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OPTIMAL DESIGN OF TUNED MASS DAMPERS BY PERFORMANCE–COST ANALYSIS

Rita Greco[1], **Giuseppe C. Marano**[3,2] **and Alessandra Fiore**[2]

[1] DICATECh, Technical University of Bari
Via Orabona 4, 70125 Bari, Italy
rita.greco@poliba.it

[2] DICAR, Technical University of Bari
Via Orabona 4, 70125 Bari, Italy
{alessandra.fiore, giuseppecarlo.marano}@poliba.it

[3] College of Civil Engineering, Fuzhou University
Xue Yuan Road, Fuzhou 350108, China
marano@fzu.edu.cn

Keywords: Tuned Mass Damper, Random vibrations, Cost of protection system, Multi-objective Optimization, Non-dominated Sorting Genetic Algorithm.

Abstract. *This work deals with the multi-objective optimization of single Tuned Mass Dampers, focusing on buildings subject to low-moderate earthquakes. The novelty consists in considering both economic and performance criteria. The economic objective is represented by the cost of the device, directly related to its mechanical parameters, while the ratio between the absolute acceleration of the protected structure and the unprotected one is assumed as performance parameter. This latter parameter allows in fact to evaluate the damage level and the behavior of both components and equipment. A multi-objective optimization problem is then so formulated and the Non-dominated Sorting Genetic Algorithm, in its second version, is applied to obtain the Pareto optimum solutions. Finally, by a sensitivity analysis, the optimum solutions are commented with respect to some input data.*

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1 INTRODUCTION

The limitation of vibrations effects due to environmental dynamic loads is a very important matter in the design of civil and mechanical engineering structures [1-8]. In this field, many different strategies have been proposed also with regard to safety structural problems induced by random vibrations action caused by natural or artificial loads, as for example earthquakes, wind pressure, traffic vibrations, sea waves and so on [9-16]. Generally, four groups of control systems are distinguished in literature: Active, Hybrid, Semi-active and Passive. Among these, passive systems are the most unsophisticated and the cheapest ones.

With the purpose of maximizing the Tuned Mass Damper (TMD) efficiency, over the years, numerous approaches have been proposed for the optimum design of TMD. After the work of Ormondroyd and Den Hartog [17], several optimum design methods have been introduced in literature, aimed at minimizing the vibrations induced in mechanical and structural systems by various types of excitation sources [18-23]. In most of the above studies, the main structure is generally represented by an equivalent single degree of freedom system. Similarly the performance of a TMD applied to a multi degree of freedom (MDOF) structure and optimized to control only a single mode of vibration (usually the fundamental one) has been investigated by various authors [24-25]. The optimal parameters of single and multiple TMDs for the control of MDOF structures have been studied by several researchers in the last decades [23, 26-27] also considering the uncertainties affecting structural parameters [28-29]. The above studies mainly take as objective function the efficiency of the TMD expressed by a performance index that generally is chosen as the ratio between the response (displacement, acceleration dissipated energy) of the unprotected system and the same quantity of the protected one.

Differently from previous studies in this field, in this paper an optimum design of a single TMD installed on the top floor of a structure modelled by a linear multi degree of freedom system is carried out at the aim to simultaneously minimize the protection system cost and maximize a direct index of performance of the TDM. Moreover the study is developed in a stochastic way, by introducing a Gaussian non stationary filtered stochastic process to model the ground motion at the base of the structure.

2 ANALYTICAL FORMULATION

In this study the problem of a single TMD, positioned at the top of a main structure modelled as a linear viscous elastic MDOF system, is analysed. A deterministic second order mechanical linear system with *n* degree of freedom is considered and described by using lumped masses, as shown in Figure 1. Under the hypothesis of random dynamic inputs, the dynamic system of motion can be written as:

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} + \mathbf{C}\dot{\mathbf{X}} = -\mathbf{M}\mathbf{r}\ddot{X}_b \quad (1)$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are, respectively, the deterministic mass, damping and stiffness matrices (as detailed in appendix); $\mathbf{X}$, $\dot{\mathbf{X}}$ and $\ddot{\mathbf{X}}$ are the tuned-system relative displacement, velocity and acceleration vectors referred to each degree of freedom; $\mathbf{r} = (1...,1)^T$; $\ddot{X}_b(t)$ is the seismic action and, since it is mathematically described by a stochastic process, many advantages can be reached by modeling it through a filtered white noise.

The seismic action $\ddot{X}_b(t)$ is modelled by the non-stationary Kanai Tajimi process. After introducing the space state vector, the probabilistic analysis is performed by solving the non-stationary Lyapunov matrix differential equation [30,31].

Structural optimization has been common for a long time in mechanical and aeronautical engineering. In civil engineering, it is being progressively adopted both for buildings and for bridges [21-22, 32-34].

The multi-objective optimization of the above described TMD is herein formulated as the search of a suitable set of design variables, collected in the so called Design Vector (DV) $\mathbf{b} \in \mathbf{\Omega_b}$, $\mathbf{\Omega_b}$ being the admissible domain, which minimize two objective functions (*OFs*). The first OF to be minimized is the ratio between the standard deviation of the top floor absolute acceleration of the protected structure $\sigma_{\ddot{y}_N}$ and the one of the unprotected structure $\sigma^0{}_{\ddot{y}_N}$:

$$OF_1 = I_{\ddot{y}_N} = \frac{\sigma_{\ddot{y}_N}}{\sigma^0{}_{\ddot{y}_N}} \tag{2}$$

This parameter is in fact a direct index of the performance of a TMD, coherently with the strategy to reduce those structural detrimental vibrations that can induce damages in contents and equipments, as required in operational performance level.

The second objective is to minimize the cost related to the use of a TMD system. In fact, especially for new and strategic buildings or for retrofitting of existing constructions, the containment of costs is one of the most important targets in a protection system design.
The mass ratio μ is assumed as a fixed quantity, while each possible DV is defined as:

$$\mathbf{b} = \left(\omega_T, \xi_T\right)^T \tag{3}$$

where $\omega_T = \sqrt{k_T / m_T}$ and $\xi_T = c_T / 2\sqrt{m_T k_T}$ are the circular frequency and the damping coefficient of the TMD respectively. In this context, without loss of generality, it is assumed that the cost of the TMD is expressed as a linear function of the mechanical design parameters k_T and c_T respectively, so that:

$$C_T^1 = k_T + \alpha_C c_T \tag{4}$$

where α_C is the cost parameter. Moreover, by considering that the damping device has in general a different (higher) unit cost in comparison to the stiffness one, the cost ter α_C is defined as the ratio between these two unitary costs:

$$\alpha_C = \frac{\text{cost}(c_T = 1)}{\text{cost}(k_T = 1)} \tag{5}$$

Equation (4) can rearranged as below:

$$C_T^1(\mathbf{b}) = \mu m_S \left[\omega_T^2 + 2\alpha_C \xi_T \omega_T \right] \tag{6}$$

μ being the ratio between the tuned and the structural masses. The protection system cost for a unit system mass m_S, finally, is given by:

$$OF_2 = I_{C_T^1}(\mathbf{b}) = \frac{C_T^1(\mathbf{b})}{m_S} = \mu \left[\omega_T^2 + 2\alpha_C \xi_T \omega_T \right] \tag{7}$$

Finally, in this work the NSGA-II [35] is adopted in order to obtain the Pareto sets and the corresponding optimum DV values for different systems and input configurations. In particular, the Real Coded Genetic Algorithm [36], the Binary Tournament Selection [37], the Simulated Binary Crossover [38] and the polynomial mutation [36] are used.

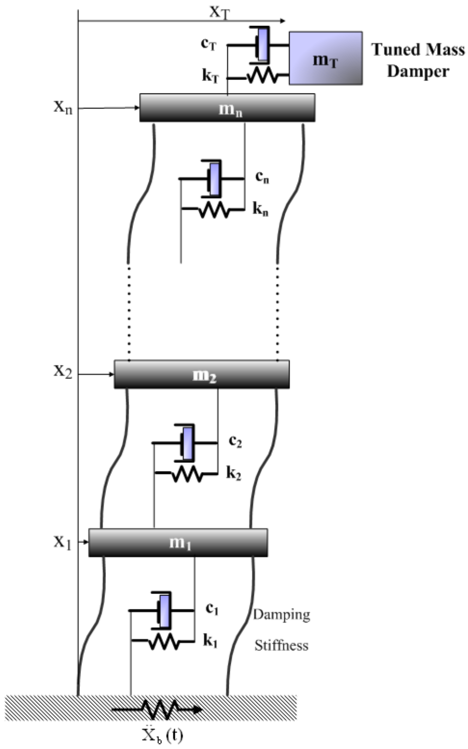

Figure 1: MDOF system equipped with a TMD.

3 NUMERICAL APPLICATION

In this section the described multi-objective optimization is applied to a10-storey building equipped at the top floor with a TMD. The mechanical properties of the structure are given in Table 1, while the setup parameters used in the analysis are reported in Table 2. The population size is 500, which allows to obtain a continuum Pareto front. The maximum iteration number is 100. The seismic model parameters are summarized in Table 3.

	Mode/level									
	1	2	3	4	5	6	7	8	9	10
T_i (*sec*)	1.3444	0.5064	0.3127	0.2299	0.1855	0.1584	0.1396	0.1245	0.1113	0.0987
$k_i(Nm)$	4.5 10^8	4.1 10^8	3.8 10^8	3.5 10^8	3.2 10^8	2.8 10^8	2.5 10^8	2.2 10^8	1.8 10^8	1.5 10^8
c_i (*Nm/sec*)	1.25 10^6	1.2 10^6	1.1 10^6	1.1 10^6	1.0 10^6	0.9 10^6	0.9 10^6	0.8 10^6	0.8 10^6	0.7$10^6$
$m_i(kg)$	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5	3.5 10^5

Table 1: Main structure mechanical characteristics.

Figure 2 shows the variability of OF_1 and OF_2 in the space of the frequency ratio $\rho_\omega = \omega_T / \omega_1$ (i.e. the TMD frequency over the main system fundamental frequency) and the TMD damping ratio ξ_T. By comparing these surfaces an opposite behaviour can be noted: as

OF_1 decreases, until a minimum point (this corresponds to the best TMD performance), OF_2 monotonically increases. This consideration confirms the impossibility to achieve the multiple OF minimization in an absolute sense, both in terms of structural performance, minimizing the absolute acceleration, and of protection cost minimization.

In Figure 3, the optimum design variables and the contour lines of the OF surfaces are plotted overlapped, in order to more clearly underline the location of the Pareto solutions with respect to the single optimization one.

Figure 4 shows the Pareto front (b) and the corresponding optimum DV components ξ_T^{opt} and ρ_ω^{opt} (a). In particular Figure 4b points out an asymptotic limit, which simultaneously corresponds to the highest effectiveness of the TMD and the highest cost $I_{C_T^1}^{OPT}$. This means that the advantage in terms of increase of TMD performance is negligible in comparison with the cost growing.

Input data for GA	
Maximum generation	500
Population size	100
Crossover probability	0.9
Mutation probability	0.1

Table 2: Input data for GA.

ω_f	ξ_f	S_0	μ	α_C
$9\pi rad/\sec$	0.4	$300\ \text{cm}^2/\sec^3$	2%	50

Table 3: Seismic and input parameters. [According to the non-stationary Kanai Tajimi process adopted to model the seismic action, ω_f is the filter frequency, ξ_f is the filter damping, S_0 is power spectral density intensity of the-white noise excitation at the bed rock]

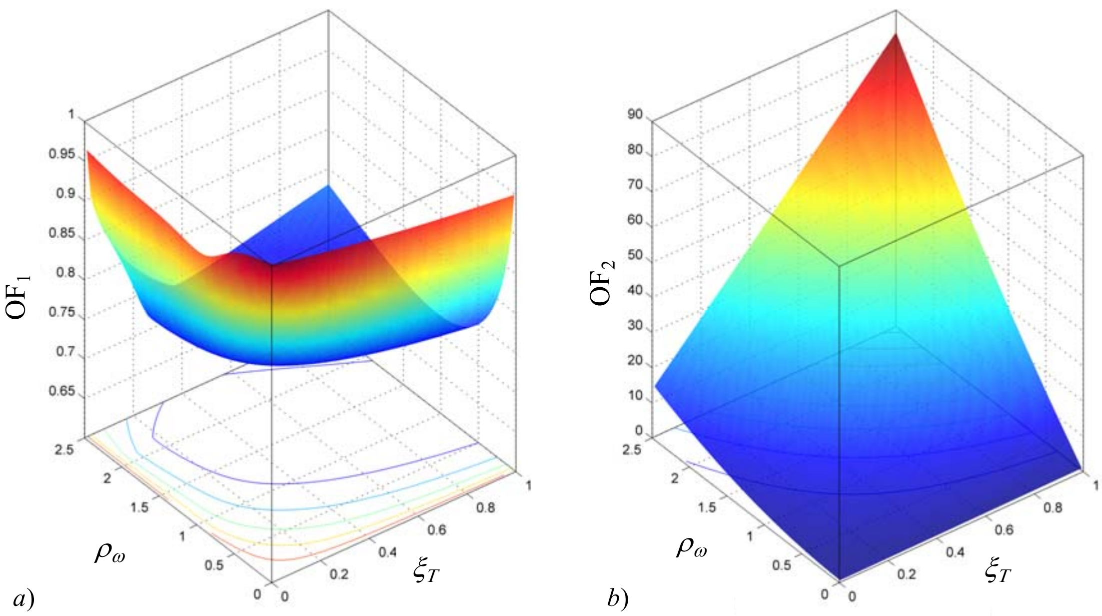

Figure 2: OF_1 (a) and OF_2 (b) versus ξ_T and ρ_ω.

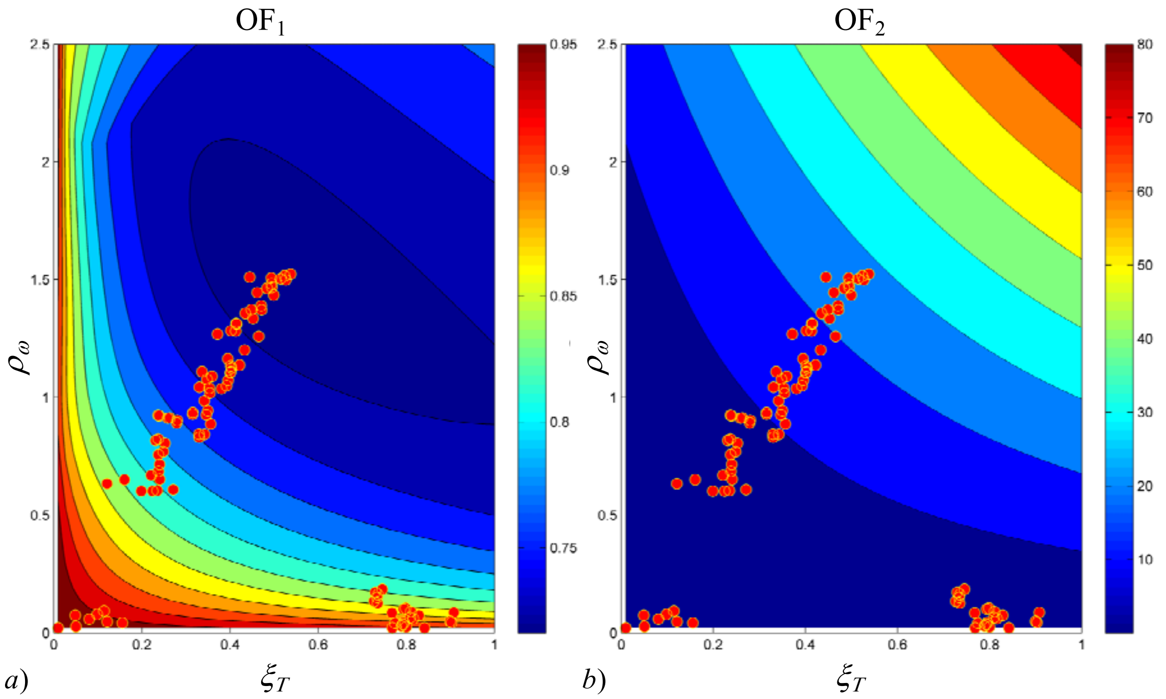

Figure 3: OF_1 (a), OF_2 (b), and Pareto solutions plotted in the space state domain.

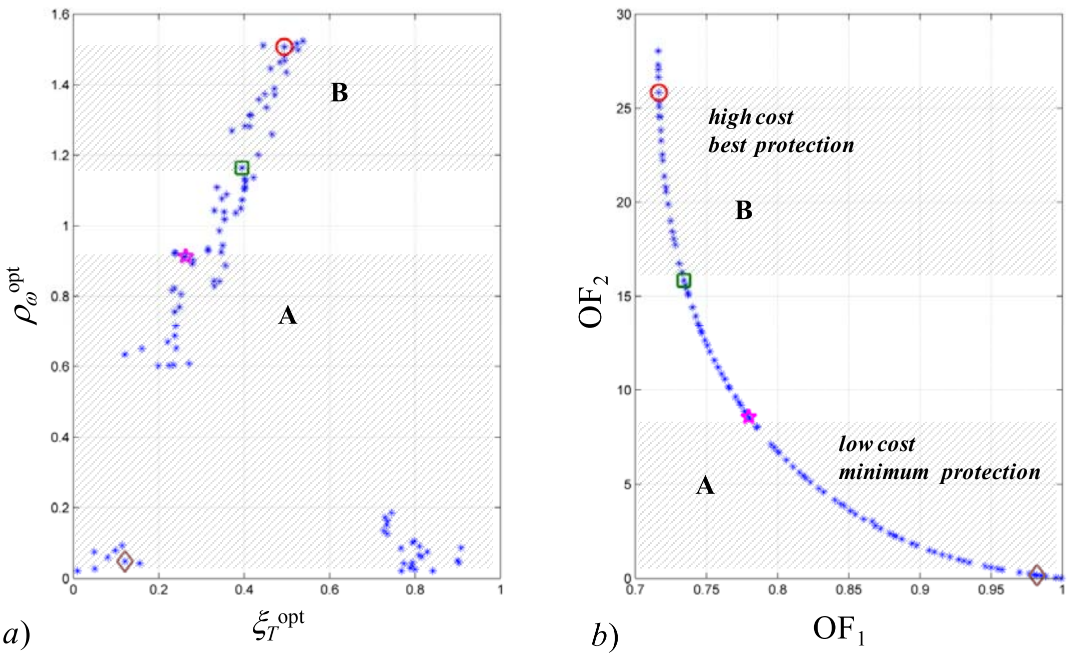

Figure 4: Optimal TMD parameters (a) and Pareto front (b).

By analysing the Pareto front and the optimum design variables, two different strategies can be distinguished at the aim to optimize the TMD (Table 4).

The first strategy corresponds to the Pareto front region between the pink star and the brown rhombus (*region A*) concerning low-medium TMD costs. In this region, the TMD increases the vibration protection efficiency essentially working by means of the tuned frequency. Starting from a low TMD cost (the lowest protection level represented by the rhombus symbol) and moving towards higher costs, the optimum TMD frequency ratio increases obtaining higher protection levels, whereas the TMD optimal damping ratio is almost constant and equal to its minimum value, as clearly indicated by the location of the optimal solutions in Figure 4 (a).

The second strategy deals with *region B* (the front portion between the green square and the red circle) characterized by high-medium costs; in this portion, the reduction of vibration levels in the main structure can be achieved by tuning the TMD frequency to the fundamental structural one (as indicated in Fig 4 (a), ρ_ω is about 0.9 at the level of the pink star). This tendency demonstrates that the best structural protection performance (indicated by the Red Circle) can be achieved by applying tuned masses with high tuned damping and frequency larger than the system one. This gain in terms of protection efficiency is paid by a very high increasing of cost.

Front portion	*Kind of strategy*	*Tuned efficiency*
A	*Strategy 1* Based on increasing the tuning optimum frequency	High protection increasing in comparison to cost increasing *Economically convenient strategy*
B	*Strategy 2* Based on increasing the tuning optimum damping	High cost increasing in comparison to protection increasing *Economically not convenient strategy*

Table 4: Main trends of extreme data.

4 CONCLUSIONS

In this work, a multi-objective optimum criterion for TDM device has been proposed, considering both economic and performance indices. The ratio between the protected and the unprotected system absolute accelerations has been considered as a first OF. A further objective, related to the cost of the protection system, has been introduced. These two OFs are antithetic, and therefore the NSGA-II has been performed to achieve the optimum Pareto solutions. Pareto fronts show that two different strategies can be distinguished at the aim to optimize the TMD: *i*) a first strategy based on the increase of the TMD frequency. This strategy leads to a high TMD performance in comparison with the cost, and therefore this strategy is economically convenient; *ii*) a second strategy based on increasing the TMD damping. In this case, the increase of the cost is high in comparison with the protection one. This strategy is economically not convenient.

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