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GENERATING ARTIFICIAL TIME HISTORIES USING A NEW COMPUTER-BASED ENVIRONMENTAL PLATFORM: OPENSIGNAL

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Abstract. *The paper is presenting a new artificial time-histories generation model which uses spectrum compatible time-histories, created using a new envelope function that can be used in regions characterized by subduction earthquakes with long duration period and paucity of earthquake records data. The method has been implemented in a new computer-based platform for ground motion processing and selection. The platform is organized into three main modules as follows: Signal Processing is designed for data processing; it allows to open raw data downloaded from main databases all over the world acquiring the principal seismic parameters. Seismic Records Selection evaluates a set of spectra or time-histories data. Various attenuation laws and correlation models are implemented to compute the Conditional Mean Spectrum and the Predicted Mean Spectrum. Real time-histories can be selected and downloaded by PEER and ESMD databases using waveform matching, spectrum matching or energetic methods. A physics-based stochastic model is implemented for the generation of synthetic time-histories as well as a wavelet-based stochastic models for artificial time-histories. Site Response Analysis concerns modelling the local site effects of the ground motion propagation using a hybrid approach based on an equivalent linear model. The soil behavior is modeled assuming that both shear modulus and damping ratio vary with shear strain amplitude. Hence, the hysteretic behavior of the soil is described using the shear modulus degradation and damping ratio curves.*

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1 INTRODUCTION

The exponential increment of computational power is progressively moving the state-of-practice in earthquake engineering toward the use of dynamic non-linear time history analysis with respect to response spectrum analysis. All these methods need as prerequisite the selection of a proper suite of earthquake ground motions to be reliable. In fact, among all possible sources of uncertainty (e.g., structural material properties, modelling approximations, design and analysis assumptions, etc.), the selection of earthquake ground motion has the highest effect on the variability of the structural response [1] [2] . Two selection criteria are mainly used for such aim: (i) researches based on parameters obtained by disaggregated seismic hazard maps at a specific site, such as the magnitude, *M*, and the source-to-site distance, *R*, the soil type, V_{s30}, the source mechanism and the duration, (ii) researches based on spectral matching , such as design code spectrum (DS), seismic scenario determined from a ground motion prediction relationship (PMS) [3], uniform hazard spectrum (UHS), and conditional mean spectrum (CMS) [4]. Using DS and UHS might bring to over-softening and overdamping during the analysis [5]; therefore, a matching procedure based on the CMS has been developed and presented for the Italian territory in this paper. The lack of real records in several areas of the world leads to cases studies where is necessary resorting to artificial or synthetic time-histories. Three methods have been implemented: according to the grade of knowledge, records can be generated starting from (i) the target spectrum, (ii) the seismic parameters (M, R, V_{s30}), (iii) the geometric spreading and *Q* function. The local seismic response has been modelled using an equivalent linear model, assuming that both the shear modulus and the damping ratio vary with the shear strain amplitude. So, the hysteretic behaviour of the soil is described using the shear modulus degradation and damping ratio curves. A large number of computer programs, public and commercial, are available at the Observatories and Research Facilities for European Seismology (ORFEUS) data center1, such as SMARTS 2.0, Shake-91, DIMAS, and PickEv 2000, and at the Pacific Earthquake Engineering research center2, such as SIMQKE-I and SIMQKE-II. Most of existing public signal processing software are developed to analyse a single seismic earthquake record at a time (e.g., Seismosignal – available at http://seismosof.com/or view wave available at http://iisee.kenken.go.jp/staff/kashima/viewwave.html). For multiple records analysis, commercial software are needed, such as Bispec [5], but they have the inconvenient that they are not freely available in the market. Iervolino et al. [6] developed a Matlab-based software called REXEL, which allows ground motion selection using the Italian database, Itaca [7]; however, the proposed software is not able to perform multiple signal processing and it is not able to build the CMS on the Italian territory automatically, by using as input the GIS coordinates. Similar software have also been developed by Corigliano et al. [8] who implement a software called ASCONA for the selection of compatible natural ground motions. Recently, also Katsanos and Sextos [9] developed a Matlab-based software environment integrating finite element analysis with earthquake records selection which works with the PEER database. The presented platform includes many features, which are not available in the programs mentioned above: signal processing, soil response analysis, the computation of the Conditional Mean Spectra. This paper presents the application of CMS in a classic power spectrum-based model with the innovation of random envelope functions in order to generate spectrum-compatible ground motions. The method has been applied in a software called OPENSIGNAL [10] for processing and selection of seismic records, freely available at the following link:

http://areeweb.polito.it/ricerca/ICRED/Software/OpenSignal.php

2 SIGNAL PROCESSING

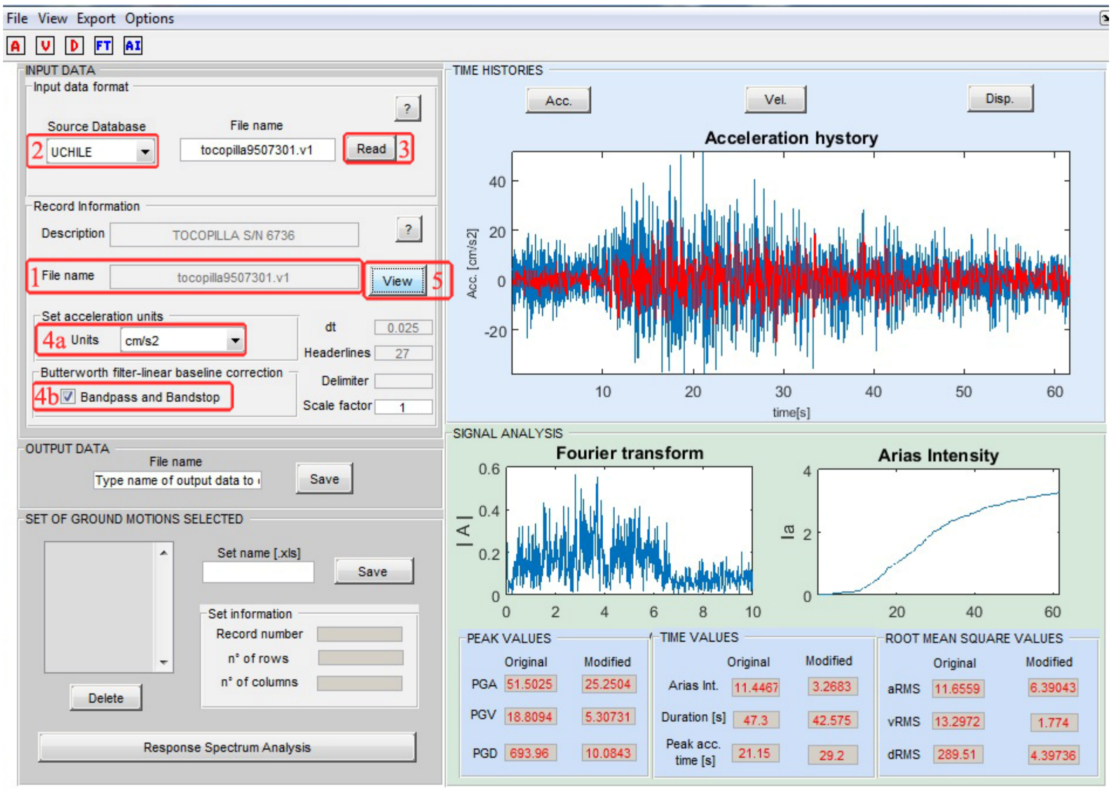

Figure 1 Signal Processing window.

Many ground motion parameters, such as peak displacements and velocities, are used often in different field of earthquake and geotechnical engineering [11]; however, their values are affected by the noise of the earthquake ground motion. The influence of noise in ground motion records is evident at low and high frequencies where the signal-to-noise ratio is usually lower compared to the mid spectrum. Then, filtering operations became the primary tool for correcting the ground motion records and consequentially it has become standard practice to cut low and high frequencies by looking at the spectra of the Fourier amplitude spectra and the signal-to-noise ratio. The Butterworth filter is the most used in seismic applications and it is designed to have a flat frequency response in the pass-band range, while it is equal to 0 in the stop-band range. Analytically, the frequency response amplitude of the Butterworth filter is given by

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}} \tag{1}$$

where ω is the generic angular frequency, ω_c represents the cut-off frequency, and N is the order of the filter. The source of this error as mentioned above, is generated by the high and low frequency noise which contaminates the signal. Baseline corrections can be applied using different techniques, in the time domain to remove unwanted trends and in the frequency domain to remove unwanted frequencies. In the frequency domain, the noise is most easily removed by the use of a bandpass filter, like the low-pass filter which is set up to values <0.1.

In addition, the effect of aliasing can be eliminated by filtering the original ground motion beyond the Nyquist frequency (12.5 Hz). In most of the databases, ground motion records are already filtered, but there might be cases in which some records are unfiltered and in that case, the filtering option in the environmental platform can be used. **Error! Reference source not found.** shows *Signal Processing* window of computer environment. The number identifies the steps to be followed in order to obtain a correct processing procedure and they are summarized below:

1. Import the record with "Open" or insert the name.dat of the file.
2. Select the correct database.
3. Push "Read" for reading the file and extracting of the main information.

4a. Select the measure units for the accelerations.

4b. Tip the "Bandpass and Bandstop" for filtering the signal.

5. Push "View" for loading the acceleration history and plotting the signal properties and the time histories.

In the *Input Data Format* (upper left), the records are uploaded and read automatically for the selected ground motion databases (PEER, ESMD, UCHILE and ITACA) or using the option "free format." The Butterworth filter by modifying can be set according to minimum frequency, f_{min}, maximum frequency, f_{max}, and order, n. The effect of the filter is shown in *Time Histories* panel where the time histories of accelerations, velocities, and displacements, both filtered and unfiltered are displayed. In the *Signal Analysis* block (low right corner in Figure 1), the main parameters of earthquake records (e.g., peak ground acceleration, velocity and displacement, duration, etc.) both peak and root mean square values are calculated and saved both for filtered and unfiltered data; Arias Intensity and Fourier Transform graphs are plotted in specific frames . Finally, all processed data and records can be exported in other common formats, such as MS Excel and txt (bottom left in Figure 1).

2.1 Response Spectral Analysis

Once the set of ground motions are selected and filtered, the Elastic Response Spectra (acceleration, velocity, displacement, etc.) can be computed for a given value of damping ratio. Furthermore, the mean and median acceleration response spectra of the uploaded set of records with the associated range of dispersion (±σ) can be also evaluated and plotted using also log and semi-log scales.

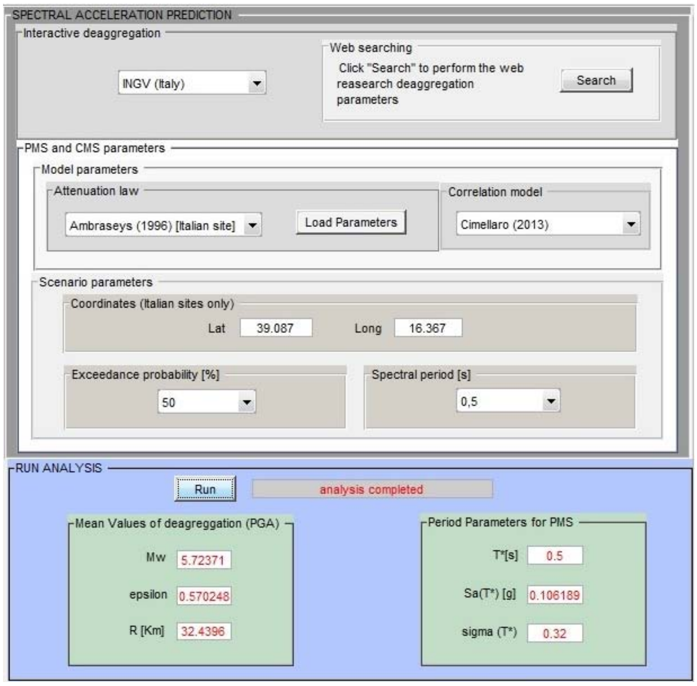

Figure 2 Computation of CMS in Italian territory; scenario parameters panel allows its computation according to GIS coordinates. CMS for other countries is available with several attenuation laws and correlational models [12][21].

3 TARGET SPECTRA

Different types of target spectrum can be defined during the ground motion selection process. The design spectrum (*DS*) can be evaluated according to the Italian seismic standards, the NTC 2008 [13] for any point in the Italian territory, once the parameters are defined (e.g., nominal life, soil category, damping ratio, over strength factor q to describe the inelastic behaviour, etc.). Additionally, the *DS* according to the European seismic standard, EC8 [14], and to the US standards [15] can be evaluated inserting the proper parameters. Furthermore, the platform allows evaluating for a given probability of exceedance the *UHS*, and the Predicted Mean Spectrum (*PMS*) using different ground motion prediction equations (GMPE), which are currently available: Ambraseys et al., Campbell and Bozorgnia, Boore and Atkinson, and Chiou and Youngs [3][16][17][18]. Two additional attenuation laws have been recently inserted to define the *CMS* for Chilean sites [19] and the Regional Indian attenuation equations for Indian sites [20]. However, the novelty of the proposed system architecture is that it allows evaluating the CMS [21][22] for the first time on the entire Italian territory automatically knowing the GIS coordinates (Figure 2) and in any other site worldwide knowing the proper parameters.

4 REAL TIME-HISTORIES SELECTION

Ground motion selection and scaling procedures are applied in order to obtain a set of motions that are usually used in dynamic elastic and even non-linear response history analysis. The proposed framework retrieves records from the PEER-NGA strong motion database (PEER – available at http://ngawest2.berkeley.edu/), the European strong motion database (ESMD – available at http://www.isesd.hi.is/ESD_Local/frameset.htm). Three selection criteria are available: Waveform Matching, Spectral Matching, Energetic criteria.

Waveform Matching can be obtained selecting some specific seismological parameters obtained by the disaggregated seismic hazard maps at a specific site, such as the moment magnitude, Mw, the fault distance or Joyner–Boor distance (R or RJB, expressed in kilometer), the fault mechanism, the soil type according to EC8, and the waveform parameters (e.g., peak ground acceleration, peak ground velocity, peak ground displacement).

Spectral Matching requires an additional step, the definition of the target spectrum and the type of matching to be carried out. The current available options in the platform are three: (i) Single period, (ii) Multi periods (up to three values), and (iii) Mean Deviation. A selected percentage error is defined in all cases to vary the number of earthquakes selected. The second step is the selection of the Target Spectrum among the CMS, the DS, the UHS, the PMS, or any User Defined (UDS) response spectrum. After the selection of the target spectrum, the search of the records between the ground motion databases available in the computer environment is performed. Both horizontal and vertical components of ground motion can be considered in both research methods. Spectrum-compatible records can be located among the last research on Waveform matching.

5 SYNTHETIC TIME-HISTORIES SELECTION

The combination of seismological models of the spectral amplitude of ground motion with the engineering notion that high-frequency motions are basically random [23] is the basis of *SMSIM* code. Following the generic formulation proposed in [23], the total spectrum of the motion at a site can be divided into contributions from earthquake source (*E*), path (*P*), site (*G*) and type of motion (*I*):

$$Y(M_0, R, f) = E(M_0, f)P(R, f)G(f)I(f) \tag{2}$$

Referring to [23] for detailed information about the meaning and formulation of each contribute, Figure 3 illustrates the window of selection of input parameters. Beta is the shear-wave velocity in the vicinity of the source. Geometric spreading is represented by three linear segment, described by three slopes and two points in common. Spectrum control is controlled by the choice of source type (single or double corner), Sigma is the stress drop related to the magnitude Mr and Mf the derivate of log(sigma) with respect to magnitude, fb/fa is the ratio between corner frequencies. The whole path-attenuation is given by a piecewise continuous set of three straight lines in log Q and log f space (s1 and s2 are the slopes of first and third lines that have values Qr1 and Qr2 at frequencies fr1 and fr2, ft1 and ft2 are the transition frequencies). Wfa and Wfb defines the weight of fa and fb concerning the source duration. The diminution factors regard the application of a filter. Parameters for Western North America and Eastern North America are uploaded and available by default. The site amplification is fixed with a generic formulation for generic rock site. An arbitrary number of independent Time-histories can be generated; for each time-history, several parameters are computed and displayed (e.g. velocity, displacement, Fourier amplitude, Arias intensity).

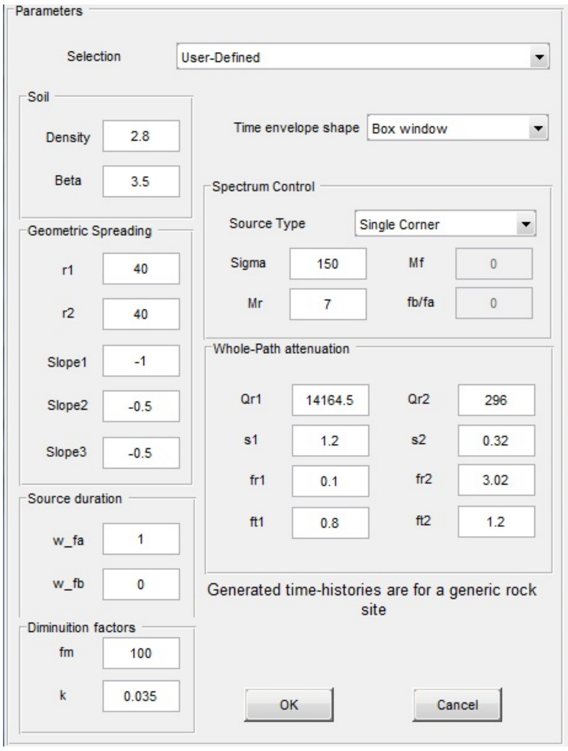

Figure 3 Required parameters for generation of synthetic time-histories.

6 ARTIFICIAL TIME-HISTORIES SELECTION

6.1 Wavelet-based method

The decomposition of ground motion time-histories into wavelet packets and the reconstruction of time-histories from wavelet packets has been applied in the creation of a stochastic ground-motion model [24]. The wavelet packet is defined as

$$c_{j,k}^{i} = \int_{-\infty}^{\infty} x(t)\psi_{j,k}^{i}(t)dt \tag{3}$$

The inverse wavelet packet is

$$x(t) = \sum_{i=1}^{2^j} \sum_{k=1}^{2^{N-j}} c_{j,k}^{i}\, \psi_{j,k}^{i}(t) \tag{4}$$

Employing two groups of wavelet packets, 13 parameters quantify time and frequency characteristics of the acceleration time histories. Such parameters are predicted as a function of

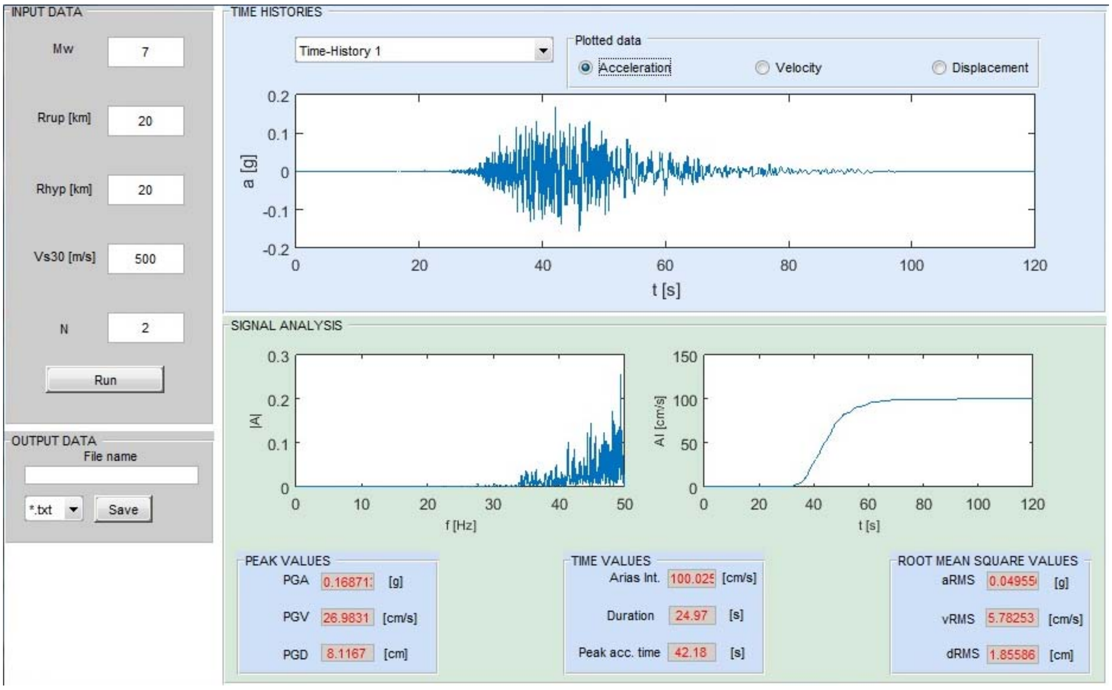

Figure 4 Window of generation of artificial time-histories. Synthetic time-histories share a coincident layout with few differences in the INPUT DATA panel.

four predictor variables: the moment magnitude (Mw), the hypocentral distance (Rhyp), rupture distance (Rrup), and average shear-wave velocity within 30 m depth (VS30). In turn, the predictor variables are obtained by a two-stage regression analysis. In Yamamoto [24] it has been proposed a range of values that provides a good match from the GMPEs; those values are reported and recommended inside the code. As in synthetic time-histories, the generation of independent time-histories is unlimited and the signal processing is analogous (Figure 4).

6.2 Power spectral density based method

Spectrum-compatible ground motions are generated by the power spectral density PSD, which is related to a target spectrum [25]:

$$PSD(\omega_n) = \frac{1}{\omega_n\left(\frac{\pi}{4\varsigma} - 1\right)}\left(\frac{\omega_n^2 S_V^2}{r_{s,p}^2} - \int_0^{\omega_n} PSD(\omega_n) d\omega\right)^{1/2} \quad (5)$$

where ω_n is the natural pulse, $r_{s,p}$ is the peak factor for a ground motion of duration s and probability p, ς is the damping factor. Referring to [25] for detailed information about the formulation of peak factor, PSD can be used for generation of stationary time-histories:

$$A_n(\omega_n) = \sqrt{2PSD(\omega_n)\Delta\omega} \quad (6)$$

$$z(t) = \sum_n A_n \sin(\omega_n t + \phi_n) \quad (7)$$

A stationary ground motion is obtained by summing several sine functions with a random phase ϕ_n, which provides independence among several generated ground motions. Non-

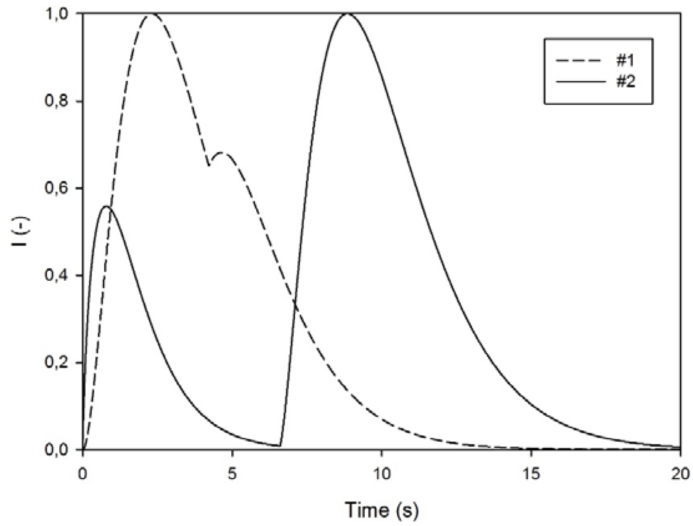

Figure 5 Two examples of implemented random envelope functions. They are generated by random shifting and scaling the second curve in a selected range and overall scaling of the whole function.

stationarity is achieved by an envelope function *I(t)* multiplied for the stationary time-history. The novelty of the procedure is a code generating envelope functions (Figure 5) from overlapping two or more Saragoni and Hart functions [26] in order to taking into account the random shape of real ground motions. A process of iterative adjustment improves the spectral matching by

$$PSD(\omega_n)^{i+1} = PSD(\omega_n)^i \left(\frac{S_V}{S_V^i}\right)^2 \tag{8}$$

Such procedure is implemented into the software by selecting the target among the available spectra with a signal-processing equal to other synthetic and artificial models, providing a useful tool for generating realistic spectrum-compatible time-histories (e.g. regions with undersized source databases).

7 SITE RESPONSE ANALYSIS

The proposed method uses an equivalent linear model (hybrid approach), which describes the soil behavior assuming that both the shear modulus and the damping ratio vary with the shear strain amplitude. Therefore, the hysteretic behavior of the soil is described using the shear modulus degradation curve (G - γ) and the damping ratio curve (ξ - γ). In the proposed platform, clay, sand, and rock degradation curves are available by default [27].

The dynamic equations of the system:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{I\}\ddot{u}_g \tag{9}$$

(9)(10) is solved using Newmark method:

$$\dot{u}_{i+1} = \dot{u}_i + [(1-\gamma)\Delta t]\ddot{u}_i + (\gamma\Delta t)\ddot{u}_{i+1} \tag{10}$$

$$u_{i+1} = u_i + \Delta t\dot{u}_i + \left[\left(\frac{1}{2} - \beta\right)\Delta t^2\right]\ddot{u}_i + \beta\Delta t^2\ddot{u}_{i+1} \tag{11}$$

Where [M], [C], and [K] are the mass, damping, and stiffness matrices, while $\{\ddot{u}\}$, $\{\dot{u}\}$ and $\{u\}$ are the vectors of absolute nodal accelerations, velocities, and displacements; Δt defines the time step, β and γ are both assumed equal to 1/4. Evaluating the nodal displacements and the corresponding shear deformations γ at a given time instant t, the γ values are inserted in the curves to update the shear modulus G and the damping ratio ξ, which are used to define the new stiffness and damping matrices defined at the same time instant t. The proposed method has some limitation at large shear strain deformations, because in that case, the soil presents a non-linear behavior and both stiffness and damping depend on the number of loading–unloading cycles. Nevertheless, it was observed that at medium deformations, the non-linear behavior of the soil is not significantly influenced by the load path [28]. Thus, the proposed hybrid approach can lead to reliable results for the range of medium deformations.

8 CONCLUSION

The paper presents a new model of generation of artificial time-histories. Employing a classic method based on the analysis of the power spectrum density, an arbitrary number of independent stationarity artificial spectrum-compatible time-histories are generated.

The novelty of the method consists in two features: (i) the application of the Conditional Mean Spectrum, which leads to an improvement of the energetic content, (ii) the achievement of non-stationarity obtained by random envelope functions, which provide realistic shape to the final time-histories. The random envelope functions are created by the overlapping of two or more envelope functions and varying the shape and position of these. Such model is implemented in a software platform for processing and selection of seismic records, called “OPENSIGNAL,” freely available for the public. The platform consists of a number of modules, integrated in a unified environment and aimed for: selection of ground motion records, signal processing, response spectra analysis, soil spectra analysis and generation of synthetic/artificial time-histories.

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