding the influence of type-1 mechanism and if it is possible to preliminarily decide, on the basis of geometrical characteristics, loads and steel grade which one of the three undesired mechanism governs the design of the structure.

In order to answer these questions a parametric analysis has been led by varying geometry, loads and steel grade on MR-Frames with the scope to observe whether there a type mechanism which mostly governs the collapse of structure.

As regards the parameters involved, the geometry has been varied in term of number of storeys, number of bays, storey height and bay length. In particular the number of storey adopted goes from 3 to 8 while the number of bays goes from 2 to 6. Storey height have been assumed equal to 3 m, 4 m and 5 m while bay length has been considered equal to 3 m, 3.5 m, 4 m, 4.5 m, 5 m, 5.5 m, 6 m, 6.5 m and 7 m. Permanent loads, $g_k$, has been adopted equal to 4 kN/m2 while live loads $q_k$ has been assumed as equal to 2 and 3 kN/m2. Loads have been combined following this relation: $g_k+0.3q_k$. In addition, the influence light of the loads acting on the beams goes from 1 m to 6 m. Beams have been designed on the basis of the acting loads with a minimum and maximum bending moment of $q_d l^2/16$ and their shapes have been selected from the IPE standard profiles. The adopted steel grades are S275, S355, S450. Seismic forces distribution has been assumed proportional to the storey height by neglecting their magnitude because it is irrelevant for the design result of the procedure.

The whole of the analysed number of cases is equal to 176960 but 12690 of these ones have beams incompatible with the assigned loads, so that the real number of analysed cases is equal to 162270.

After the application of TPMC to the analysed cases the following results have been observed: the number of cases governed by a mechanism typology different by the type-1 without the occurrence of the technological condition are 197 while the number of cases that are governed by a mechanism typology different by the type-1 after the occurrence of the technological condition are 22260. It means that, as preliminarily stated, at the occurrence of the technological condition the sum of required column plastic moments for $i_m > 1$ is affected by the increase of the first right hand side term of Eq. (16) and the increase of the subtracting second term at right hand side of Eq. (16) and for this reason type-2 and type-3 mechanism become determinant for the design of the structure.

However, by stopping the attention on the procedure without the occurrence of technological condition is important to observe that only the 0.12% of the analysed cases is governed by a collapse typology different from type-1 and for this reason, it is possible to confirm that type-1 mechanism governs, in the most of the cases, the design of MR-Frames.

Many advantages belong to this observation: first of all the chance to use a simplified version of TPMC to design MR-Frames. This simplified version starts from the preliminary assumption that column sections are pin-jointed at their bases. As a consequence, Eq.(15) becomes:

$$
\sum_{i=1}^{n_c} M_{c,i,1} \geq \frac{2 \sum_{k=1}^{n_s} \sum_{j=1}^{n_p} M_{b,Rd,j,k} + \left(\gamma_1^{(3)} - \gamma(g)\right) \delta_u \sum_{k=1}^{n_s} F_k h_k}{\sum_{k=1}^{n_s} F_k h_k h_1 \sum_{k=1}^{n_s} F_k} - 1
$$

and Eqs. (16), (17), and (18), become:
\[ \sum_{i=1}^{n_c} M_{c,ii}^{(1)} \geq (\alpha^{(g)} + \gamma_{im}^{(1)} \delta_u) \left( \sum_{k=1}^{i_m} F_k h_k + h_{im} \sum_{k=i_m+1}^{n_s} F_k \right) - 2 \sum_{k=1}^{i_m-1} \sum_{j=1}^{n_b} M_{b,Rd,jk} \] (21)

needed to avoid type-1 mechanisms;

\[ \sum_{i=1}^{n_c} M_{c,ii}^{(2)} \geq (\alpha^{(g)} + \gamma_{im}^{(2)} \delta_u) \sum_{k=i_m}^{n_s} F_k (h_k - h_{im-1}) - 2 \sum_{k=i_m}^{n_s} \sum_{j=1}^{n_b} M_{b,Rd,jk} \] (22)

needed to avoid type-2 mechanisms;

\[ \sum_{i=1}^{n_c} M_{c,ii}^{(3)} \geq (\alpha^{(g)} + \gamma_{im}^{(3)} \delta_u) \frac{(h_{im} - h_{im-1})}{2} \sum_{k=i_m}^{n_s} F_k \] (23)

needed to avoid type-3 mechanisms; where \( \alpha^{(g)} \) does not depend on the sum of the reduced plastic moments of columns at the first storey \( \sum_{i=1}^{n_c} M_{c,i1} \) and it is provided by the following relation:

\[ \alpha^{(g)} = \frac{\sum_{k=1}^{n_s} \sum_{j=1}^{n_b} 2B_{jk} R_{b,jk}}{\sum_{k=1}^{n_s} F_k h_k} \] (24)

being Eqs. (16), (17), and (18) independent of the sum of the reduced plastic moments of columns at the first storey, \( \sum_{i=1}^{n_c} M_{c,i1} \), the column sections at first storey are provided starting by Eq. (20) while the column sections at each storey required to avoid undesired mechanism are directly provided starting by Eq. (21), (22) and (23), without the preliminary design of column sections at first storey. In this way, each storey is independent of the other storey and in particular, of the first one, so that, even if the technological condition occurs the column sections at each storey do not need to be revised. In addition, the solution provided in the framework of the hypothesis of pin-jointed column bases (simplified TPMC) constitutes a safe side solution with reference to the original TPMC [36], only in the case of type-1 mechanism governing the design procedure.

4 CONCLUSIONS

- In this paper the state of the art of Theory of Plastic Mechanism Control (TPMC) has been presented with reference to MR-Frames.
- The analysis of the influence of geometry, loads and steel grade for the development of a specific collapse type of steel MR-Frames is reported.
- The kinematic theorem of plastic collapse extended to the concept of mechanism equilibrium curve has been applied to a large number of steel frames with different geometrical configurations and different loads and steel grades.
- The 88% of the analysed cases is governed by type-1 mechanism.
- In this last case a simplified version of TPMC could be applied.

REFERENCES

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