

ON THE USE OF GENETIC ALGORITHMS TO ASSESS THE SEISMIC RESISTANCE OF PLANAR FRAME STRUCTURES

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Abstract. *Limit analysis represents a widely adopted strategy for the assessment of the bearing capacity and collapse mechanism of frame structures. Within this framework, in this paper an original strategy for the effective evaluation of the ultimate resistance and the corresponding failure mechanism of planar frame structures subjected to lateral loads is presented. The methodology is based on the generation of the elementary collapse mechanisms to be combined following a collapse load factor minimization criterion. When a large number of possible combinations has to be investigated a prompt procedure able to quickly converge to the actual collapse load factor and to jump out of local minima is needed. For this reason a procedure based on genetic algorithms is here adopted and a dedicated user-friendly software was developed in the NetLogo programming environment. Validations of the proposed procedure with respect to nonlinear static analysis are reported together with some significant sensitivity analyses with respect to load distribution parameters. The results demonstrate the reliability of the procedure and can provide useful information also in view of seismic design optimization strategies.*

1 INTRODUCTION

The plastic analysis of frame structures has been studied in the relevant literature mainly by means of two different approaches. The first of these is the structural analysis based on the finite element method in which, by means of iterative algorithms, the nonlinear static analysis is conducted on structures till the structural collapse [1]. Since the structural response is computed at each step of the loading history, the analysis can be very time-consuming.

On the other hand, limit analysis provides information on the carrying capacity and collapse mechanism of structures. It is a reliable strategy, faster than nonlinear static and dynamic analyses, thus making this method widely employed by practitioners. The conventional strategy is based on the fundamental kinematic theorem which analyses all the possible collapse mechanisms of a structure and the related collapse loads. The theorem states that the absolute lowest value among the considered mechanisms corresponds to the actual failure mode and the associated load to the actual collapse load. This method therefore does not require the direct computation of stiffness matrix and it is not necessary to apply the complete history of loading. Within the framework of limit analysis many proposals have been presented, mainly by means of linear programming [2]-[4].

Among those approaches a clever and frequently used method is that firstly developed by Neal and Symonds [5],[6] which is based on the analysis of a small number of independent collapse mechanisms (the so called elementary mechanisms), and on the subsequent search of the combination of those mechanisms which guarantees the lowest load factor. The mechanism associated to the lowest load factor represents the actual failure mechanism of the structure. However, this method requires a tedious work of combining the elementary mechanisms aiming at finding the combination associated to the lowest load multiplier. The latter aspect induced some authors to automate both steps of generating the elementary mechanisms and combining them; in particular, heuristic algorithms based on natural computation [7],[8] and genetic algorithms have been often employed [9] for a fast computerization of this methodology. Nevertheless in the existing literature only the case in which all the loads increase proportionally is taken into account.

In this paper the method proposed in [5],[6] is extended to account for a seismic load condition in which there is the contemporary presence of permanent vertical loads on the beams and incremental horizontal forces at each floor. In the described seismic load scenario the plastic hinges may occur at the two ends of the columns and also in any section along the beam. In the paper the exact location of the plastic hinges along the beam is calculated by means of the strategy proposed in [10].

The proposed methodology is able to provide fast and reliable information on the resistance and collapse mechanisms of frame structures subjected to seismic load.

A second novelty of the paper is the development of an original software code in the agent-based programming language NetLogo [11]. The developed code allows, without performing any analytical resolution of equations or matrices, to build into a virtual metrical space and to visualize in the user interface, every single mechanism and the correspondent collapse load by means of a user-friendly approach.

Then, the elementary mechanisms are combined and the minimum collapse load is obtained by means of an optimization procedure based on genetic algorithms which guarantees a fast convergence.

Several applications have been performed both with reference to the classical plastic analysis approach (all the loads increase proportionally), and with a seismic point of view considering a system of horizontal forces whose magnitude increases while the vertical loads are assumed to be constant. The values of the collapse load and the plastic hinge distribution at

collapse, obtained by means of the proposed method for seismic applications, have been compared to the correspondent results provided by nonlinear push over analysis showing a very good correspondence. In addition, a parametric study to assess the influence of the magnitude of the permanent loads acting on beams is showed.

2 COMBINATION OF ELEMENTARY COLLAPSE MECHANISMS

A generic planar regular frame, characterized by the number of floors N_f and the number of columns N_c , is here considered. The frame is clamped at its base, therefore the degree of hyperstaticity of the structure, denoted by h , is given by

$$h = 3 \left[N_c - 1 + (N_f - 1)(N_c - 1) \right] \quad (1)$$

The plastic moments are assumed to be $M_{b,ij}$ for beams and $M_{c,ij}$ for columns, being i and j the generic floor and column of the frame, respectively. The loads acting on the frame are, at each floor, $F_{h,i}$ the concentrated horizontal forces and either by vertical incremental forces $F_{v,ij}$, applied in the mid span, or by permanent vertical distributed loads q_{ij} . Plastic hinges in can be located in s “critical sections” correspondent to each joint and to a certain section of each beam which can coincide with its middle, in case of concentrated forces, or it can vary along the span in case of distributed loading. Floor heights and bays are indicated with L_j and H_i , respectively. The layout of a generic frame is showed in Figure 1.

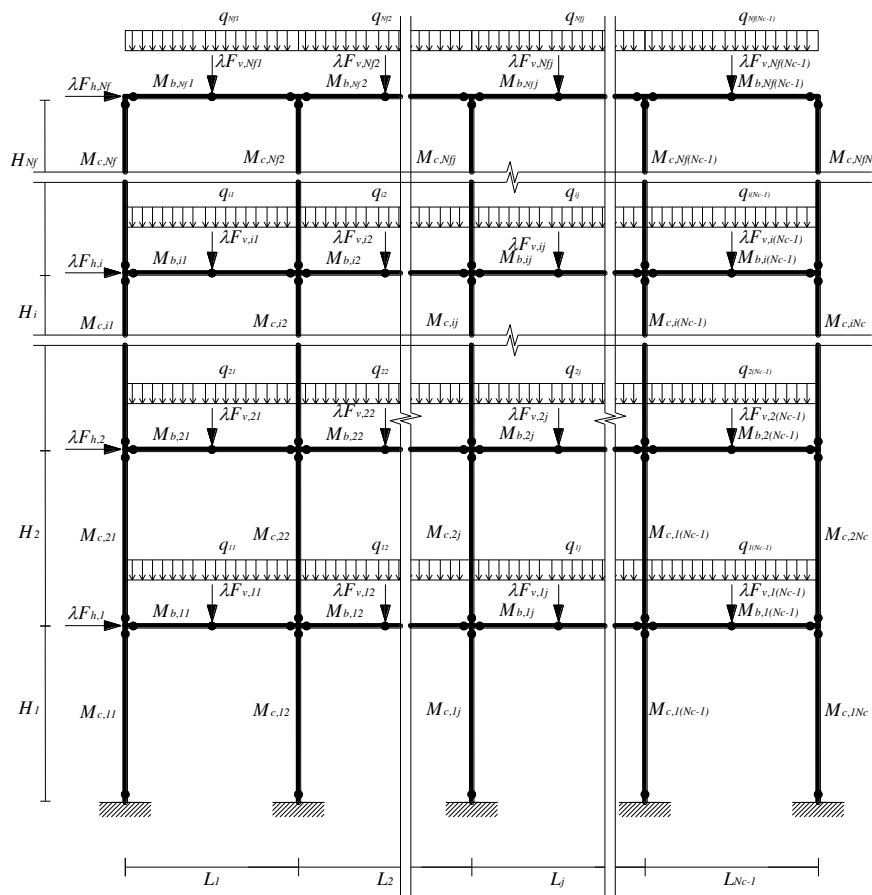


Figure 1: Layout of a generic planar frame.

As introduced by Neal and Symonds [5],[6] a small number $e=s-h$ of independent elementary collapse mechanisms can be taken into account and these may be classified in three different types: floor, beam and node mechanisms. In Figure 2 the three independent elementary collapse mechanisms for a generic frame are reported for clarity.

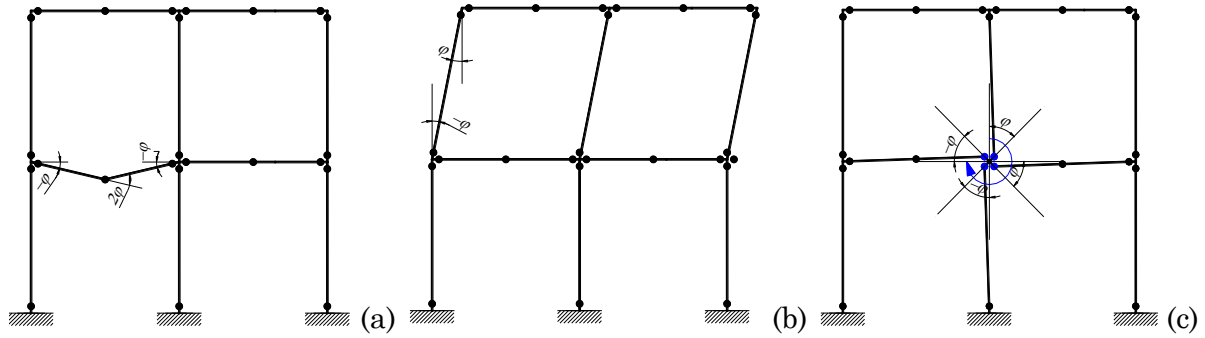


Figure 2: Elementary collapse mechanisms: (a) beam, (b) floor and (c) node mechanisms.

In the following two subsections the classical approach of the method of combination of elementary mechanisms applied to proportional loads is briefly recalled first, then the extension to account permanent loads is presented.

2.1 Proportional loads

When all the applied loads increase proportionally for each collapse mechanism the external work W_{ext} is associated to all the vertical and horizontal concentrated forces $F_{h,i}$ and $F_{v,ij}$ for the correspondent displacements $d_{h,i}$ and $d_{v,ij}$, while the internal work (plastic dissipation) is referred to the plastic hinges and is the product of the plastic moments of beams and columns for the related rotations.

The elementary mechanisms are successively combined by adding the rotations related to each critical section and then multiplying each total rotation for the plastic moment of the related element, while the external work is the sum of the elementary ones. Therefore the collapse load multiplier λ_c is the minimum ratio

$$\lambda_c = \frac{W_{int}}{W_{ext}} \quad (2)$$

2.2 Seismic load

In seismic conditions the vertical loads are assumed to be distributed and of constant value therefore for each collapse mechanism the collapse load multiplier λ_c can be calculated as:

$$\lambda_c = \frac{W_{int} W_{extV}}{W_{extH}} \quad (3)$$

where W_{extH} and W_{extV} represent respectively the work associated to the amplified horizontal forces and to permanent vertical load. In this case either the external or the internal work for each beam mechanism must be calculated considering that, as shown in Figure 3, a plastic hinge can be opened at a position x_{ij} from the left end of the beam dependent on the magnitude of the uniform load acting on the beam. The superimposition of the uniformly distributed vertical loads and of the horizontal forces implies that, by increasing the horizontal forces the first plastic hinge opens at the right end of the beam, while the second hinge occurs at the left end when the distributed load is equal to the limit value [10]:

$$q_{lim,ij} = \frac{4M_{b,ij}}{L_j^2} \quad (4)$$

When the vertical load q_{ij} exceeds such a limit value the abscissa of the potential along-span plastic hinge is given by

$$x_{ij} = L_j - 2\sqrt{\frac{M_{b,ij}}{q_{ij}}} \quad (5)$$

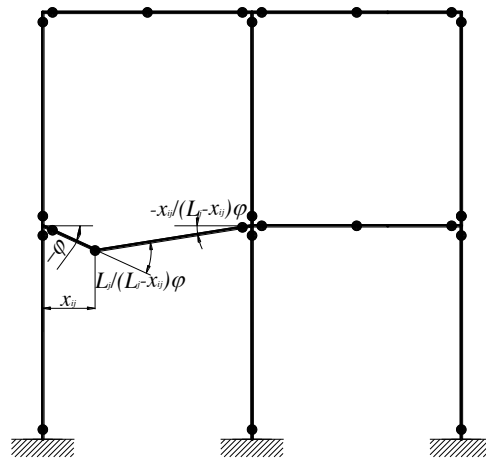


Figure 3: Beam mechanism in case of permanent loads.

For the evaluation of the external and internal work in a combined mechanism, the considerations described in the case of proportional load hold.

Analysing all the possible combinations of elementary mechanisms, the minimum value of λ_c must be sought in order to obtain the real collapse load.

3 OPTIMIZATION PROCEDURE BASED ON GENETIC ALGORITHM

In this paper a genetic algorithm strategy for the optimization procedure which seeks the minimum value of λ_c is employed.

A “genetic algorithm” is an adaptive stochastic method based on an opportune combination of random mutations and natural selection of chromosomes, that is arrays of integer values which correspond to a specific configuration of the problem under study, aiming at numerical-

ly finding optimal values of some specific function. The algorithm acts over a population of P potential solutions by applying, iteratively, the “survival of the fittest” principle: in such a way it produces a sequence of new generations of individuals that evolves towards a stationary population where the large majority of surviving solutions approach the real solution of a practical problem.

Within the specific problem here treated, each number of a chromosome (called gene) represents how many times a given elementary mechanism enters in the combination. Therefore, if a total amount $N = N_{fm} + N_{bm} + N_{nm}$ of elementary mechanisms (being N_{fm} , N_{bm} and N_{nm} the number of floor beam and node mechanisms respectively), a generic chromosome C_i of the population represents a weighted combination of those mechanisms and can be coded in the string $C_i = (c_1, c_2, c_3, \dots, c_k, \dots, c_N)$ with $c_k \in [0, c_{max}]$, being c_{max} the maximum number of times the k -th mechanism is involved in the combination ($c_k = 0$ means that the mechanism is not involved at all). The load factor λ_i of the corresponding combination (chromosome) can be consistently computed. The overall number of possible different chromosomes is thus $P_{max} = (c_{max} + 1)^N$, a quantity which rapidly increases with N even for small values of c_{max} . Aiming at the minimization of λ_i an opportune “fitness function” to be maximized is here defined as follows

$$f(\lambda_i) = K - \lambda_i \quad (6)$$

where K is an arbitrary constant great enough to have $f_i > 0$. Starting from the initial population of P chromosomes (typically $P = 100$), a new generation is created from the old one, where chromosomes that have a higher fitness score are more likely to be chosen as “parent” than those that have low fitness scores. The selection method adopted in this paper is called “tournament selection”, with a tournament size of three. Either one or two parents (depending on the crossover-rate variable, see later) are chosen to create children: with one parent, the child is simply a clone of the parent; with two parents, the process is the digital analogue of sexual recombination (crossing-over). Once the new generation is created, there is also a chance that random mutations will occur at level of the single genes c_k of the child chromosomes, and some of them will be changed into new ones (always chosen in the interval $[0, c_{max}]$). By iteratively repeating this process several times, chromosomes with the highest fitness will be progressively selected in the space of all the possible combinations and will quickly spread among the population reducing the diversity of the individuals.

One of the main goals of this paper is to develop an original code for both the calculation of all the elementary collapse mechanisms of a given planar frame and the implementation of the genetic algorithm described above for the determination of the combination of elementary mechanisms with the minimum collapse load factor for the structure considered. For this purpose, a very powerful software has been adopted, that is NetLogo, which is a freeware multi-platform environment with an owner high level programming language and with a very ductile and versatile user interface. The idea is to harness the power of the NetLogo graphical user interface and the versatility of its agent-oriented programming language in order to write an original code for the complete and automatic analysis of the limit behaviour of structures under earthquake excitations. In Figure 4 the layout of the user interface in NetLogo is reported. The interface is visually organized in two parts: (i) the *central-left* part, dedicated to the input parameters, to the setup of the frame and to the calculation of its elementary mechanisms; the chosen frame is then visualized in the World of NetLogo; (ii) the *right* part, dedicated to the plastic analysis of the frame through the genetic algorithm.

All the options of the genetic algorithm can be set. In particular, the *crossover-rate* slider controls the percentage of each new generation that has to be created through recombination

of the genes of two parents' chromosomes and the percentage ($100 - \text{crossover-rate}$) to be created through cloning of one parent's chromosome, while the mutation-rate slider controls the percentage probability of mutation, which applies to each genes of all the chromosomes in the new generation. Typical values of these two quantities are about 80% for the crossover rate and about 1% for the mutation rate. On the other hand, typical values for the population size are between 25 and 100 chromosomes, while usually c_{max} is not greater than 3. At the end of the simulation it is also possible to visualize the combination of elementary mechanisms corresponding to the winning chromosome (that is the actual collapse mechanism). However, it is preferable to launch the multi-event version of the algorithm, which runs the same genetic routines several times with different random initial populations, in order to escape from local minima of the collapse load and to have more chances to reach a global one.

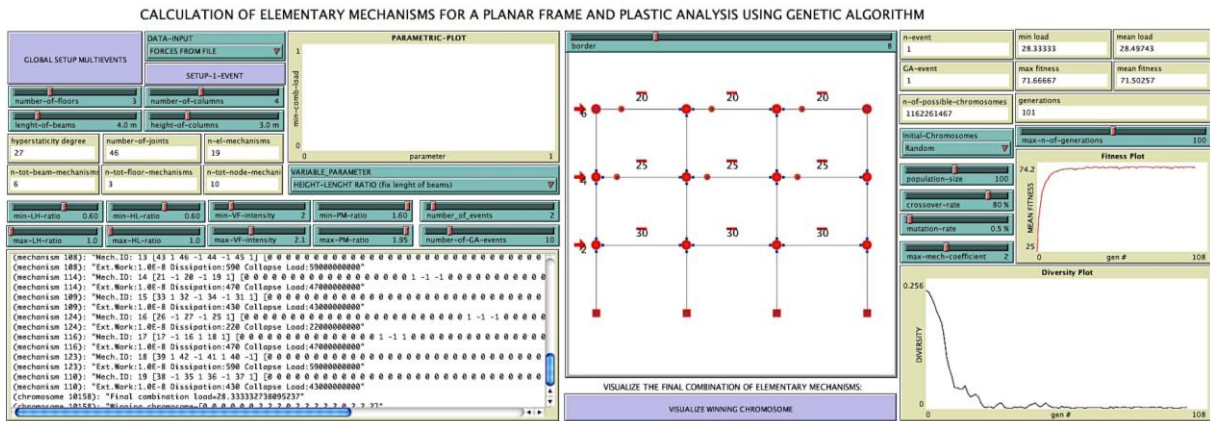


Figure 4: Layout of the NetLogo user interface.

4 NUMERICAL APPLICATIONS

In this section several applications will be presented, aiming at validating the proposed approach and performing a parametric study in order to assess how the magnitude of the permanent loads can affect the collapse load and mechanisms of frame structures under seismic conditions.

In the first sub-section the case of frames subjected to a proportionally increasing load distribution is treated to validate the proposed approach on two frames already studied in the literature.

In the second sub-section the case of seismic collapse load is faced with. In this second case, a distribution of increasing horizontal loads (seismic contribution) is considered together with a constant vertical load distribution (gravitational loads) and the results are compared with a classic pushover analysis by means of a FEM software [12]. In addition a parametric study is performed to assess the sensitivity of the ultimate load with respect to the magnitude of the gravitational loads. In all the considered cases, the relevant collapse mechanisms are reported and discussed.

4.1 Proportionally increasing loads

The applications reported in this subsection can be found in [13] where the ant colony algorithm is employed as optimization procedure, and represent a validation of the proposed approach. The two analyzed frames are reported in Figure 5, which are in the following addressed as Frame A and Frame B respectively.

The previously described NetLogo code provided for Frame A the load multiplier of $\lambda_c=2.1538$ (vs 2.15 reported in [13]), which is the minimum value over 10 runs. The setup for the main parameters of the genetic algorithm was the following: a population size of 100 chromosomes (with $c_{max} = 2$) was considered, a time range of 100 generations with 80% of crossover rate and 1% of mutation rate. These values will be adopted for almost all the simulations in the paper. Figure 6a shows the corresponding collapse mechanism of the NetLogo code which is the same one obtained in [13].

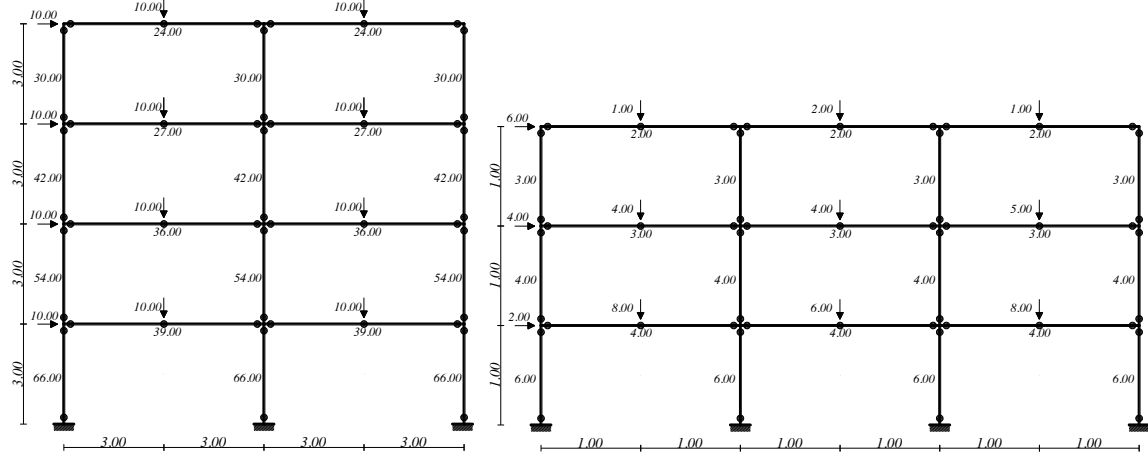


Figure 5: Layout of the two analyzed frames.

It is worth to point out that potential along span plastic hinges are reported with an orange circle irrespectively of the actual participation of the hinge to the overall collapse mechanism. Only in presence of relative rotation in correspondence of its location, the plastic hinge can be considered participating to the failure mechanism.

For the Frame B the load multiplier of 1.8734 (vs 1.873 reported in [13]) over 10 runs was obtained with the proposed approach while the collapse mechanism is reported in Figure 6b. Again, the collapse mechanism and the ultimate load factor are in good agreement with those available in the literature.

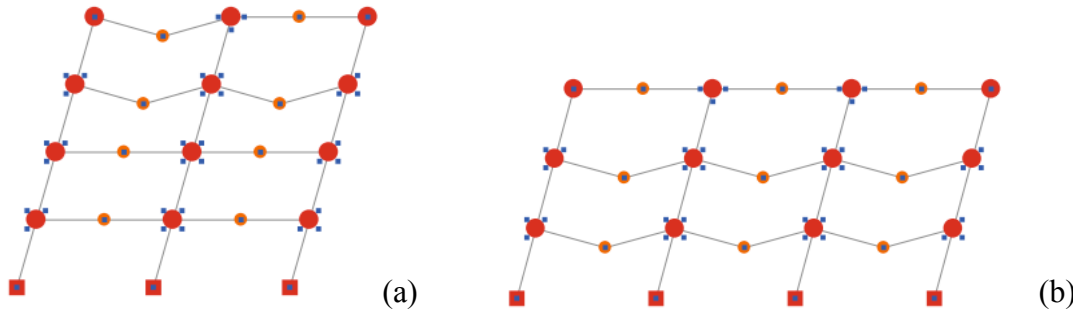


Figure 6: Collapse mechanisms obtained for the two frames. (a) Frame A, (b) Frame B.

4.2 Seismic loads

In this section the case of a frame subjected to permanent distributed vertical loads acting on the beams and increasing horizontal floor forces is treated. The considered frame is a three storey and three bay one whose geometry is reported in Figure 7. In the same figure the plastic moments are reported next to each beam and column and the values of the horizontal and vertical forces are also indicated.

As already described, the location of this potential along span plastic hinge depends on the length and on the plastic moment of the beam, as well as on the intensity of the permanent

vertical load acting on it. In particular, when the acting load is lower than a limit value, no along span plastic hinge can occur (i.e. plastic hinges can occur only at the two ends of the beam), while when the acting load is higher than the limit load the along span plastic hinge can occur at a specific section. These potential plastic hinges may anyway not be involved in the failure mechanism. In order to show all the possible scenarios, in the reported applications the cases of permanent vertical loads lower and higher than the limit values are considered as well as the case in which the acting load is exactly equal to the limit value. In particular, in the following reported applications the vertical load acting on the beams of the first floor has been chosen to be equal to the limit value ($q_{lim,1} = 30$) while for the second and third floors it is assumed to be greater ($q_{lim,2} = 22.5$, $q_{lim,3} = 15$). Correspondingly, in terms of possible along span plastic hinges, none can occur at the beams of the first floor, while at the second and the third floors the along plastic hinges can occur at the abscissae 0.205 and 0.536 from the left ends of the beam, respectively.

In this case the comparison with the proposed procedure has been performed with a classic pushover approach in terms of ultimate load. A numerical model was implemented in the well known FEM software SAP2000 [12]. The considered model employs lumped nonlinearities, simulated by means of perfectly plastic hinges. Since in a pushover analysis the location of plastic hinges has to be set 'a priori', in the beams the critical sections have been set every 10 cm.

For the considered frame, the FEM approach leads to a collapse multiplier of the horizontal loads equal to $\lambda_{c,SAP} = 28.1645$, while with the proposed approach the collapse multiplier is equal to $\lambda_c = 28.28$. It is worth to notice that, since the forces are gradually incremented in a pushover analysis, the actual collapse multiplier λ_u of a structure represents an upper bound for such an approach. Furthermore, due to the kinematic theorem of plastic analysis λ_u represents a lower bound for a limit analysis approach. Therefore it has to be $\lambda_{c,SAP} \leq \lambda_u \leq \lambda_c$.

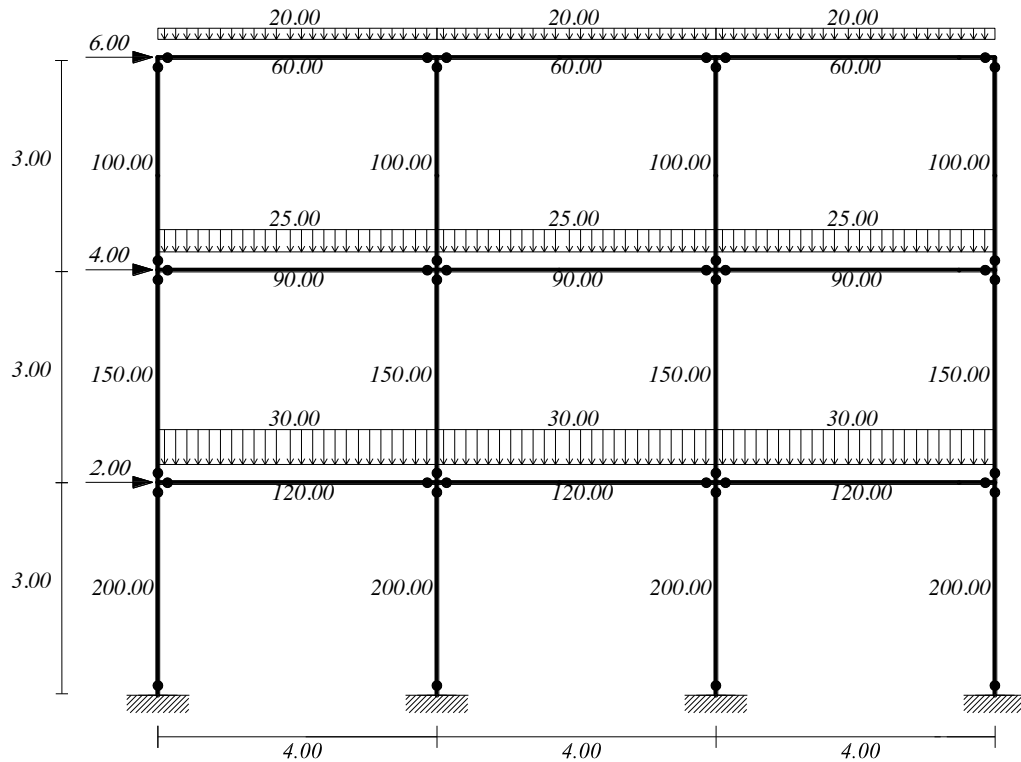


Figure 7: Geometrical and mechanical layout of the frame studied in seismic conditions.

Figure 8 reports a comparison of the collapse mechanisms for the considered frame obtained with the proposed approach (Figure 8a) and with a classic pushover approach (Figure 8b) and shows that some of the potential plastic hinges along the span of the beams on the third floor do not rotate in the total collapse mechanism. The collapse mechanisms again are almost coincident.

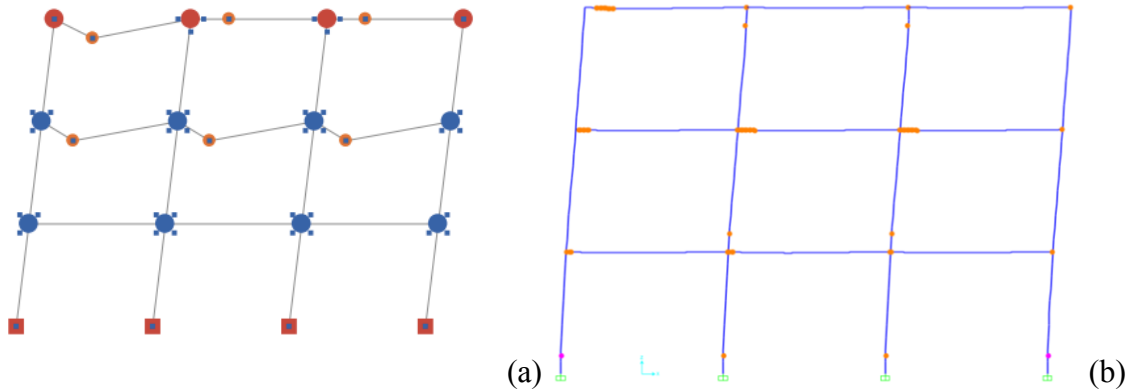


Figure 8: Collapse mechanisms of the frame under seismic conditions: (a) proposed model (b) pushover analysis.

In Figure 9 the multiplier α of the permanent loads is considered variable in the range [0-3]. In particular the conditions $\alpha = 0$ and $\alpha = 1$ represent, respectively, the cases of no permanent loads acting on beams and permanent load of the benchmark case (Figure 7). In the top left panel, the locations of the plastic hinges at each floor versus α are reported, in terms of their normalized abscissae x_{ij}/L , showing that above a certain threshold (different for each floor), increasing the value of α the hinges progressively move towards the right end of the beam. On the top right panel of Figure 9 the trend of the obtained ultimate load versus α is shown in comparison with the correspondent values provided by pushover analysis. The results show a good agreement between the FEM and the proposed approach. It is worth to notice that as the multiplier α is lower than a transition value equal to 1.06, the permanent load acting on all beams does not imply any plastic rotation located at the hinges along the axis of the beams and the collapse mechanism does not change, therefore the ultimate load is constant in this range. Then, when α increases the ultimate load decreases progressively and plastic hinges also open along the beams producing relative rotations. It can be easily seen, from the collapse mechanisms reported in the same figure, that increasing the value of α the positions of the plastic hinges in the beams move towards the right end.

5 CONCLUSIONS

An automatic approach for the evaluation of plastic loads and failure modes of planar frames, based on the generation of elementary collapse mechanisms and on their linear combination, has been presented.

The proposed approach, representing an extension of the method of combination of elementary mechanisms, accounts for permanent distributed vertical loads in addition to horizontal concentrated increasing forces and makes use of an original software developed in the agent-based programming language NetLogo, which is here applied for the first time to structural engineering. The code is able to automatically determine and visualize all the elementary mechanisms in planar frames and, by means of an optimization procedure based on genetic algorithms, to calculate, with great accuracy and in a very short computing time, the collapse load and the related mechanism.

Several applications have been performed either with reference to the classical plastic analysis approach, in which all the loads increase proportionally, or with a seismic point of view considering a system of horizontal forces whose magnitude increases while the vertical loads are assumed to be constant.

The performed applications have been compared either to some of the available results provided in the literature, in the case of proportional loads, or to the results provided by non linear pushover analysis in the seismic approach.

The seismic applications represent an original contribution towards the limit behaviour of structures under earthquake excitations, since general trends of seismic behaviour of planar frames can be deduced from the obtained results.

To this purpose, a parametric study has been performed aiming at evaluating the influence of the intensity of permanent vertical weights on the ultimate collapse load of planar frames. In particular it has been shown that the collapse load decreases, and the location of the plastic hinges on the beams involved in the collapse mechanism move towards the right end of the beam, for increasing values of the permanent vertical loads. The achieved results, with reference to the parametric studies, not only may provide significant information on the seismic performance of frame structures but also represent a useful tool in their optimal design.

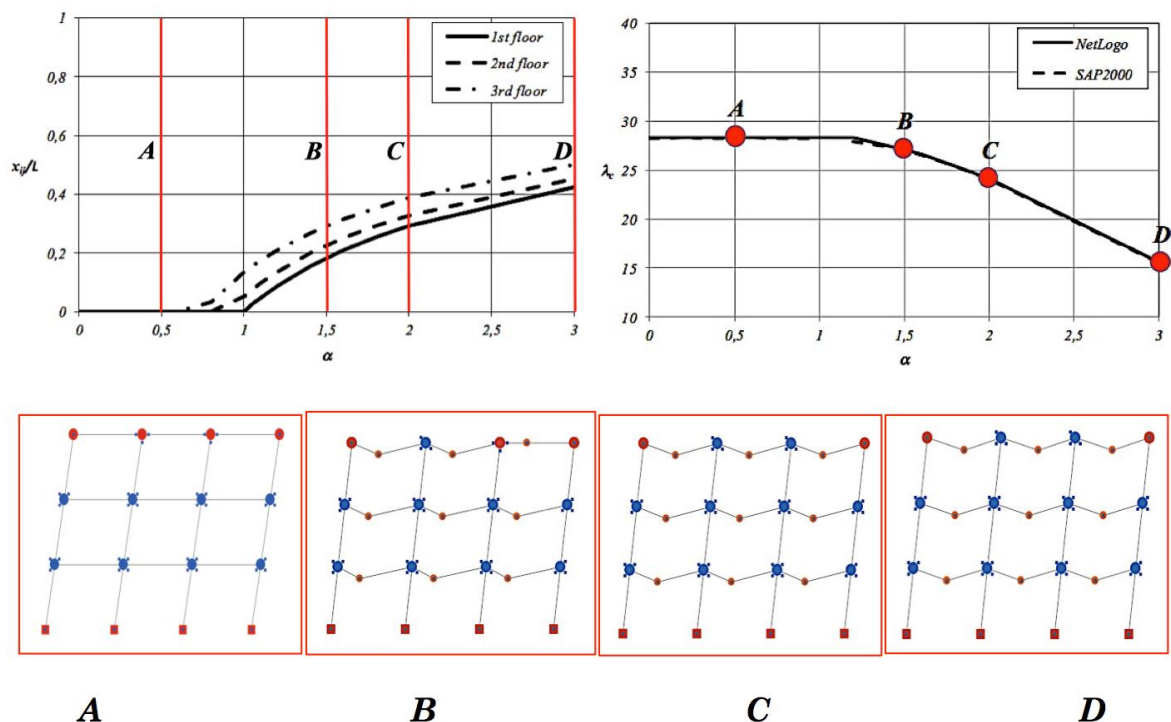


Figure 9: Location of plastic hinges on the beams (top left panel) and collapse load multiplier (top right panel) vs intensity of the permanent vertical loads; Bottom panels: selected collapse mechanisms.

REFERENCES

- [1] Bergan P.J, Sørense T. A comparative study of different numerical solution techniques as applied to a nonlinear structural problem. *Comput Methods Appl Mech Eng* 1973; 2(2):185–201. J.T. Oden, T. Belytschko, I. Babuska, T.J.R. Hughes, Research directions in computational mechanics. *Computer Methods in Applied Mechanics and Engineering*, **192**, 913-922, 2003.

- [2] Charnes A, Greenberg HJ. Plastic collapse and linear programming, *Summer Meeting of the American Mathematical Society*; 1959.
- [3] Livesley RK. Linear programming in structural analysis and design. In: Gallagher RH et al., editors. Optimum structural design. New York: Wiley; 1977 [Chapter 6].
- [4] Cohen MZ, Maier G, editors. Engineering plasticity by mathematical programming. Proceedings of the NATO Advanced Study Institute, University of Waterloo, Waterloo, Canada, 2–12 August 1977. Pergamon Press Ltd; 1979.
- [5] Neal BG, Symonds PS. The rapid calculation of plastic collapse loads for a framed structure. In: Proceedings of the institution of civil engineers London, vol. 1, part 3; 1952. p. 58–100.
- [6] Neal BG, Symonds PS. The calculations of collapse loads for framed structures. *J Inst Civil Eng* 1951; 35:21–40.
- [7] Goldberg DE. Genetic algorithms in search, optimization and machine learning. Addison-Wesley, USA; 1989.
- [8] Holland JH. Adaptation in natural and artificial systems. MIT Press, USA; 1992.
- [9] Kaveh A, Khanlari K. Collapse load factor of planar frames using a modified genetic algorithm. *Commun Numer Methods Eng* 2004; 20:911–25.
- [10] Mazzolani F, Piluso V. Plastic design of seismic resistant steel frames. *Earthquake Engineering and Structural Dynamics*. 1997; 26:167-191.
- [11] U. Wilensky NetLogo, <http://ccl.northwestern.edu/netlogo>, Center for Connected Learning and Computer-Based Modeling. Northwestern University, ^[1]~~SEP~~Evanston, IL 1999.
- [12] CSI Analysis Reference Manual for SAP2000, Computers and Structures Inc., 2007.
- [13] M. Jahanshahi, M. Pouraghajana, M. Pouraghajanb, Enhanced ACS algorithms for plastic analysis of planar frames, *Comp. Meth. Civil Eng.*, Vol.4 No.1 (2013) 65 - 82