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RELATION OF FREQUENCY BANDWIDTH TO CRACK LENGTH IN STRESSED ROCKS

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Abstract: *An equation for the relationship between the crack length and frequency bandwidth of Acoustic Emission (AE) in stressed rocks has been obtained by a mathematical method. The result of the present study shows an inverse relationship between the crack length and the bandwidth of the acoustic emissions. This contradicts the classical equation used in the literature which shows that the crack length is inversely related to the main frequency of the AE waves. The research proposes that the main frequency of the AE waves would depend on the cracking mechanism and so upon the rock properties, stress and the environmental factors. By the derived equation and appropriate experiments, the limits of the crack length can be found. The study shows that there is valuable information in the frequency characteristics of the AE signals which can be exploited.*

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1 INTRODUCTION

Fault movement during earthquake causes seismic waves which have been studied from different aspects in literature. It seems that some researchers, by theoretical models and analytical methods, have concluded that these waves are among (or close to) low-pass signals (have a maximum value at zero frequency [1]), and so they concluded that the cutoff frequency of these signals have an inverse relation with the period of the signal and therefore with the fault length [1]. Later this conclusion was generalized to the acoustic emission (AE) due to micro cracks to conclude that the crack length has an inverse relation with the dominant frequency of the AE signal ([2], [3], [4], [5]). Despite that, no field or laboratory observation has demonstrated a low pass nature for the AEs, and instead a band pass signal has been observed, but the inverse relation between the cutoff frequency and the crack length has been used extensively by researchers.

In this paper, a mathematical method is used to analyze AEs in rock, and the relation between the crack length and the bandwidth frequency of AE signals is derived. The result shows that in contrast to what had been claimed in the conventional equation presented before (e.g. in [2], [6]), in which the length of the crack determines the main frequency of the emissions, the crack length widens the frequency spectrum of the AEs, and so it affects the bandwidth of the signal not the central frequency.

2 FREQUENCY ANALYSIS

The analysis given here can be applied to any continuous signal with a limited period. Therefore, this method can be utilized to analyze a few different types of cracks, including stable or confined cracks defined in [2], which are formed by loading stress on rocks. Also, it should be noted that real signals in nature are limited in period, so they have an unlimited frequency spectrum. Therefore, in the analysis given below, which mainly uses the frequency bandwidth of the signals, one of the common definitions of signal bandwidth such as half power bandwidth, can be selected as the effective Bandwidth of the signal, and without loss of generality the rest of the spectrum could be considered zero.

The previous researches show that the frequency spectrum of the AE signals starts from a few hundred hertz, and extends up to a few megahertz.

The mechanism of the fracture in rock materials can be related to the physical and mechanical properties of rock, stress, and environmental factors. For the purpose of frequency analysis, we assume that the mechanism of cracking creates a wave x(t), with an arbitrary spectrum X(f) and a central frequency f_m, as shown below:

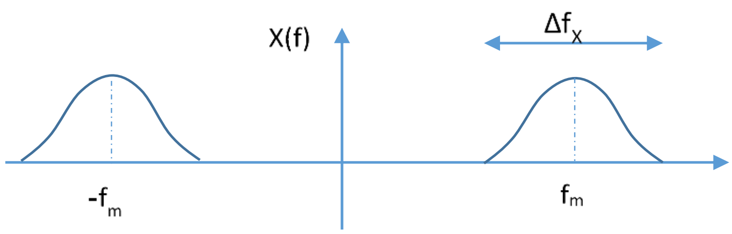

Figure 1: Frequency spectrum of a wave with arbitrary shape generated by the cracking mechanism

Note that x(t) is independent of the crack length because the crack initiates from a point and grows afterwards, while under appropriate conditions it can grow indefinitely. The crack stops sometime after it is initiated, and signal x(t) becomes limited in time by T. Therefore, it can be said that the source signal s(t) (received by transducer) is equal to signal x(t) multiplied by a square wave a(t) with period T (Figure 2), i.e.:

$$s(t) = x(t)a(t) \quad 1$$

When the edge of the crack does not oscillate, the period of the signal is proportional to the crack length:

$$T = \frac{L}{v_c} \quad 2$$

where v_c is the velocity of the crack propagation, and L is the crack length.

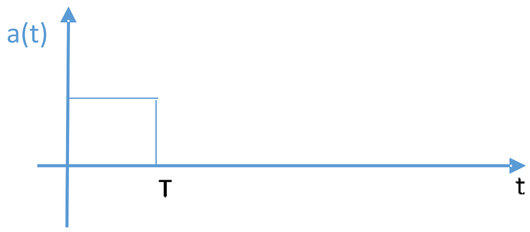

Figure 2: a single square wave corresponding to a crack with length $L = v_c T$.

In 1, it is implied that the period of x(t) is not less than that of s(t); in other words, it is assumed that if the stress is maintained, the crack can extend and elongate, which is a logical assumption. As said before, the AE waves due to micro-cracking are band-pass signals, therefore x(t) should also be considered a band pass signal.

The Fourier transform of the square function shown in Figure 2 is a Sinc function whose bandwidth Δf_A is inversely proportional to period T, and so to the crack length:

$$|A(f)| = |T sinc(fT)| \quad (3)$$

$$\Delta f_A = \frac{2}{T} = \frac{2v_c}{L} \quad (4)$$

From (2), the Fourier transform of an AE signal due to a crack with length L is equal to the convolution of the Fourier transforms of two functions x(t) and a(t):

$$S(f) = X(f) * A(f) \quad (5)$$

Where X(f) and A(f) are the Fourier Transforms of x(t) and a(t) respectively. The bandwidth of s(t) is then equal to the sum of the bandwidths of these two signals:

$$\Delta f_s = \Delta f_X + \Delta f_A \quad (6)$$

So, the effect of the crack length on the propagated AE is a widening of its frequency spectrum by Δf_A, but the central frequency of the spectrum would remain the same as that of A(f). Figure 3 shows the Fourier transform of a typical signal s(t) based on an arbitrary form of x(t):

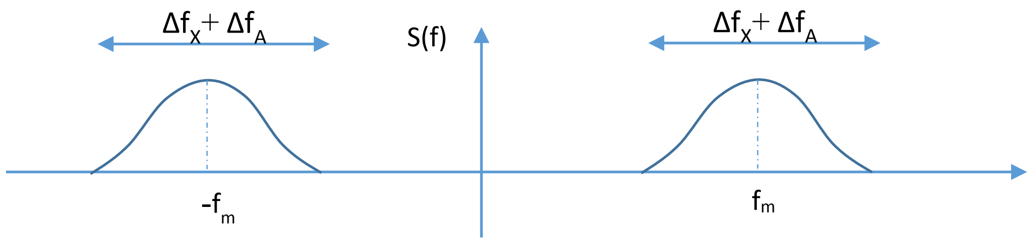

Figure 3: Frequency spectrum of the signal source s(t) generated by the cracking mechanisms with the spectrum shown in Figure 1.

Therefore, it seems that unlike what has been stated previously, that the main frequency of AE signals have an inverse relation with the crack length, the effect of the crack length is on the bandwidth of the AE signal and not on its central frequency.

Consequently, for the long cracks with low frequency crack mechanism, we would expect a low central frequency, and a small bandwidth for the related AE wave. For the long cracks with high frequency cracking mechanism, a high central frequency, and small BW are expected for the AE signals. Similar arguments can be made for small cracks with low and high frequency cracking mechanisms. Thus, by low frequency cracking mechanism, small cracks would generate low frequency and wide band AE signals; while small cracks with high frequency cracking mechanism, would generate high frequency, and wide band AE waves.

Example: As an example, assume that the cracking mechanism creates a sinusoidal waveform with frequency f_m, and that the cracking stops after T seconds, and forms a crack with length L. In this case, s(t) would be a wave form as shown below:

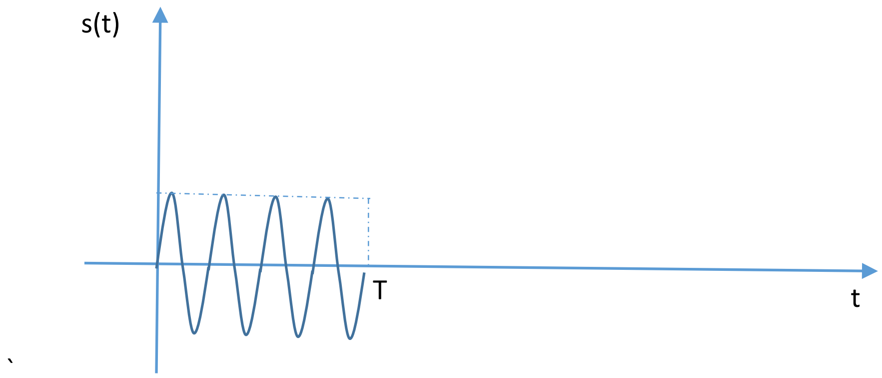

Figure 4: a typical signal s(t) with a sinusoidal cracking mechanism

The spectrum of s(t) would then be the result of convolving the Sinc function with an impulse at $f = f_m$, as shown below:

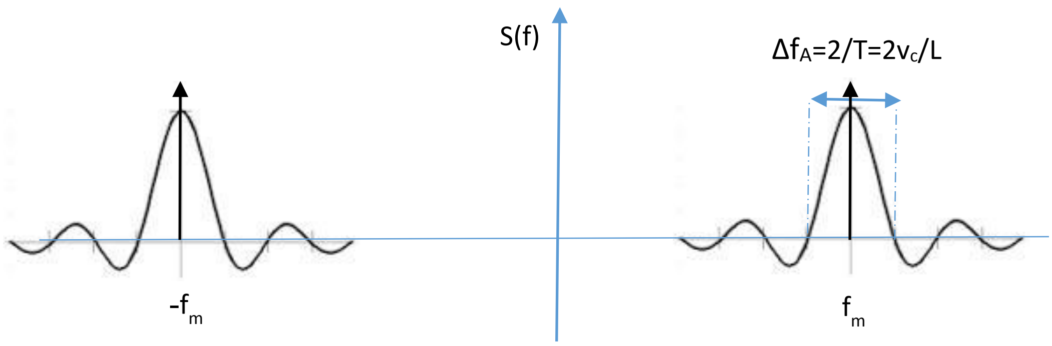

Figure 5: frequency spectrum of a source signal with sinusoidal cracking mechanisms and the crack length *L*

3 DISCUSSION

In a hypothetical ideal case of a homogenous and isotropic rock, constant stress, consistent environmental factors and infinite volume of the material, we can expect that cracking mechanism forms a periodic wave with similar cycles, in which all cycles have equal amplitude and duration. In this case, a limited portion of the corresponding AE signal (e.g .one period of the resulted periodic signal) would represent the whole AE wave due to the cracking mechanism.

If the cracking mechanism generates a periodic AE waveform with similar cycles (which can be perceived in an ideal case), it can be said that forming the crack has occurred in smaller and similar continuous steps. The concept of existence of the smaller steps in the crack formation is in agreement with the concept of the critical length (L_{cr}) defined in [2]. He defined the critical crack length as the minimum length for a Griffith crack to be able to radiate energy. In the abovementioned ideal case, wherein the AE wave is periodic, it might be said that when the crack initiates, first a mini-crack with length L_{cr} is formed, and then the crack extends by another critical length L_{cr}, and this process goes on while the crack extends.

4 CONCLUSIONS

In this study, the frequency characteristics of the AE signals were investigated. First, the concept of the basic crack and the basic length and period were introduced. The concept of the basic length is consistent with the critical crack length which was presented by Armstrong [2]. Then by a mathematical method, the relation between the bandwidth of the AE signals with the crack length and the bandwidth of the waves generated by the cracking mechanisms was derived. Frequency analysis shows that the crack length on the frequency characteristics of the AE signals affects the bandwidth of the signal, not its main frequency. As the crack length increases, the bandwidth of the signal decreases. The research proposes that the main frequency of the AE signal depends on the cracking mechanisms, which in turn is determined by environmental factors, mechanical and physical properties of rocks, and stress.

By the equations derived in the present study, and with the appropriate experiments which give us the frequency limits of the AE signals, the limits of the cracks' lengths can be obtained. The study shows that valuable information resides in the frequency characteristics of the AE signals which must be utilized and exploited.

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