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EXPANSION OF THE LUMPED PARAMETER METHOD TO NONLINEAR, SOIL-STRUCTURE INTERACTING DYNAMIC SYSTEMS BY MEANS OF A MULTI-OBJECTIVE OPTIMIZATION ALGORITHM

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Abstract The computational demand of the soil-structure interaction (SSI) analysis for the design and assessment of structures as well as for the evaluation of life cycle cost and risk exposure has led the civil engineering community to the development of a variety of methods towards the model order reduction of the coupled soil-structure dynamic system. Different approaches have been proposed in the past as computationally efficient alternatives to the conventional FEM simulation of the complete structure-soil domain, such as the nonlinear lumped spring, the macroelement method and the substructure partition method. Yet, with few exceptions, no approach was capable of capturing simultaneously the frequency-dependent dynamic properties along with the nonlinear behavior of the condensed segment of the overall soil-structure system, thus generating an imbalance between the modeling refinement achieved for the soil and the structure. To this end, a dual frequency- and intensity-dependent expansion of the Lumped Parameter Modeling method is proposed in the current paper, materialized through a multi-objective algorithm, capable of closely approximating the behavior of the nonlinear dynamic system of the condensed segment. The efficiency of the proposed approach is validated for the case of an existing bridge, wherein the seismic response is comparatively assessed for both the proposed method and the detailed finite element model. The above expansion is deemed a computational efficient and reliable approach for simultaneously considering the frequency and amplitude dependence of soil-foundation-superstructure effects in the framework of nonlinear response history analysis.

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1 INTRODUCTION

As has been observed in multiple occasions in the past [1], the supporting soil can play an essential role on the superstructure's behavior during a significant earthquake incident. The soil–structure interaction effect under such a dynamic hazard can lead to unexpected structural behavior, which cannot be easily predicted in advance. As a result, a realistic representation of the semi-infinite soil domain during the simulation of the soil-structure interacting system is considered an essential prerequisite to an accurate dynamic analysis.

Numerous approaches have been established in the past addressing soil structure interaction simulation from several different perspectives. The truncated soil domain finite element method is an example of a direct approach for the simulation of the aforementioned effect, where a detailed behavior of the soil-foundation system, including material and geometrical complexity, can be obtained. However, such an approach can be computationally burdening for the analysis of complex structures to such an extent, that is almost prohibitive in the framework of probabilistic assessment that requires a large number of model realizations.

In the light of the above limitations, the most common approach is to sacrifice the modeling refinement in terms of subsoil domain size and the subsequent analysis accuracy by reducing the order of the system. Primary focus is then made on refined models that are tailored to capture damage at specific structural and foundation components. Particularly for bridge structures, geometric nonlinearities that may arise from gap (i.e., joint) closure and stopper activation are also taken into consideration as they instantaneously affect the boundary conditions. Such a task can be accomplished through a partition approach of the overall dynamic system in segments, where the domain of the soil and foundation is significantly condensed on its internal degrees of freedom (DOF) or completely replaced by a simplified representation, while the superstructure is left unaltered. Numerous implementations on the substructure modeling approach have been proposed in the past among which the macroelement method tends to stand out as an effective tool capable of coping with complex constitutive laws and geometrical nonlinearities.

The latter macroelement approach, is a concept initially introduced by Montrasio et al [2] and further developed by a number of different research groups [3–5] that has successfully provided with an accurate yet low in computational effort representation of both the elastic and inelastic behavior of the soil-foundation domain. Although it emulates with great detail the different mechanisms triggered during a quasi-static excited simulation, it is common in the literature to either completely neglect or oversimplify the dynamic traits of the foundation soil domain through the use of complementary Kelvin –Voigt components. As these components are only limited to accuracy at a specific target frequency, their use can lead to significant error on the structural behavior assessment under dynamic excitation, as illustrated in [6].

A successful attempt to incorporate the frequency dependence in macroelements has been recently made by Chai et al [7]. However, as the proposed method is mainly focused on the frequency dependent traits of the system in the elastic domain, oscillations in higher intensity regions are represented by unexplored dynamic properties, thus suggesting potential misrepresentation of the soil foundation domain under specific circumstances.

To address the above limitations, a frequency-dependent macroelement method with emulated dynamic properties across various levels of increasing seismic intensity is proposed in the current study. The viscoelastic dynamic properties of the soil foundation domain along a broad frequency spectrum are successfully emulated in the time domain through the use of the Lumped Parameter Modeling method [8–11] based on the method's documented success in the elastic range, to preserve the original system's accuracy, stability and passivity.

More specific objectives of the present study are: (a) the development of the required extraction technique capable of selecting representative properties of the dynamic system along different intensity levels (b) the development of a complete procedure to reflect the aforementioned dynamic traits as a function of intensity; and (c) the numerical verification of the proposed procedure by means of a simplified single DOF supported on a shallow foundation.

2 EXTRACTION OF SYSTEM DYNAMIC PROPERTIES FOR DIFFERENT INTENSITY LEVELS

An essential step towards the expansion of the Lumped Parameter Modelling method from elastic to inelastic, frequency-dependent, dynamic systems is the derivation of representative dynamic traits of the system in different levels of excitation intensity. In the following paragraphs a dynamic trait extraction approach is presented, implementing the linearization and dynamic condensation of the soil –foundation segment of the system on different representative segment state variable combinations.

2.1 System Linearization on a selected Variables State

The mathematical representation of the superstructure – foundation – semi-infinite soil domain system can be illustrated in the classic ode formulation of equation (1) after the appropriate geometrical discretization of the original PDE equation system:

$$\mathbf{M}\cdot\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\mathbf{f}(\mathbf{u})=\mathbf{F} \tag{1}$$

The variables **M** and **C** denote the mass and damping coefficient matrices of the system, **f(u)** denotes the nonlinear force to displacement relation vector, while **F** is the external loading vector applied to the respective DOFs. It is possible to derive a linearized model of the aforementioned nonlinear dynamic system around a preselected state $\mathbf{u_o}$, through the Taylor expansion of the nonlinear returning force component **f(u)** as illustrated in equation (2). The linearized model accuracy is highly correlated with the distance of the dynamic system current variable state from the variable state $\mathbf{u_o}$.

$$\mathbf{M}\cdot\ddot{\mathbf{u}}+\mathbf{C}\dot{\mathbf{u}}+\dot{\mathbf{f}}(\mathbf{u}_o)\cdot(\mathbf{u}-\mathbf{u}_o)+\mathbf{f}(\mathbf{u}_o)=\mathbf{F} \tag{2}$$

$$\text{where } \mathbf{u}\in[\mathbf{u_o}-\boldsymbol{\varepsilon},\mathbf{u_o}+\boldsymbol{\varepsilon}]$$

The overall system can be partitioned into individual segments according to the previously presented substructure partition method, where the soil –foundation segment expected to be reduced is designated as the condensed segment. The nonlinear returning force equation terms of the condensed segment are replaced by their representative 1st order Taylor expansion corresponding to a specific variable state region $\mathbf{u}_0$=[$\mathbf{u}_0^{ss}$, $\mathbf{u}_0^{i}$, $\mathbf{u}_0^{soil}$]$^{\mathbf{T}}$.

$$\mathbf{u}=\begin{bmatrix}\mathbf{u}_{ss}\\ \mathbf{u}_i\\ \mathbf{u}_{soil}\end{bmatrix},\quad \mathbf{M}=\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{M}_{ii}^{ss}+\mathbf{M}_{ii}^{soil} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{soil}\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}\mathbf{C}_{ss,ss} & \mathbf{C}_{ss,i} & \mathbf{0}\\ \mathbf{C}_{i,ss} & \mathbf{C}_{ii}^{ss}+\mathbf{C}_{ii}^{soil} & \mathbf{C}_{i,soil}\\ \mathbf{0} & \mathbf{C}_{soil,i} & \mathbf{C}_{soil,soil}\end{bmatrix}$$

$$\mathbf{f}(\mathbf{u}) = \begin{bmatrix} \mathbf{f}_s(\mathbf{u}_{ss},\mathbf{u}_i) \\ \mathbf{f}_i^s(\mathbf{u}_{ss},\mathbf{u}_i) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dot{\mathbf{f}}_{i,i}(\mathbf{u_o}) & \dot{\mathbf{f}}_{i,soil}(\mathbf{u_o}) \\ \mathbf{0} & \dot{\mathbf{f}}_{soil,i}(\mathbf{u_o}) & \dot{\mathbf{f}}_{soil,soil}(\mathbf{u_o}) \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u}_{ss} \\ \mathbf{u}_i \\ \mathbf{u}_{ve} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{X}_i \\ \mathbf{X}_{soil} \end{bmatrix}$$

$$\mathbf{X}_i = \mathbf{f}_i^{soil}(\mathbf{u_o}) + \dot{\mathbf{f}}_{i,i}(\mathbf{u_o}) \cdot \mathbf{u}_o^i + \dot{\mathbf{f}}_{i,soil}(\mathbf{u_o}) \cdot \mathbf{u}_o^{soil}$$

$$\mathbf{X}_{soil} = \mathbf{f}_{soil}(\mathbf{u_o}) + \dot{\mathbf{f}}_{soil,i}(\mathbf{u_o}) \cdot \mathbf{u}_o^i + \dot{\mathbf{f}}_{soil,soil}(\mathbf{u_o}) \cdot \mathbf{u}_o^{soil} \quad (3)$$

The expansion of the system presented in equation (1) according to the selected segmentation and condensed segment linearization is illustrated in equation (3). The overall system's partitions are notated as *ss*, *soil*, and *i* corresponding to the superstructure segment, the soil-domain - foundation segment and the interface DOFs, respectively. The matrix system of equation (3) is expanded into the equations (4-6), where the terms corresponding to internal condensed segment DOFs are appropriately transformed in the frequency domain.

$$\mathbf{M}_{ss}\ddot{\mathbf{u}}_{ss} + \mathbf{C}_{ss,ss}\dot{\mathbf{u}}_{ss} + \mathbf{C}_{ss,i}\dot{\mathbf{u}}_i + \mathbf{f}_{s,ss}(\mathbf{u}_{ss},\mathbf{u}_i) = \mathbf{F}_s \quad (4)$$

$$\begin{aligned} &\mathbf{M}_{ii}^{ss}\ddot{\mathbf{u}}_i + \mathbf{C}_{ii}^{ss}\dot{\mathbf{u}}_i + \mathbf{f}_{s,\iota\iota}(\mathbf{u}_{ss},\mathbf{u}_i) + \mathbf{C}_{i,ss}\dot{\mathbf{u}}_{ss} + \\ &\mathscr{F}^{-1}(\mathscr{F}(\mathbf{M}_{ii}^{soil}\ddot{\mathbf{u}}_i + \mathbf{C}_{ii}^{soil}\dot{\mathbf{u}}_i + \dot{\mathbf{f}}_{i,i}(\mathbf{u}_o)\cdot\mathbf{u}_i + \mathbf{C}_{i,soil}\dot{\mathbf{u}}_{soil} + \dot{\mathbf{f}}_{soil,i}(\mathbf{u}_o)\cdot\mathbf{u}_{soil} + \mathbf{X}_i(\mathbf{u}_o))) = \mathbf{F}_i \end{aligned} \quad (5)$$

$$\begin{aligned} &\mathscr{F}(\mathbf{M}_{soil}\ddot{\mathbf{u}}_{soil} + \mathbf{C}_{soil,\mathbf{i}}\dot{\mathbf{u}}_{\mathbf{i}} + \mathbf{C}_{soil,soil}\dot{\mathbf{u}}_{soil} + \dot{\mathbf{f}}_{soil,i}(\mathbf{u}_o)\cdot\mathbf{u}_{ii} \\ &+ \dot{\mathbf{f}}_{soil,soil}(\mathbf{u}_o)\cdot\mathbf{u}_{soil} + \mathbf{X}_{soil}(\mathbf{u}_o)) = \mathbf{0} \end{aligned} \quad (6)$$

The notations $\mathscr{F}$ and $\mathscr{F}^{-1}$ denote the Fourier and the inverse Fourier transformation, respectively. Through the appropriate transformations, eq. (5) and (6) can be written as:

$$\mathbf{M}_{ii}^{ss}\ddot{\mathbf{u}}_i + \mathbf{C}_{ii}^{ss}\dot{\mathbf{u}}_i + \mathbf{C}_{i,ss}\dot{\mathbf{u}}_{ss} + \mathbf{f}_{s,\iota\iota}(\mathbf{u}_{ss},\mathbf{u}_i) + \mathscr{F}^{-1}\left(\mathbf{S}_{ii}^{soil}\mathbf{U}_i + \mathbf{S}_{i,soil}\mathbf{U}_{soil} + \mathbf{X}_i(\mathbf{u}_o)\cdot 2\pi\cdot\delta(\omega)\right) = \mathbf{0} \quad (7)$$

$$\mathbf{S}_{soil,soil}\mathbf{U}_{soil} + \mathbf{S}_{soil,i}\mathbf{U}_i + \mathbf{X}_{soil}(\mathbf{u}_o)\cdot 2\pi\cdot\delta(\omega) = 0 \quad (8)$$

Function $\delta(\omega)$ corresponds to the Dirac delta function, notation $\mathbf{S}_{ij}$ and represents the current impedance notation $-\mathbf{M}_{ij}\omega^2 + \mathbf{C}_{ij}\omega i + \dot{\mathbf{f}}_{i,j}(\mathbf{u}_o)$, while $\mathbf{U} = \mathscr{F}(\mathbf{u_j})$. The dynamic system of equation (3) can now be significantly reduced in size through the elimination of the Fourier transformed displacement vector $\mathbf{U}_{soil}$ as follows:

$$\mathbf{M}\cdot\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{f}(\mathbf{u}_o) + \mathbf{P} = \mathbf{F}$$

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_{ss} \\ \mathbf{u}_i \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ii}^{ss} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{ss,ss} & \mathbf{C}_{ss,i} \\ \mathbf{C}_{i,ss} & \mathbf{C}_{ii}^{ss} \end{bmatrix}$$

$$\mathbf{f}(\mathbf{u})=\begin{bmatrix}\mathbf{f}_{s,ss}(\mathbf{u}_{ss},\mathbf{u}_i)\\ \mathbf{f}_{s,tt}(\mathbf{u}_{ss},\mathbf{u}_i)\end{bmatrix}+\begin{bmatrix}\mathbf{0}\\ \mathscr{F}^{-1}\left(\mathbf{S}_{ve,sg}\right)*\mathbf{u}_i\end{bmatrix},\quad \mathbf{P}=\begin{bmatrix}\mathbf{0}\\ \mathscr{F}^{-1}\left(\mathbf{RF}\cdot\sqrt{2\pi}\cdot\delta(\omega)\right)\end{bmatrix}$$

$$\mathbf{RF}=\mathbf{S}_{i,soil}\cdot\mathbf{S}_{soil,soil}^{-1}\cdot\mathbf{X}_{soil}(\mathbf{u}_o)+\mathbf{X}_i(\mathbf{u}_o)$$

$$\mathbf{S}_{reduced}=\mathbf{S}_{i,i}^{soil}(\mathbf{u}_o)-\mathbf{S}_{i,soil}(\mathbf{u}_o)\cdot\mathbf{S}_{soil,soil}^{-1}(\mathbf{u}_o)\cdot\mathbf{S}_{soil,i}(\mathbf{u}_o)\tag{9}$$

The extracted, reduced, complex matrices $\mathbf{RF}(\mathbf{u_o})$ and $\mathbf{S}_{\mathrm{reduced}}(\mathbf{u_o})$ are capable of accurately emulating the local dynamic behavior traits of the foundation-soil condensed segment for small trajectories of the segment DOFs near the linearization state space $\mathbf{u}_o$.

2.1 Truncated selection of condensed system

Due to the non-finite number of different state variable sets of the soil-foundation segment, it is numerically impossible to retrieve the condensed complex matrix pairs **RF** and $\mathbf{S}_{\mathrm{reduced}}$ for each possible combination of state variables. As a result, a selection of representative state variable sets in connection to the behavior of the interface DOFs is essential. An effective yet computationally non-burdening truncation on the possible state variable combinations can be accomplished through the use of the static pattern of deformation of the condensed segment corresponding to a specific configuration of the interface DOFs state-space $\mathbf{u_i}$. The current stiffness $\dot{\mathbf{f}}_{soil,i}, \dot{\mathbf{f}}_{soil,soil}$ matrices and residual force $\mathbf{f}_i^{soil}, \mathbf{f}_{soil}$ vectors corresponding to a specific interface DOF configuration are calculated from the condensed segment static equation system under the externally imposed displacement values of $\mathbf{u_i}$. The process is repeated for a discrete number of state variable sets $\mathbf{u_i}$ within the expected domain of displacement for the interface DOFs.

3. INELASTIC LUMPED PARAMETER MODEL

Since the inelastic dynamic behavior of the condensed soil-foundation segment can now be extracted in the form of a preselected number of (reduced) complex matrix doublets **RF** and $\mathbf{S}_{\mathrm{reduced}}$, it is possible to develop a modified LP model assembly that is capable to emulate such inelastic behavior. The predefined Type 3 LP design, proposed by Saitoh [12], is selected as the basis of the newly developed inelastic LP model. The proposed LP model for the scenario of the simplest case of a single interface DOF is illustrated in figure (1).

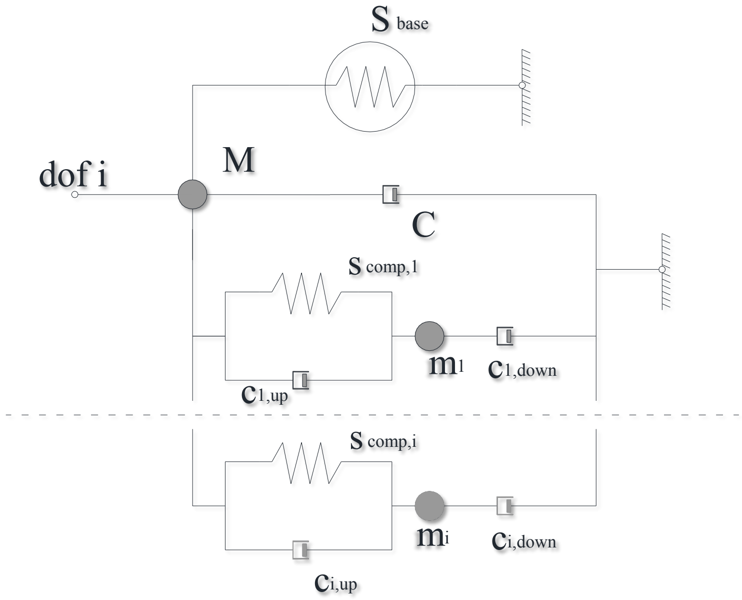

Figure 1. Physical representation of the proposed Lumped Parameter Model

As shown in the Figure, the LP model consists of one *inelastic base* component S_b capable of emulating the quasi-static properties of the condensed soil-foundation segment, while additional complementary components are accordingly calibrated to match the intensity-dependent dynamic traits of the soil-foundation system. The base component can be utilized by standard elastoplastic or hypoplastic macroelements proposed in the literature, depending on the specific foundation. In regard to the complementary components, conventional solutions are not capable of providing the sufficient accuracy during the LP model calibration process, as the assembly is now expected to emulate the dynamic behavior of the targeted system in different levels of intensity. To this end inelastic externally controlled components $s_{comp,i}$ are integrated in the LP model as presented in the following paragraphs, while the components $c_{i,up}$, $c_{i,down}$ and m_i follow the conventional definition of dashpot and mass components respectively.

3.1 Derivation of externally controlled inelastic components

The $s_{comp,i}$ components of the proposed LP assembly follow the multi-linear inelastic constitutive law illustrated in eq. (10) and their current stiffness is associated with variables representing the plastic state of the base component.

$$\dot{F}_{comp} = k_{comp}\left(\mathbf{a}_{ex}\right)\cdot \dot{u}$$

$$where \quad k_{comp}(\mathbf{a}_{ex}) = \begin{cases} k_o & if\ \mathbf{a}_{ex} < \mathbf{c}_o \ or \ \ Unloading \\ \vdots & \\ k_n & if \quad \mathbf{c}_{n-1} < \mathbf{a}_{ex} < \mathbf{c}_n \end{cases} \tag{10}$$

The variable Vector $\mathbf{a}_{ex}$ corresponds to the base component S_b parameters at the current state of the system, $\mathbf{c}_i$ are constant value vectors controlling the regions for which $\mathbf{a}_{ex}$ corresponds to different spring stiffness k_i, while u is the change rate of the component's displacement.

The selection of an externally controlled component lies within the nature of the extracted dynamic traits. As presented in the previous sections, each extracted doublet of **RF** and $\mathbf{S}_{\text{reduced}}$ has been affiliated with a specific combination of interface DOF displacements. The interface DOF controlled components within the LP model assembly possess an independent stiffness value corresponding to each interface displacement combination. As a result, the calibration process can be significantly more efficient in comparison to the use of conventional multi-linear inelastic springs. The components are numerically implemented to the LP assembly through the explicit Runge-Kutta iterative method.

3.2 LP model Impedance Functions

Through the definition of the base and complementary components of the assembly it is now possible to generate the Matrix Expression of the proposed LP model in regard to a single DOF interface, as illustrated in eq. (11).

$$\mathbf{f}_{LP}(\mathbf{u}) = \begin{bmatrix} f_{base}(\mathbf{u}) + f_{comp,1}(\mathbf{u}) \cdots + f_{comp,N}(\mathbf{u}) \\ f_{comp,1}(\mathbf{u}) \\ \vdots \\ f_{comp,N}(\mathbf{u}) \end{bmatrix} \qquad \mathbf{M}_{LP} = \begin{bmatrix} M & 0 & \dots & 0 \\ 0 & m_1 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & m_N \end{bmatrix}$$

$$\mathbf{C}_{LP} = \begin{bmatrix} C + c_{up,1} \dots + c_{up,N} & -c_{up,1} & \dots & -c_{up,N} \\ -c_{up,1} & c_{up,1} + c_{down,1} & & \vdots \\ \vdots & & \ddots & 0 \\ -c_{up,N} & \dots & 0 & c_{up,N} + c_{down,N} \end{bmatrix} \tag{11}$$

The variable vector **u** corresponds to the displacement of the LP model's DOFs, $\mathbf{M}_{\text{LP}}$ represents the mass matrix of the LP model, $\mathbf{C}_{\text{LP}}$ represents the damping matrix of the LP model while $\mathbf{f}_{\text{LP}}$ denotes the nonlinear force to displacement relation vector.

$$S_{LP}^{j} = K^{j} - M \cdot \omega^{2} + C \cdot \omega i + \sum_{i=1}^{N} \left(\frac{(k_{comp,i}^{j} + c_{up,i} \cdot \omega i)(-m_i \cdot \omega^{2} + c_{down,i} \cdot \omega i)}{k_{comp,i}^{j} + c_{up,i} \cdot \omega i - m_i \cdot \omega^{2} + c_{down,i} \cdot \omega i} \right) \tag{12}$$

$$RF_{LP}^{j} = X + \sum_{i=1}^{N} \left(\frac{(k_{comp,i}^{j} + c_{up,i} \cdot \omega i) \cdot x_i}{k_{comp,i}^{j} + c_{up,i} \cdot \omega i - m_i \cdot \omega^{2} + c_{down,i} \cdot \omega i} \right)$$

$$where: X = f_{base}(\mathbf{u}^{j}) + (K^{j}) \cdot u_{base}^{j} + \sum_{i=1} \left(f_{comp,i}(\mathbf{u}^{j}) - k_{comp,i}^{j} \cdot u_i^{j} \right)$$

$$x_i = f_{comp,i}(\mathbf{u}^{j}) - k_{comp,i}^{j} \cdot u_{base}^{j} + k_{comp,i}^{j} \cdot u_i^{j} \tag{13}$$

In a similar manner to the previously presented linearization process of the soil foundation segment, it is possible to derive a linearized model of the LP dynamic system around a preselected state $\mathbf{u}^j$, through the Taylor expansion of the nonlinear returning force vector $\mathbf{f}_{LP}$. Through the affiliation of the interface DOF displacements to a variable state combination of the LP model, it is possible to extract the model's $\mathbf{S}_{LP}$ and $\mathbf{RF}_{LP}$ matrix doublets, representing the dynamic behavior of the LP model in accordance to a specific interface DOF displacement combination. The impedance and returning force functions S_{LP} and RF_{LP} for the single interface DOF LP model are illustrated in equations (12-13).

4. PROPOSED MODEL CALLIBRATION

4.1 Model order reduction through LP model calibration

The model order reduction approach selected for the current study is materialized through the derivation of a reduced model capable of approximating the behavior of the targeted complex system. It is possible to formulate the model order reduction approach in the optimization problem illustrated in equation (12).

$$\begin{aligned} &\min_{S_r} f(M, M_r) \\ &\textit{subject to } \; g(M, M_r) \le 0 \\ &\qquad\qquad h(M, M_r) = 0 \end{aligned} \tag{14}$$

The variable M represents the targeting dynamic system ODE formulation, M_r is the reduced order system possessing a lower number of state variables in comparison to M, function f specifies the behavioral similarities between the targeted and reduced system, while functions g and h are inequality and equality constraints of the optimization problem, imposing attractor behavior and maintaining specific structure properties of the system M.

The optimization problem of (eq. 14) can be reformulated on the specific scenario of the SSI system order reduction, through the appropriate derivations of the previously presented objective function and imposed constraint terms. In that case, the soil foundation segment of the overall system is significantly condensed through the use of the inelastic LP model presented in the previous section as an adequate replacement. The order reduction problem takes the following multi objective optimization form as illustrated in equations (eq. 15-16).

$$\begin{aligned} &\min_{\mathbf{x}} \; (f_1(x), \ldots f_N(x)) \\ &\textit{subject to } \; \mathbf{x} \ge 0 \end{aligned} \tag{15}$$

$$\begin{aligned} f_j(\mathbf{x}) = &\sum_{i=1}^{N} \left[\mathrm{Re}\left(S^j_{Tar}(\omega_i)\right) - \mathrm{Re}\left(S^j_{LP}(\omega_i, \mathbf{x})\right) \right]^2 + \sum_{i=1}^{N} \left[\mathrm{Im}\left(S^j_{Tar}(\omega_i)\right) - \mathrm{Im}\left(S^j_{LP}(\omega_i, \mathbf{x})\right) \right]^2 \\ &+ \sum_{i=1}^{N} \left[\mathrm{Re}\left(RF^j_{Tar}(\omega_i)\right) - \mathrm{Re}\left(RF^j_{LP}(\omega_i, \mathbf{x})\right) \right]^2 + \sum_{i=1}^{N} \left[\mathrm{Im}\left(RF^j_{Tar}(\omega_i)\right) - \mathrm{Im}\left(RF^j_{LP}(\omega_i, \mathbf{x})\right) \right]^2 \end{aligned} \tag{16}$$

Index j denotes a specific interface DOF variable state, S^j_{tar} and S^j_{LP} indicates the impedance functions of the targeted and reduced LP system, respectively, on a given variables state *j*, while RF^{j}_{Tar} and RF^{j}_{LP} indicates the returning force complex functions of the targeted and reduced LP system respectively on a given variables state *j*.

4.2 Proposed algorithmic approach for the optimization process

The scalarization of the multi-objective optimization problem illustrated in equations (eq. 15-16) is selected as the most effective approach, given that the problem's high number of objective functions can lead to computationally burdening solutions when a posteriori methods are introduced. The weighted-sum method initially proposed by Zadeh [13], is a popular method following an a priori preference on the selection of a Pareto optimal through appropriate, importance-related weighting of the objective functions. The initial optimization problem takes the following single objective form as illustrated in equation (eq. 17).

$$\min_{\mathbf{x}} \ g(\mathbf{x}) = \sum_{j=1} w_j \cdot f_j(\mathbf{x})$$
$$subject\ to\ \ \mathbf{x} \geq 0 \qquad (17)$$

As the objective function commonly behaves in a nonlinear, non-convex manner, an arbitrary implementation of a deterministic optimization approach can lead to local minima without expanding the LP model's potential to its full capacity. As a result, the algorithm used for the solution of the optimization problem of eq. (17) consists of the combined efforts of a deterministic search method operating in a local level and a general plan operating in a global level. The local search method most efficiently suited for eq. (17) is the interior point trust region approach proposed by Coleman and Li[14]. According to this approach, the quadratic trust region sub-problem is approximately solved as the minimization of a quadratic problem subjected to an appropriate ellipsoidal constraint. The global level general plan is achieved through the multiple execution of the interior point trust region method, initiated from different stochastically generated locations inside a prediction region (Fig. 2).

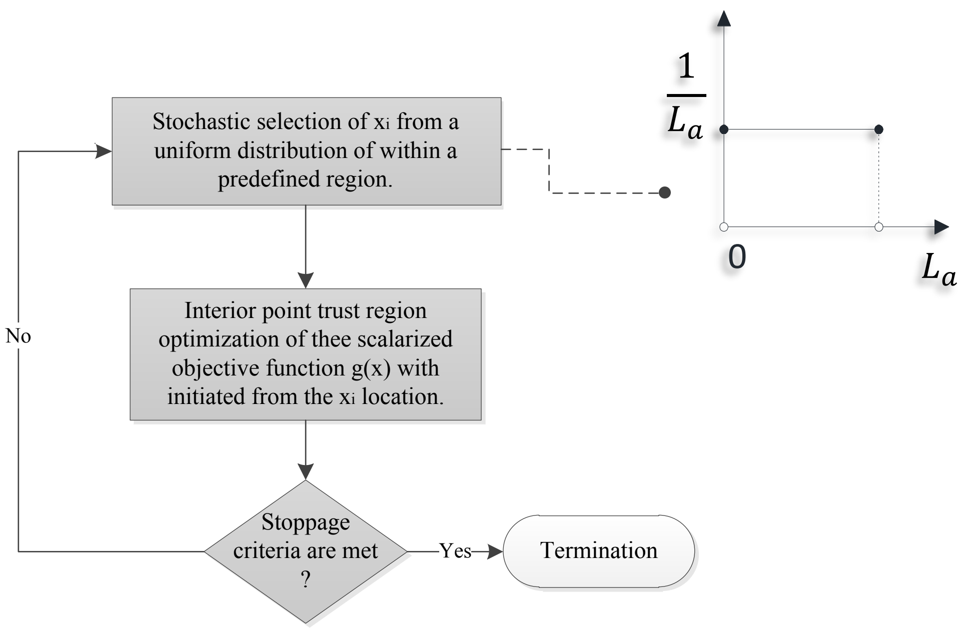

Figure 2. Multi-start algorithm used for the calibration process

The termination criteria for the proposed optimization scheme consists of an iterative evaluation of the objective function value for the normalized static stiffness of the LP model, along with a maximum boundary on the number of the overall sampling points x_i.

5. SINGLE INTERFACE DOF NUMERICAL VERIFICATION

As presented in the previous paragraphs, the proposed procedure can provide a computationally viable alternative to the direct FEM approach for the solution of the SSI problem. However, the procedure is founded on several assumptions, which can render its efficiency questionable. To this end, it is essential to verify the proposed procedure through a realistic example involving a strip foundation on homogenous clay soil as illustrated in Fig. 3. In addition to the proposed inelastic LP model approach (M1), a simplified case of the presented LP assembly with conventional *elastic* complementary components, solely tuned to the elastic impedance function of the condensed segment (M2) is also implemented for reasons of efficiency comparison.

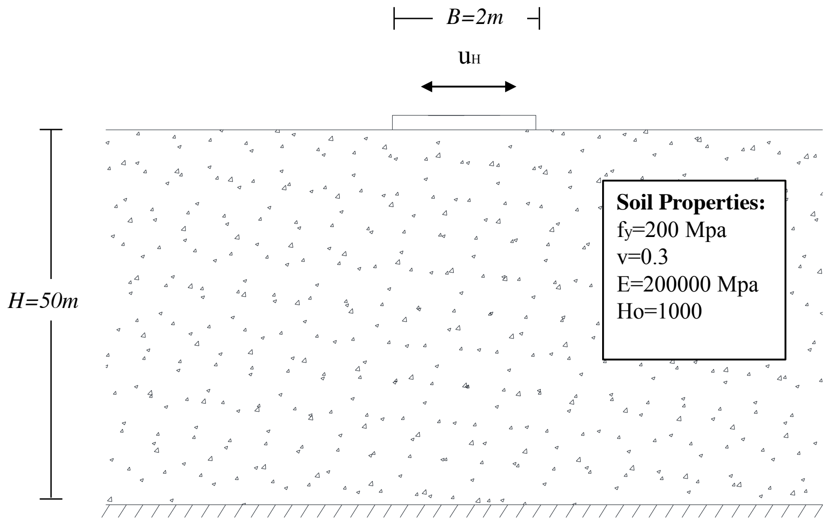

Figure 3.: Shallow foundation over uniform soil profile example (plane strain conditions)

As a targeted behavior is essential to the case study, a Finite element model of the soil foundation segment has been constructed. The soil foundation system is simulated as a plane strain model of a rigid strip foundation over a uniform clay profile. An elastoplastic constitutive law is implemented for the representation of the soil according to an associative plasticity flow with isotropic hardening and a Von-Mises yielding criterion. The soil constitutive law properties are summarized in Table I. The overall FEM model includes a 100m x 50m truncated region of the semi-infinite soil domain, where absorbing boundaries are introduced at the side of the model according to [15], while a rigid bedrock is assumed at a depth of 50m.

Table I . Properties of the selected soil materials

Yielding Criterion	E(MPa)	Hardening Param. H_o	Poisson's Ratio ν	Yield Stress in Pure shear f_y (MPa)
Von Mises	200000	1000	0.3	200

For the current study and for simplicity purpsoses, only one DOF is considered at the interface between the soil foundation segment and the superstructure, i.e., corresponding to the horizontal displacement of the centroid of the strip foundation. As a result, a single DOF elastoplastic constitutive law is utilized as the base component of the LP model of both the proposed method (M1) and the conventional complementary component method (M2). The selected constitutive law is described by the following elastoplasticity loading/unloading conditions (eq.18) and yielding equation (eq.19).

$$\dot{\gamma} \geq 0 \text{ , } \Phi(F) \leq 0 \text{ , } \dot{\gamma} \cdot \Phi(F) = 0 \tag{18}$$

$$\Phi(F) = \left| F - H(e_p) \right| - G(a) \tag{19}$$

where F denotes the generalized force of the base component, while Φ is the yielding criterion of the elastoplastic constitutive law. With regard to the terms of the yielding function, H corresponds to the kinematic hardening function, G to the isotropic hardening function and e_p and a are the isotropic and kinematic hardening parameters. A polynomial form is selected for the representation of the hardening function G and H, where the polynomial factors are calibrated according the quasi-static behavior of the FEM model. Results of the base component efficiency for quasi static loading are illustrated in Figure 4.

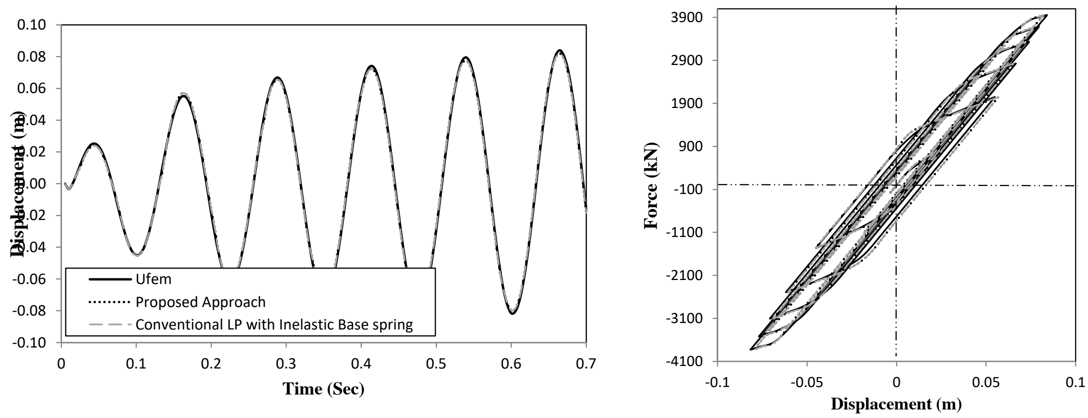

Figure 4.: Efficiency of the simplified and proposed LP models under quasi-static excitation (a) displacement to pseudo-time, and (b) force to displacement relation

For this single interface DOF the system's dynamic traits are extracted from the FEM model for different values of the interface DOF displacements in the range of 0 to 60mm. The condensed segment's impedance function and returning force matrix doublets $S_{reduced}$ and RF are extracted through the appropriate linearization and dynamic condensation of the system in the frequency domain, in regard to the preselected interface DOF. The extracted properties are used as a target for the calibration of the complementary components of the proposed method (M1), while the simplified method (M.2) is tuned according to the elastic impedance function of the targeted system.

5.3 Time domain verification of the proposed procedure

As both the proposed (M1) and the conventional (M2) LP methods fulfil the acceptable accuracy criteria for the quasi-static behaviour of the targeted soil-foundation system, a dynamic case study analysis is performed in the following paragraphs. The dynamic response of the soil-foundation interface DOF under harmonic excitations on a frequency range of 0.5-4Hz is compared between the two LP model reduction methods and the actual FEM model as illustrated in Figures (6-9).

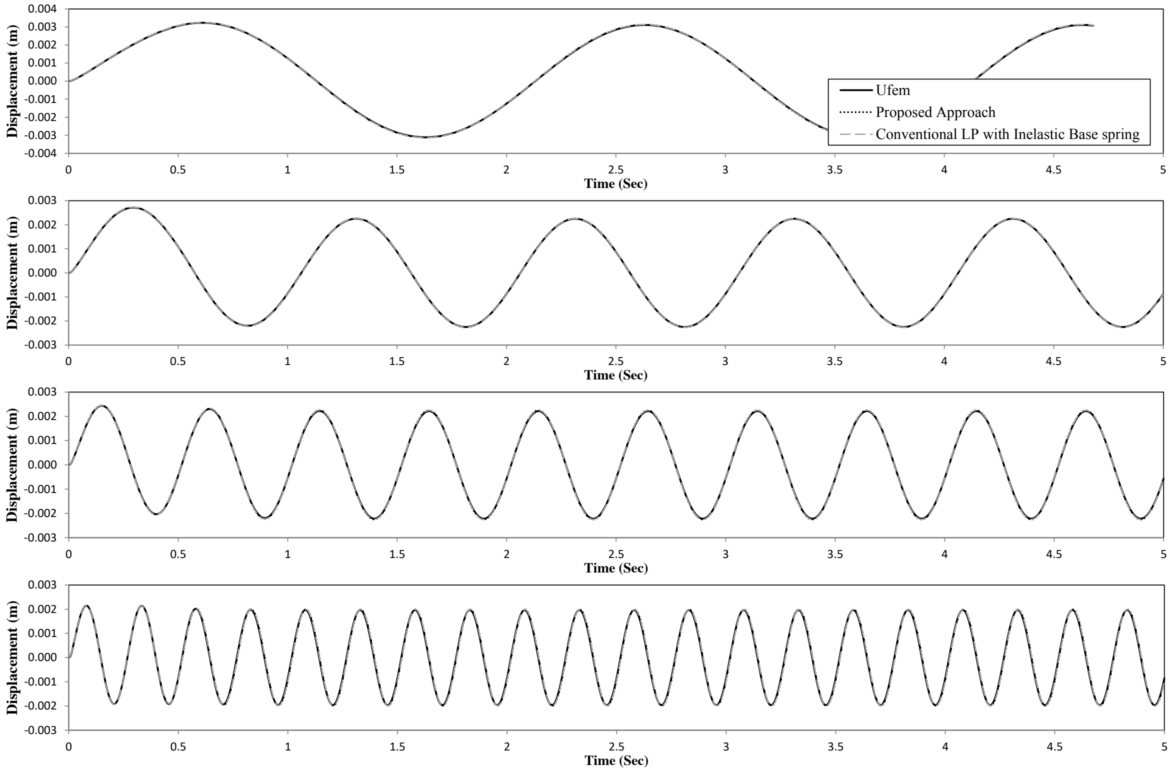

Figure 6. Low intensity dynamic response of LP models: displacement to time relationship

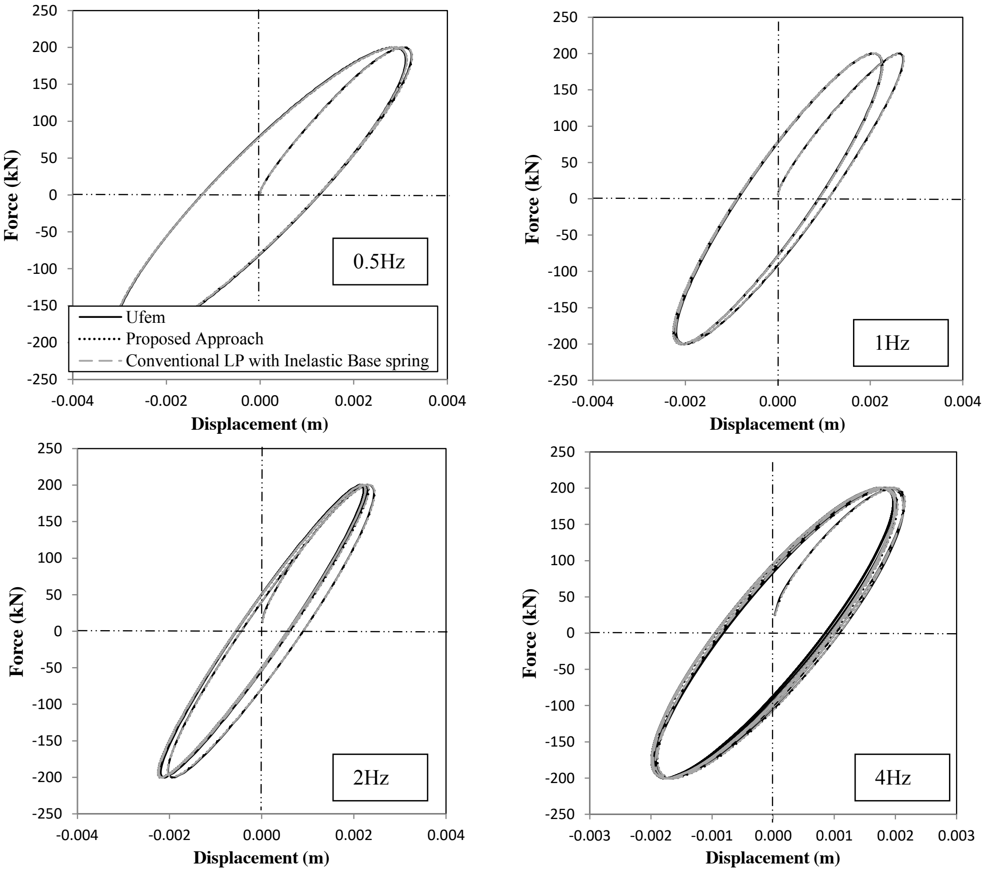

Figure 7. Low intensity dynamic response of LP models: force to displacement relationship

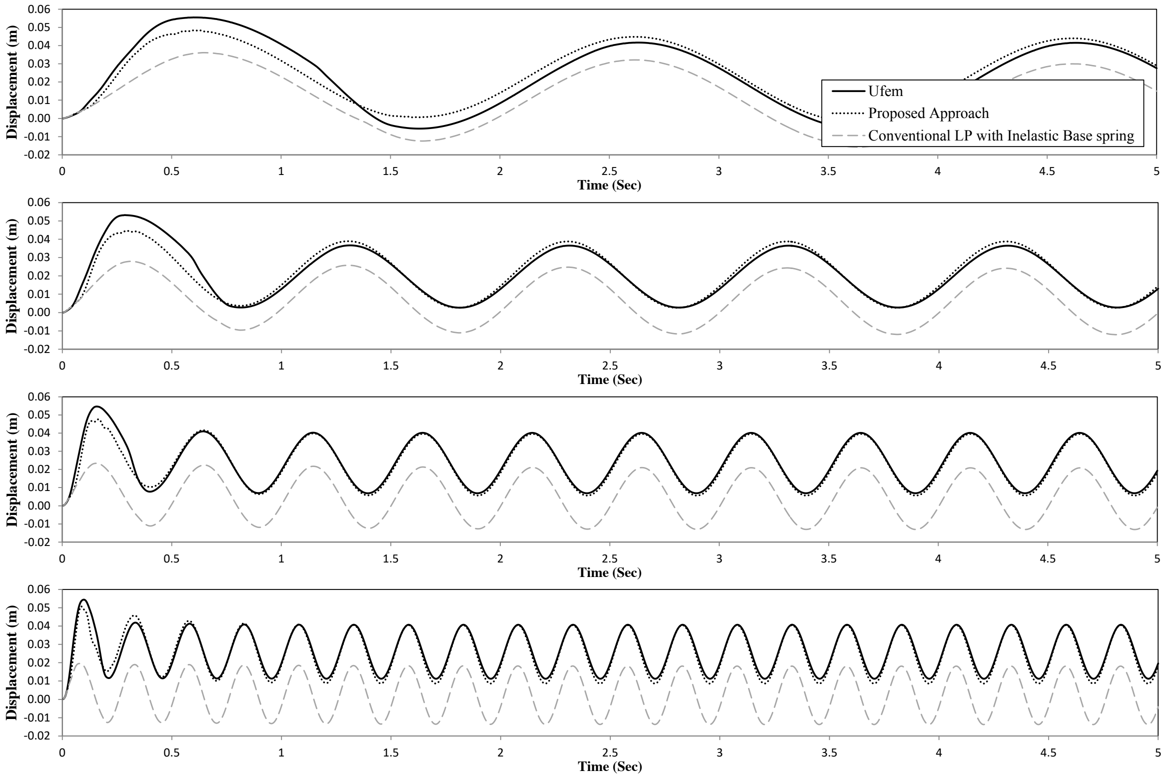

Figure 8. High intensity dynamic response of LP models: displacement to time relationship

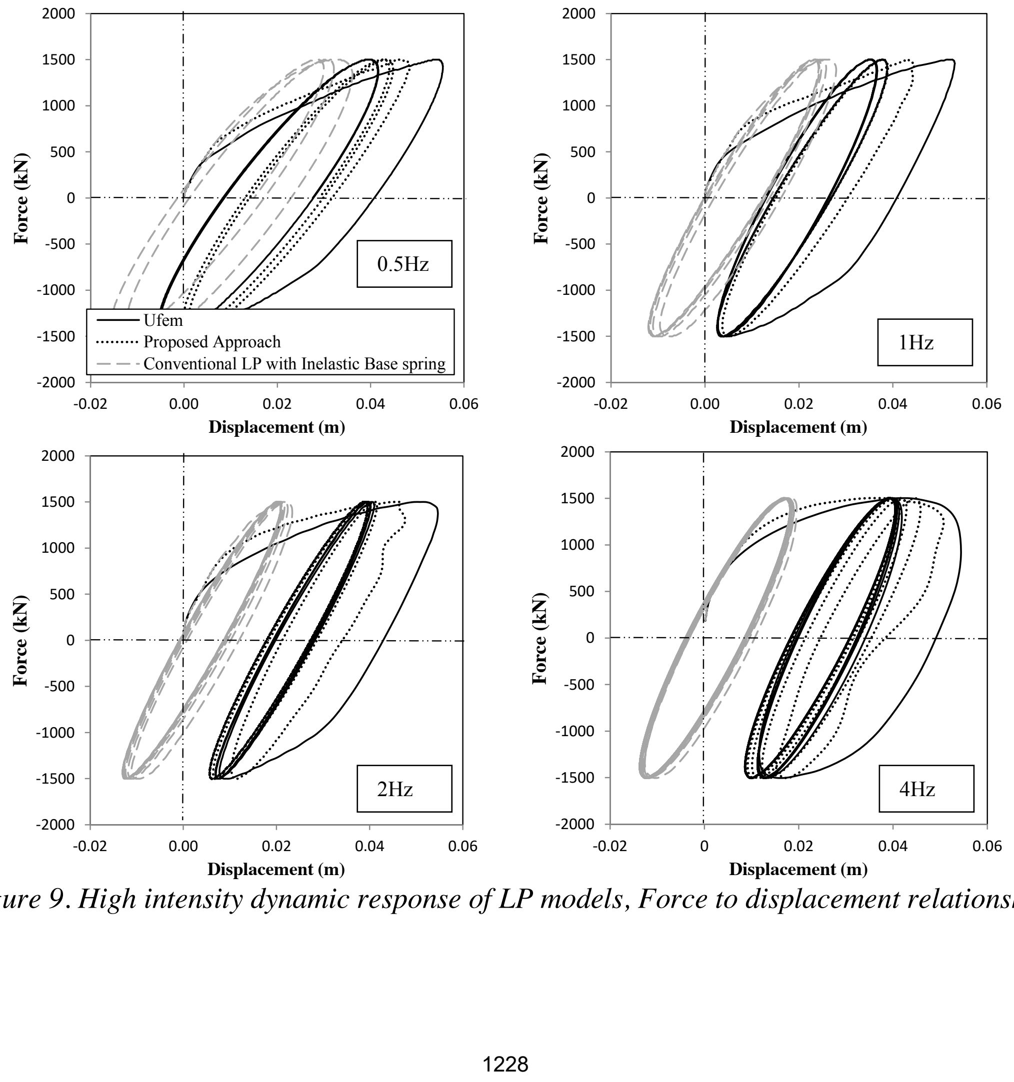

Figure 9. High intensity dynamic response of LP models, Force to displacement relationship

Figures 6-9 illustrate that both the proposed method and the simplified approach emulate the elastic dynamic behavior of the targeted FEM model in an accurate manner, as elastic dynamic properties are successfully targeted by both models during the calibration process. Particularly for the scenario of higher excitation intensity, the subsoil region demonstrates permanent plastic deformation as evident in Figures 8-9). It is observed that within the selected frequency range 0.5-4Hz, the post yield behavior of the simplified LP approach (M2) is clearly less efficient in comparison to the proposed method. This can be attributed to the fact that M2 method in only tuned at a limited degree, hence, the conventional complementary components are not able to maintain a behavioural accuracy outside the intensity and/or frequency regions within which calibration was performed. Additionally, as expected, the observed error is higher for higher frequencies where the static components of the reduced methods is less dominant.

5. CONCLUSIONS

A lumped parameter modelling method has been proposed in the current paper, capable of accurately emulating the dynamic behaviour of the soil foundation system. In contrast to methods presented in the existing literature, the proposed method copes with the frequency depended properties of the soil-foundation system within a breadth of intensities by means of the expansion of the lumped parameter model framework to inelastic dynamic systems. The procedure is numerically verified in both quasi-static and dynamic excitations by comparing a refined FE model, the complete and the simplified version of the proposed approach. Even though the results are herein limited to single DOF representations of simplified soil foundation systems, they clearly demonstrate the efficiency of the proposed frequency- and intensity-dependent LP model for the purpose of SSI analyses. Future work is essential to the expansion of the proposed method for the case of more complex soil-foundation systems and seismic loading.

6. REFERENCES

1. Mylonakis G, Gazetas G. Seismic Soil-Structure Interaction: Beneficial or Detrimental? *Journal of Earthquake Engineering* 2000; **4**(3): 277–301. DOI: 10.1080/13632460009350372.
2. Montrasio L, Nova R. Settlements of shallow foundations on sand: geometrical effects. *Géotechnique* 1997; **47**(1): 49–60. DOI: 10.1680/geot.1997.47.1.49.
3. Chatzigogos CT, Pecker A, Salençon J. Macroelement modeling of shallow foundations. *Soil Dynamics and Earthquake Engineering* 2009; **29**(5): 765–781. DOI: 10.1016/j.soildyn.2008.08.009.
4. Salciarini D, Tamagnini C. A hypoplastic macroelement model for shallow foundations under monotonic and cyclic loads. *Acta Geotechnica* 2009; **4**(3): 163–176. DOI: 10.1007/s11440-009-0087-2.
5. Li Z, Kotronis P, Escoffier S, Tamagnini C. A hypoplastic macroelement for single vertical piles in sand subject to three-dimensional loading conditions. *Acta Geotechnica* 2016; **11**(2): 373–390. DOI: 10.1007/s11440-015-0415-7.
6. Lesgidis N, Sextos A, Kwon OS. Influence of frequency-dependent soil-structure interaction on the fragility of R/C bridges. *Earthquake Engineering & Structural Dynamics* 2017; **46**(1): 139–158. DOI: 10.1002/eqe.2778.
7. Chai S hyeon, Ghaemmaghami AR, Kwon OS. Numerical modelling method for inelastic and frequency-dependent behavior of shallow foundations. *Soil Dynamics and Earthquake Engineering* 2017; **92**(August 2016): 377–387. DOI: 10.1016/j.soildyn.2016.10.030.
8. Wolf J. Consistent lumped-parameter models for unbounded soil: Physical representation. *Earthquake Engineering & Structural Dynamics* 1991; **32**(December 1989): 11–32.
9. Paronesso A, Wolf JP. Global lumped-parameter model with physical representation for unbounded medium. *Earthquake Engineering & Structural Dynamics* 1995; **24**(5): 637–654. DOI: 10.1002/eqe.4290240503.
10. Saitoh M. Simple model of frequency-dependent impedance functions in soil-structure interaction using frequency-independent elements. *Journal of Engineering Mechanics* 2007(October): 1101–1114.
11. Lesgidis N, Kwon O sung, Sextos A. A time-domain seismic SSI analysis method for inelastic bridge structures through the use of a frequency-dependent lumped parameter model. *Earthquake Engineering & Structural Dynamics* 2015: 2573. DOI: 10.1002/eqe.2573.

12. Saitoh M. On the performance of lumped parameter models with gyro-mass elements for the impedance function of a pile-group supporting a single-degree-of-freedom system. *Earthquake Engineering & Structural Dynamics* 2012; **41**(4): 623–641. DOI: 10.1002/eqe.1147.
13. Zadeh L. Optimality and non-scalar-valued performance criteria. *Automatic Control, IEEE Transactions on* 1963; **8**(1): 59–60. DOI: 10.1109/tac.1963.1105511.
14. Coleman TF, Li Y. An Interior Trust Region Approach for Nonlinear Minimization Subject to Bounds. *SIAM Journal on Optimization* 1996; **6**(2): 418–445. DOI: 10.1137/0806023.
15. Lysmer, Kuemyer - 1969 - Finite dynamic model for infinite media.