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REDUCTION OF SEISMIC LOADING ON STRUCTURES INDUCED BY PILES IN INHOMOGENEOUS SOIL

Emmanouil Rovithis[1], Raffaele Di Laora[2], Maria Iovino[3] and Luca de Sanctis[3]

[1] Institute of Engineering Seismology and Earthquake Engineering EPPO-ITSAK
Thessaloniki, Greece
e-mail: rovithis@itsak.gr

[2] Università degli Studi della Campania "Luigi Vanvitelli"
Aversa (CE), Italy
raffaele.dilaora@unicampania.it

[3] Università degli Studi di Napoli "Parthenope"
Napoli, Italy
maria.iovino@uniparthenope.it, luca.desanctis@uniparthenope.it

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Abstract. *The kinematic response of flexible piles in inhomogeneous soil is explored under harmonic and seismic excitation. The system under investigation consists of a long fixed-head pile embedded in viscoelastic soil with stiffness varying continuously with depth. A generalized power-law function is employed to describe the variable soil stiffness. The problem is treated numerically by means of a Beam-on-Dynamic-Winkler-Foundation (BDWF) model based on a layer transfer-matrix (Haskel-Thompson) formulation. This study aims at: (a) investigating numerical and modeling aspects related to Winkler analyses of soil-pile kinematic interaction in a non-homogeneous soil; (b) exploring soil-pile kinematic interaction as affected by varying subsoil conditions under harmonic oscillations; (c) providing a novel closed-form expression for the kinematic interaction factor of pile-head over free-field response as a function of a single dimensionless frequency controlling the physical phenomenon and (d) elucidating the beneficial role of piles in the reduction of seismic loading imposed on pile-supported structures in terms of spectral acceleration for different pile-soil configurations and earthquake motions. Results show that: (a) large-diameter piles in soils with very low stiffness at shallow depths may substantially reduce the seismic acceleration imposed on structures and (b) the role of pile diameter becomes less important for strongly inhomogeneous soils. The latter indicates that even small-diameter piles may induce substantial filtering of ground motion if embedded in soft clays, which may be of importance in pile design practice.*

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1 INTRODUCTION

Seismic analysis of structures supported by piles is usually performed by taking the free-field motion as the base excitation, i.e. neglecting pile-soil kinematic interaction effect. By contrast, piles may change substantially the free-field motion and, thus, the seismic action on structures. The accumulated theoretical and experimental evidence demonstrates that kinematic interaction usually reduces the seismic demand at the base of the structure compared to that associated with the free-field condition (Tajimi, 1977; Kawamura, 1977; Otha et al., 1980; Gazetas 1984; Di Laora & de Sanctis, 2013; Rovithis et al. 2015; Bilotta et al. 2015). This reduction, referred to as 'filtering effect', may be important for structures on soft soils, where piles represent the most common design option to avoid a bearing capacity failure and/or excessive settlements (Randolph; 1994; Russo et al. 2004). Di Laora & de Sanctis (2013) examined the problem of the filtering mechanism on a rational basis, through analytical and numerical tools. With reference to homogeneous or two-layered soils, the above study proposed ready-to-use formulae of a reduction factor for design spectra based on fundamental pile and soil properties. On the other hand, Towhata (1996), based on a detailed in-situ investigation of dynamic properties of soft deposits, demonstrated that shear wave propagation velocity may vary continuously with depth even for complex stratifications involving different soil materials. The problem of kinematic interaction for piles embedded in a soil with soil stiffness varying with depth has been investigated in a number of publications. Kaynia & Kausel (1991), as an example, showed that piles in inhomogeneous media filter to a greater extent the high frequency components of the seismic motion compared to a homogeneous medium. This observation was also verified by Fan et al. (1991) as part of a parametric FE study on kinematic pile response in homogeneous and inhomogeneous media. Recently, Rovithis et al. (2015) examined the effect of soil-inhomogeneity on the kinematic response of fixed-head piles embedded in a Gibson-type soil over a rigid base. Results undertaken by a rigorous approach showed that the filtering effect in this case may be much larger compared to the homogeneous case. The above finding triggered a further investigation on the role of soil inhomogeneity in the seismic performance of structures founded on piles that is presented herein. Specifically, this article deals with the problem of the Foundation Input Motion at the base of pile-supported structures under the assumption of soil shear stiffness varying continuously with depth, according to a generalized power law function, but remaining constant in horizontal planes in the far field. The problem is tackled numerically by means of a Beam on Dynamic Winkler Foundation (BDWF) model in conjunction with a layer transfer-matrix approach known as the Haskell-Thompson (Thomson 1950) technique. Pile-head to free-field kinematic interaction coefficients are derived under harmonic base excitation for a variety of subsoil conditions. A novel closed-form expression for the kinematic interaction factor of pile-head over free-filed is proposed as a function of a single dimensionless frequency governing the kinematic response of piles in the frequency domain. Finally, kinematic pile response is examined in time domain to highlight the beneficial effect of piles embedded into inhomogeneous soil in reducing spectral accelerations.

2 PROBLEM DEFINITION

The system under consideration consists of a fixed-head pile embedded in a continuously inhomogeneous viscoelastic soil layer resting on a rigid base (Figure 1). The pile is modeled as a linearly elastic cylindrical solid beam of diameter d, length L, Young modulus of Elasticity E_p and mass density ρ_p. Soil mass density, ρ_s, Poisson's ratio, ν_s, and hysteretic damping ratio, β_s, are considered constant with depth, whereas shear modulus $G_s(z)$ is assumed to increase according to the generalized power law function:

$$G_s(z) = G_{sd}\left[a + (1-a)\frac{z}{d}\right]^n \quad (1)$$

where $a = (G_{so} / G_{sd})^{1/n}$ and *n* are dimensionless inhomogeneity factors, G_{so} is the shear modulus of soil at ground surface (z = 0) and G_{sd} refers to the shear modulus of soil at the depth of one pile diameter (z = d). Naturally, for values of the inhomogeneity factor *n* close to zero or G_{so}/G_{sd} ratio close to 1, Eq. 1 describes a homogeneous medium (i.e. $G_{so} = G_{sd}$), whereas for *n*=1 a Gibson-type soil (Gibson 1974) that is representative of soft to moderately overconsolidated cohesive soils may be modeled. The soil-pile system is subjected to S-waves propagating vertically.

To account for stiffness degradation of soil with increasing shear strain, the value of the shear modulus was reduced to 1/3 of the corresponding low-strain value, as suggested by EC8 (CEN 2003) for strong seismic shaking. Therefore, the shear modulus (G_{sd}) at one pile diameter in the above equation can be calculated through the relationship involving the low-strain shear modulus at one-diameter depth G_{sdo} (= $3G_{sd}$), a, n, ρ_s and $V_{s,30}$:

$$V_{s,30} = \sqrt{\frac{3G_{sd}}{\rho_s}}\frac{30}{d}\frac{(a-1)(n-2)}{2\left\{-a^{1-n/2} + \left[a + (1-a)\frac{30}{d}\right]^{1-n/2}\right\}} \quad (2)$$

In order to investigate pile filtering effect, different soil-pile configurations were analyzed by considering a pile diameter (d) of 1 m or 1.5 m, whereas the average shear wave velocity V_{s30} was set equal to 100 and 200 m/s, thus corresponding to soil type D and C according to EC8. The above $V_{s,30}$ values were considered for four subsoil conditions referring to a constant (n = 0), a linear (a = 0.5, n = 1), a parabolic (a = 0, n = 0.5) and a proportional (a = 0, n = 1) distribution of soil stiffness with depth. In this manner, a set of sixteen soil-pile configurations (Table 1) were examined in the realm of elastodynamic considerations. In all cases, the pile's length (L) and Young's modulus (E_p) were set equal to 20 m and 30GPa, respectively. The soil density (ρ_s) and Poisson ratio (ν_s) of soil were considered equal to 2t/m^3 and 0.3, respectively, while β_s=10%.

3 HARMONIC RESPONSE

3.1 Homogeneous soil

With reference to a BDWF model of the soil-pile system, the ratio I_u between the acceleration (or the absolute displacement) atop a fixed-head infinitely long pile, a_p, and that at the surface of a homogeneous halfspace in free-field conditions, a_{ff}, is (Flores-Berrones & Whitman, 1982):

$$I_u = \frac{a_p}{a_{ff}} = \frac{k_x + i\omega c_x}{E_p I_p (k^4 + 4\lambda^4)} \quad (3)$$

where (k_x+iωc_x) is the complex-valued dynamic impedance of the Winkler medium, E_pI_p is the pile's flexural stiffness, k (= ω/V_s) is the soil wavenumber and λ is the Winkler wavenumber that is given by the well-known formula:

$$\lambda = \left[\frac{k_x + i\omega c_x}{4E_p I_p}\right]^{1/4} \quad (4)$$

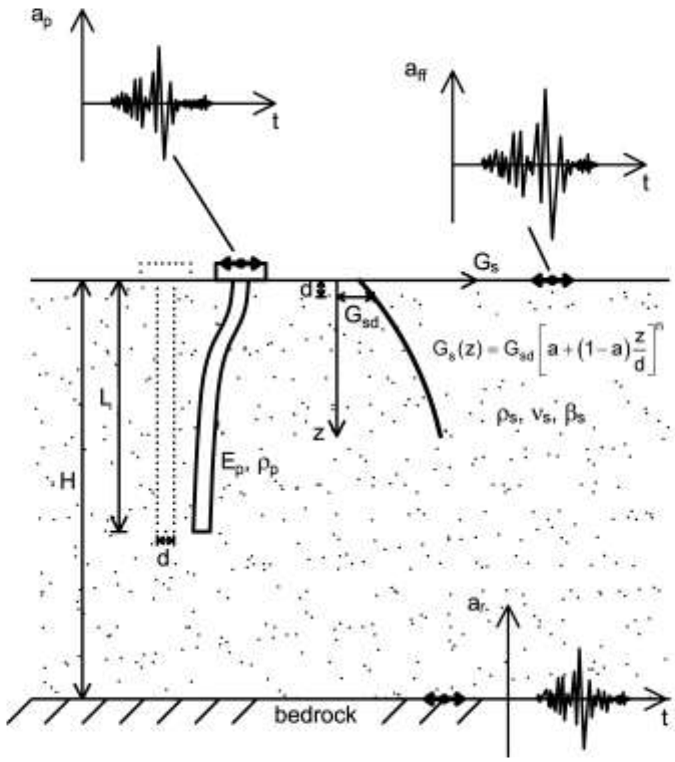

Figure 1: Single elastic fixed-head pile in a continuously inhomogeneous layer over rigid rock.

Case	Soil profile	Inhomogeneity factors		$V_{s,30}$	d
		a	n	[m/s]	[m]
1	Homogeneous	0.0	0.0		
2	Linear	0.5	1.0	100	
3	Parabolic	0.0	0.5		
4	Proportional	0.0	1.0		1.0
5	Homogeneous	0.0	0.0		
6	Linear	0.5	1.0	200	
7	Parabolic	0.0	0.5		
8	Proportional	0.0	1.0		
9	Homogeneous	0.0	0.0		
10	Linear	0.5	1.0	100	
11	Parabolic	0.0	0.5		
12	Proportional	0.0	1.0		1.5
13	Homogeneous	0.0	0.0		
14	Linear	0.5	1.0	200	
15	Parabolic	0.0	0.5		
16	Proportional	0.0	1.0		

Table 1: Soil-pile configurations under investigation

Soil material damping can be incorporated in the solution by using the standard substitution $V_s \rightarrow V_s^* \cong V_s\,(1 + i\xi)$. Pertinent expressions for k_x and c_x have been reported in many scientific contributions, among which Roesset (1980), Dobry et al. (1982), Gazetas & Dobry (1984), Makris & Gazetas (1992). It is noted that k_x referring to the stiffness of the Winkler springs is given by the Young's modulus of soil (E_s) times a proportionality coefficient δ [i.e. $k_x = \delta E_s$] (Roesset 1980, Dobry et al. 1982, Gazetas and Dobry 1984). The effect of δ is discussed in the sequel as part of a sensitivity analysis. Comprehensive reviews of literature expressions for stiffness and damping coefficients may be found in the recent works by Anoyatis & Lemnitzer (2016) and Karatzia & Mylonakis (2016b).

Anoyatis et al. (2013) showed that the kinematic interaction factor I_u of a flexible fixed-head pile in a homogeneous layer, with shear wave velocity V_s, is controlled by the single dimensionless frequency $a_{o\lambda}$ ($= \omega/\lambda V_s$):

$$I_u = \left[1 + \frac{1}{4} a_{o\lambda}^4\right]^{-1} \tag{5}$$

The above study also showed that if the "static" expression of the wavenumber λ (i.e. disregarding the imaginary part of Eq.4 related to hysteretic and radiation damping) is adopted, thus resulting in a real-valued $a_{o\lambda}$, the analytical solutions are in closer agreement with rigorous Finite Element (FE) analyses.

Eq.5 offers valuable insight in the physics of the soil-pile kinematic interaction phenomenon. Indeed, it is easy to verify that $a_{o\lambda}$ represents the ratio of the pile characteristic wavelength $1/\lambda$ over the wavelength in the soil λ_s ($=2\pi\ V_s/\omega$) and, thereby, the filtering action of piles is due to their inability to follow shorter wavelengths in the soil.

3.2 Inhomogeneous soil

For a soil with stiffness varying continuously with depth, an average wavenumber μ $[L^{-1}]$ may be defined as the average value of λ within the active length L_a of the pile (Mylonakis 1995, Rovithis et al. 2013):

$$\mu = \frac{1}{L_a}\int_0^{L_a} \lambda(z)dz \tag{6}$$

where the active pile length L_a in the case of a generalized inhomogeneous soil profile such as that adopted in this study may be calculated by the expression (Di Laora & Rovithis, 2015; Karatzia & Mylonakis, 2016a):

$$L_a = d\,\frac{1}{1-a}\left\{\left[a^{\frac{n+4}{4}} + \frac{5}{16}(n+4)(1-a)\left(\frac{\pi}{2}\frac{E_p}{E_{sd}}\right)^{\frac{1}{4}}\right]^{\frac{4}{4+n}} - a\right\} \tag{7}$$

The above solution was obtained for δ = 2. However, for typical values of pile-to-soil stiffness ratios, L_a can be taken equal to 10 pile diameters, as a first approximation. If μ and λ are treated as real-valued functions (Gazetas & Dobry 1984) and the Winkler springs modulus $k_x(z)$ is assumed to follow the same variation with depth as the soil Young's modulus $E_s(z)$ does (Mylonakis & Roumbas 2001), the solution of the integral in Eq. 6 may be expressed as (Di Laora and Rovithis 2015):

$$\mu = \frac{4\lambda_d}{d^{\frac{n}{4}} L_a (4+n)(a-1)} \left[(ad)^{\frac{4+n}{4}} - (ad + L_a - aL_a)^{\frac{4+n}{4}} \right] \tag{8}$$

where λ_d corresponds to the (static) wavenumber of a pile in a homogeneous layer with Young's modulus equal to E_{sd}:

$$\lambda_d = \left[\frac{k_d}{4E_p I_p} \right]^{1/4} \tag{9}$$

and k_d (= δE_s) refers to the spring coefficient at one pile-diameter depth related to the corresponding Young's modulus of soil (E_{sd}). It is easy to verify that μ tends to λ if the soil stiffness is constant with depth (n = 0), while for the special case of Gibson soil with zero stiffness at surface (a = 0 and n = 1), μ takes the form:

$$\mu = \frac{4}{5} \lambda_d \left(\frac{L_a}{d} \right)^{\frac{1}{4}} \tag{10}$$

Towards the identification of the key dimensionless parameter governing the kinematic response of piles in inhomogeneous soils, in analogy to homogeneous soils (where the dimensionless parameter was found analytically) a dimensionless frequency a_{eff} was introduced by Di Laora & Rovithis (2015) for describing dynamic effects in pile-head bending moments as:

$$a_{eff} = \frac{\omega}{\mu V_{s,av}} \tag{11}$$

where $V_{s,av}$ refers to an average shear wave velocity up to a depth equal to one half of the active pile length:

$$V_{s,av} = \frac{L_a / 2}{\int_0^{L_a/2} \frac{dz}{V_s(z)}} = V_{sd} \frac{(L_a / 4d)(a-1)(n-2)}{\left[a + (1-a)\frac{L_a}{2d} \right]^{1-n/2} - a^{1-n/2}} \tag{12}$$

where $V_s(z)$ is the shear wave velocity compatible with the mobilized shear modulus $G_s(z)$. For a Gibson soil with zero stiffness at surface, $V_{s,av}$ is given by:

$$V_{s,av} = \frac{V_{sd}}{2\sqrt{2}} \left(\frac{L_a}{d} \right)^{0.5} \tag{13}$$

The above frequency parameter will be used later in the paper to identify a single curve for describing pile-to-soil acceleration ratio for long piles in generalized inhomogeneous soil.

3.3 Winkler modeling issues

This section provides a comparison between the BDWF model and rigorous FE analyses carried out by the commercial software ANSYS (ANSYS 10.0). Details on the FE model can be found in Di Laora and de Sanctis (2013). The Winkler analyses were performed by employing two values of δ while considering both the complex-valued dynamic spring stiffness (resulting in a complex-valued λ according to Eq. 4) and its static counterpart. Results are plotted in Figure 2 for the soil-pile cases 5, 6, 7 and 8 reported in Table 1, referring to $V_{s,30}$ = 200m/s and d = 1m. It can be noticed that in the case of homogeneous soil, employing the static value of λ with δ = 1.2 is the best choice for assessing pile-to-soil acceleration ratio, in agreement with the findings by Anoyatis et al. (2013). For inhomogeneous soils, two issues are noteworthy: First, for $f < 3f_1$, with f_1 being the fundamental frequency of the soil, the real-valued λ still provides more accurate results, while for higher frequencies the complex value seems more appropriate; and second, the optimum value of δ depends on the excitation frequency and the subsoil conditions. Nevertheless, the exact value of δ is not critical for a reliable estimation of pile filtering effects.

On the basis of the above sensitivity analysis and upon considering that: (i) the main scope of this study is to capture the filtering effect in the time domain and (ii) the main frequency content of the free-field motions at surface, reported in the ensuing, is below $3f_1$, the static value of λ with δ = 2 was finally adopted for the BDWF analyses, as a reasonable compromise to obtain rather accurate results in all cases.

3.4 New normalization scheme

Mention has already been made to the dimensionless frequency (Eq.11), which describes pile-soil kinematic interaction under harmonic excitation in any inhomogeneous soil

Figure 3 depicts the absolute value of I_u against a_{eff} for the 16 soil-pile configurations under study. More specifically, Figure 3a refers to constant (n=0) distribution of soil stiffness with depth while Figures 3b, 3c and 3d refer to linear (a=0.5, n=1), parabolic (a=0, n=0.5) and proportional (a=0, n=1) soil stiffness variable, respectively. Each curve corresponds to a different soil-pile configuration in terms of $V_{s,30}$ and pile diameter (d).

The fact that the sixteen different curves are practically coincident indicates that the parameter μ is extremely effective in describing the combined effect of pile diameter, soil average Young's modulus and its distribution with depth (through the inhomogeneity parameters a and *n*) on kinematic pile response in inhomogeneous soils. While the above aspect is trivial for the homogeneous soil, where the interaction is controlled by the Winkler wavenumber λ, it represents an element of novelty for the generalized inhomogeneity.

Also plotted in Figure 3 is a simplified expression which can be used for practical applications, by simply substituting $a_{o\lambda}$ in Eq. (5) with a_{eff}:

$$I_u = \left[1 + \frac{1}{4} a_{eff}^4\right]^{-1} \tag{14}$$

The above expression requires a choice for the dimensionless parameter δ. Alternatively, a new parameter which does not refer necessarily to a Winkler model may be defined as:

$$a_{eff,La} = \frac{\omega L_a}{V_{s,av}} \tag{15}$$

An analogous approximate expression for I_u implementing the above parameter may, therefore, be written as:

$$I_u = \left[1 + \frac{64}{10000} a_{eff,La}^4\right]^{-1} \tag{16}$$

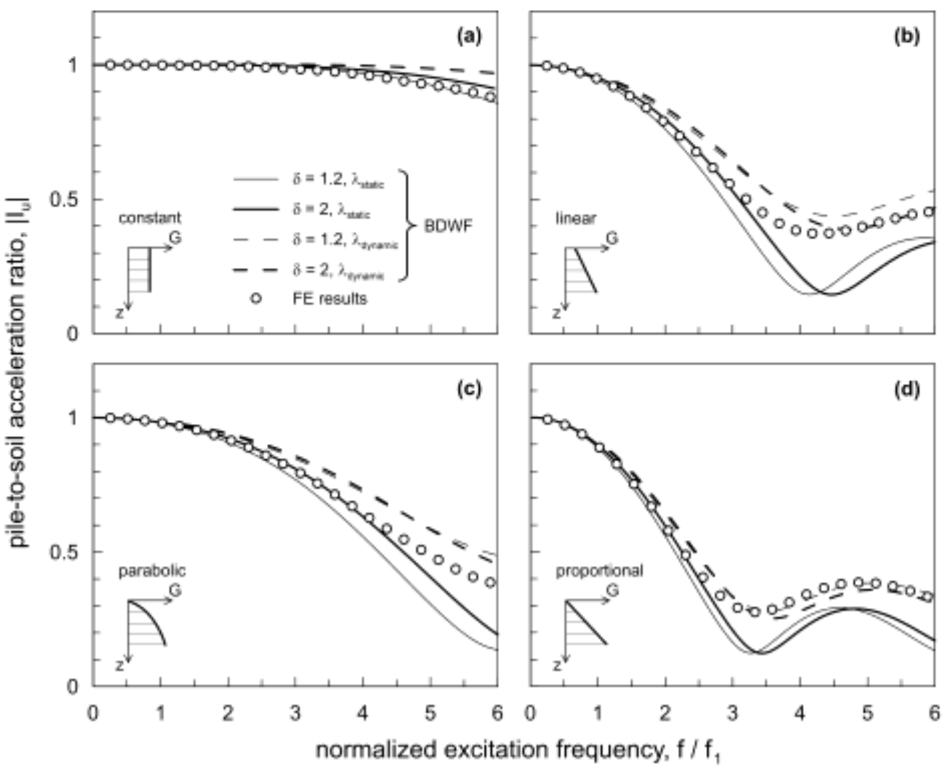

Figure 2: Effect of λ and δ on kinematic interaction factor I_u: for (a) Constant (n=0), (b) Linear (a=0.5, n=1), (c) Parabolic (a=0, n=0.5) and (d) Proportional (a=0, n=1) distribution of soil stiffness with depth. All plots refer to $V_{s,30}$ = 200 m/s and d = 1 m.

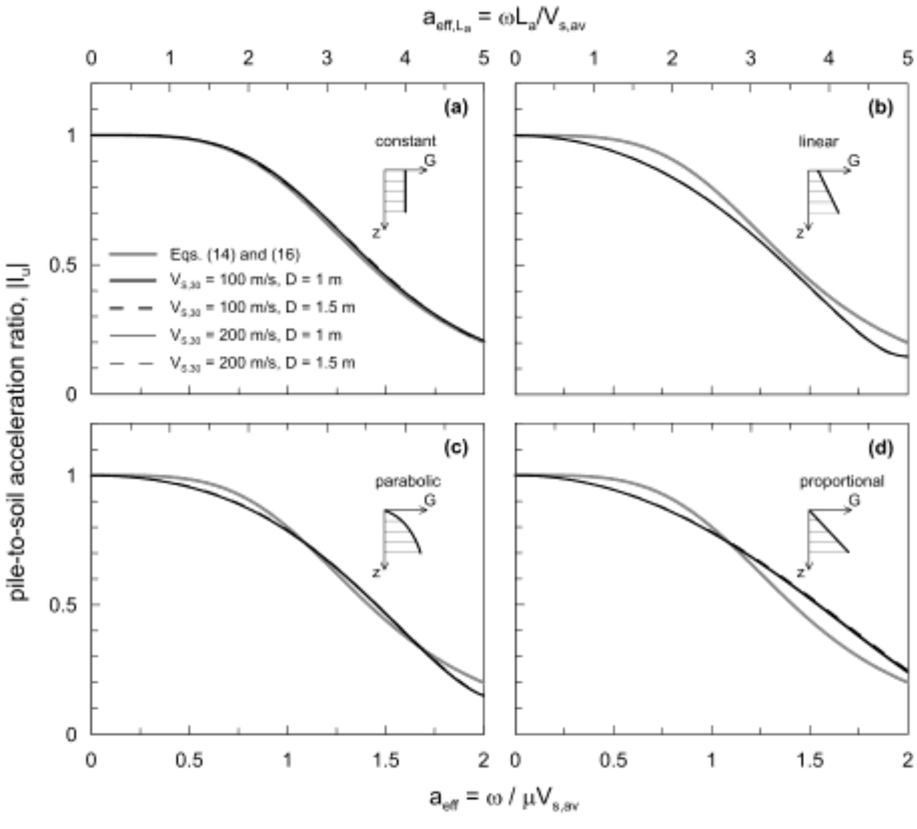

Figure 3: Pile-to-soil acceleration ratio against the dimensionless parameter a_{eff} for (a) constant (n=0), (b) linear (a=0.5, n=1), (c) parabolic (a=0, n=0.5) and (d) proportional (a=0, n=1) distribution of soil stiffness with depth.

4 PILE-INDUCED REDUCTION OF SEISMIC LOADING ON STRUCTURES

The Winkler formulation was then employed to derive acceleration spectral ratios between pile-head and free-field surface motion under seismic excitation. To this aim, a set of nine earthquake recordings were specified as input motion at the base of the soil profile. The corresponding acceleration time histories normalized by the peak acceleration amplitude at the bedrock level are plotted in Figure 4. Further details on the specific set of recordings can be found in Di Laora & de Sanctis (2013). Upon deriving the transient response at the pile-head and the free-field surface, the corresponding acceleration response spectra were computed for each one of the selected earthquake motions. Then, mean spectral acceleration ratios ξ, defined as the average response spectrum of the pile-head motion ($S_{a,p,av}$) over the average response spectrum of the free-field motion($S_{a,s,av}$) were derived for each soil-pile system. Results are synthesized in Figure 5, for the $V_{s,30}$ and pile diameter combinations reported in Table 1.

The following points are noteworthy:

(a) The pile-induced filtering effect on the seismic loading imposed on structures is more pronounced for soft soils, large-diameter piles and increasing degree of soil inhomogeneity at fixed $V_{s,30.}$

(b) For all the cases, the spectral ratio is minimized at distinct periods depending on the combination of pile diameter, soil stiffness and degree of soil inhomogeneity that characterizes each soil-pile system. However, further research is needed to identify the exact values of those periods as affected by the above model parameters and a possible interplay with the input motion.

(c) While the role of pile diameter in filtering effect is crucial for homogeneous soil, it becomes less important for increasing soil inhomogeneity.

This last point may be explained by noticing that a_{eff} (or $a_{eff,La}$) is progressively insensitive to pile diameter with increasing n. To give a physical interpretation, on one hand an increase in pile diameter increases pile characteristic length (i.e. pile is stiffer and resists more soil motion) while on the other hand the effective stiffness of soil along pile active length increases as well, decreasing filtering effect.

5 CONCLUSIONS

The kinematic response of a long fixed-head pile in viscoelastic soil with stiffness varying continuously with depth was explored numerically in both frequency and time domain by means of a Beam-on-Dynamic-Winkler-Foundation (BDWF) formulation of the soil-pile system. Following a sensitivity study on Winkler modeling aspects, the filtering action of the pile was investigated for different subsoil conditions, low-strain soil stiffness and pile diameter. With reference to the frequency domain analysis, a novel closed-form expression for the kinematic interaction factor of pile-head over free-field response was proposed as a function of a single dimensionless frequency controlling the physical phenomenon. Transient response analyses indicated the beneficial role of the pile in reducing the seismic loading imposed on structures regardless of the subsoil conditions. The above was established by means of pile-head over free-field spectral acceleration ratios for a wide range of soil-pile systems. The role of salient features involved in the physical problem was identified with possible implications in design practice, especially for piles embedded in strongly inhomogeneous soils.

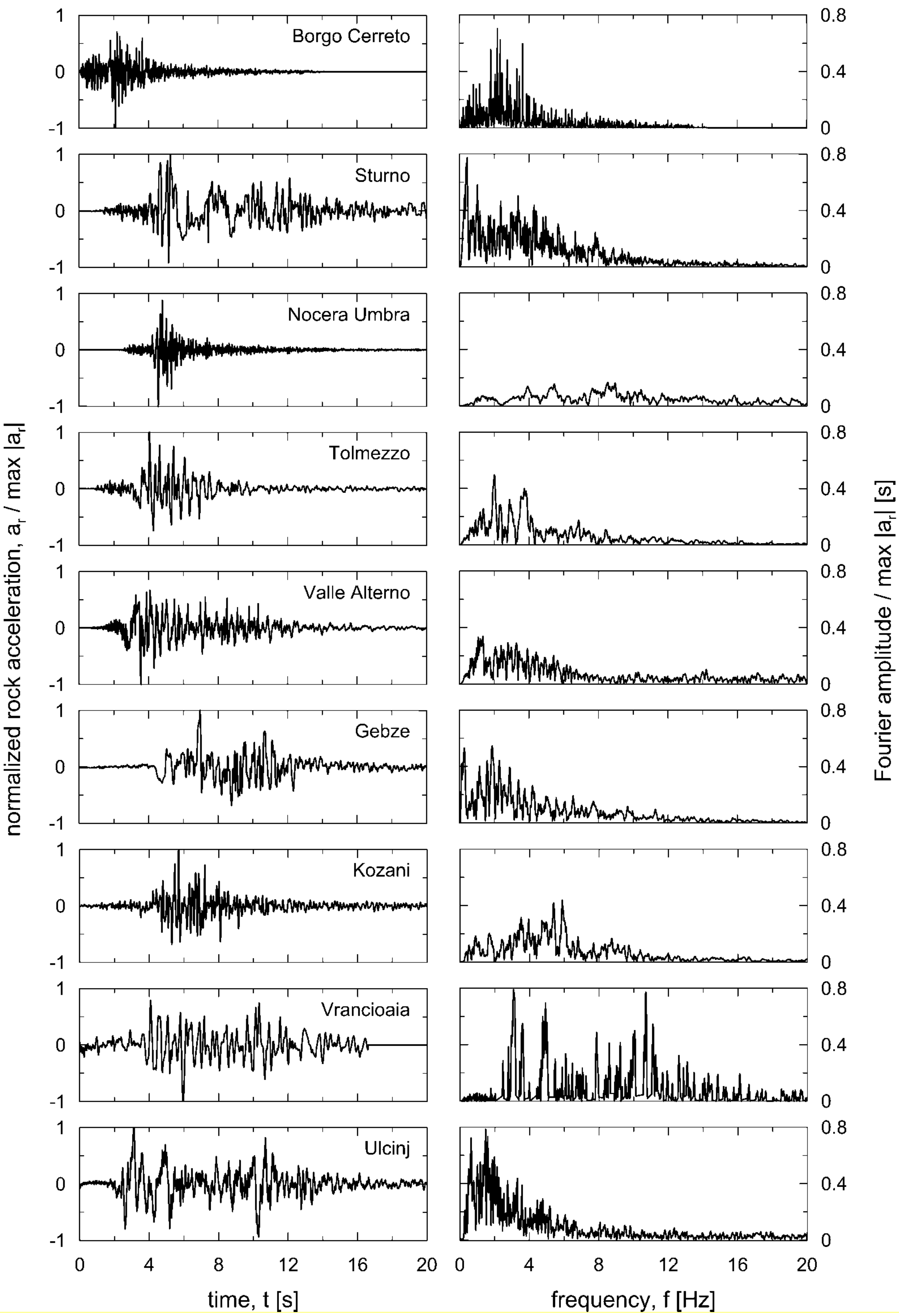

Figure 4: Earthquake records employed as bedrock signals.

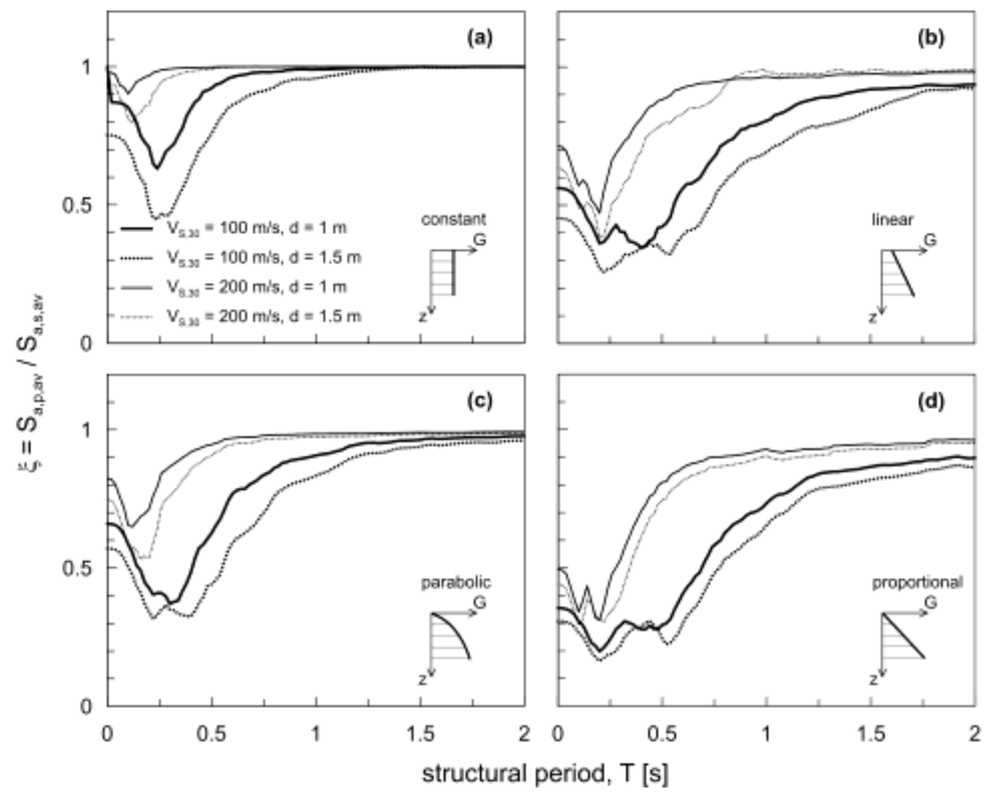

Figure 5: Mean spectral acceleration ratios defined as the average response spectrum of the pile-head motion ($S_{a,p,av}$) over the average response spectrum of the free-field motion ($S_{a,s,av,}$) for (a) constant (n=0), (b) linear (a=0.5, n=1), (c) parabolic (a=0, n=0.5) and (d) proportional (a=0, n=1) distribution of soil stiffness with depth.

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