Available online at www.eccomasproceedia.org
Eccomas Proceedia COMPDYN (2017) 1231-1239

ECCOMAS
Proceedia

COMPDYN 2017
6th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, M. Fragiadakis (eds.)
Rhodes Island, Greece, 15–17 June 2017

DYNAMICS OF AN UNDERGROUND PIPELINE WITH SLIPPING CONTACT AT SOIL-PIPELINE INTERFACE UNDER SEISMIC EXCITATION: ANALYTICAL AND NUMERICAL INVESTIGATION OF COUPLED PROBLEMS

Mukhady Sh. Israilov[1] **and Shakhzod M. Takhirov**[2]

[1] Director of Scientific Research Institute of Mathematical Physics and Seismodynamics, Grozny, Russia
e-mail: israiler@hotmail.com

[2] Eng. Manager of Structures Laboratory, Civil and Environmental Engineering Department, University of California, Berkeley; 337 Davis Hall, UC Berkeley, Berkeley 94720
e-mail: takhirov@berkeley.edu

Keywords: Seismic waves, underground pipeline, slippage at interface, numerical and analytical methods.

Abstract. *As an extension of the previously proposed method of "one-dimensional" elastic deformation of the soil, a new solution of coupled seismic oscillations of underground pipeline and surrounding elastic soil in a non-ideal contact are obtained. Namely, it is assumed that slippage at the interface can occur and when it happens the shear stress is proportional to the relative displacement of the soil and that of the pipeline (dry friction condition). In a case when the incident seismic wave is a longitudinal wave propagating along the pipeline, a closed-form analytical solution of the nonstationary problem is obtained. The dependence between the maximum stresses and the pipe's coefficient of friction is investigated. In a supersonic case, when the wave propagation velocity in the pipe is less than that of the soil, at first sight paradoxical effect was identified. This effect is related to the possibility of resonance while reducing shear stresses at the interface. The effect occurs only if there is a slippage between the pipe and the soil ("seesaw effect").This approximated approach was verified via numerical simulations.*

Peer-review under responsibility of the organizing committee of COMPDYN 2017.
doi: 10.7712/120117.5488.18510

1 INTRODUCTION

There is a large number of approximate approaches and methods ([1-4], for example) for determining the response of long underground structures (pipelines and tunnels) and their dynamic stresses under seismic impact. Since it is difficult to solve the soil-pipeline interaction problem in exact formulation, some assumptions have to be made. In these approaches and their modifications, it is assumed that the motion of the medium is specified (seismic wave) and does not change due to the presence of an obstacle. The interaction between the pipeline and the surrounding medium is taken into account by using a model in which the constants and functions are determined experimentally.

More realistic formulation of coupled dynamic problem of soil-pipeline interaction in the approximation of one-dimensional deformation of the medium is proposed in [5, 6]. However, in [5, 6] only the case of adhesion is considered where the soil and pipeline displacements at the interface are the same. Meanwhile, for some types of soils, the possibility of pipe slippage relative to soil has been shown experimentally [4, 7]. The objectives of this work are to formulate and solve coupled problems of pipeline seismodynamics in such cases, as well as to study the effect of sliding on the maximum tension–compression stresses in the pipeline, which determine its seismic performance.

2 FORMULATION OF THE PROBLEM. SOLUTION FOR THE CASE OF COULOMB FRICTION SLIDINGGENERAL SPECIFICATIONS

A straight-line pipe is modeled by an infinitely long thick-walled cylinder with external and internal radii *a* and *b*, respectively. Movement of the pipeline and its surrounding elastic medium (soil) is caused by the propagation of a longitudinal seismic wave in the soil in the direction of the pipeline axis, which is taken to be the Z-axis of the cylindrical coordinate system (r, θ, z) as presented in Figure 1.

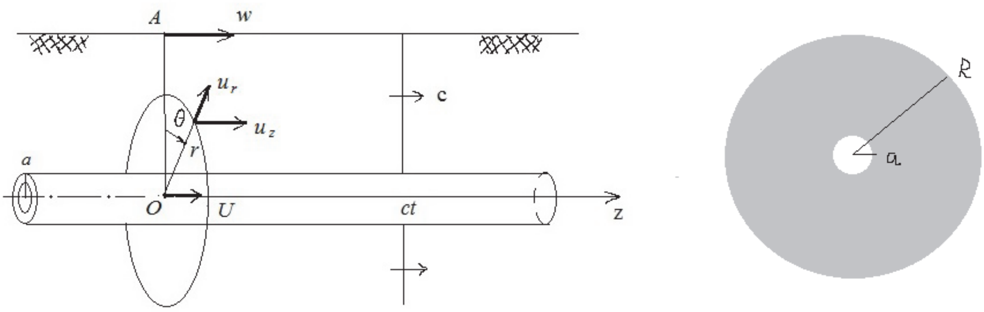


Figure 1: Problem layout.

This implies that at a relatively large distance from the pipeline $r = R$, the displacements of the medium are equal to the displacements in the incident longitudinal wave:

$$r = R: u = 0,\ v = 0,\ w = w_0(c_1 t - z)H(c_1 t - z). \qquad (1)$$

Here u, v, and w are the components of the displacement vector $\boldsymbol{u}$ along the axes r, θ, and z, respectively, $R > a$ is the depth of the pipeline (usually, R is usually few orders greater than a), $H(\xi)$ is the Heaviside step function, $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ is the velocity of propagation of longitudinal waves, λ and μ are Lamé constants, and ρ is the density of the elastic soil.

The boundary conditions at the soil- pipeline interface are given by the following

$$r = a:\ u = 0,\ v = 0,\ \sigma_{rz} = \mu\partial w/\partial r = k[w(a, z, t) - U(z, t)], \tag{2}$$

where $U(z, t)$ is the axial displacement of the pipeline, determined from the equation of its longitudinal vibrations, and the constant k is the interaction coefficient (friction coefficient).

In the formulation of the third condition in (2), it is taken into account that by virtue of the first condition (2), the shear stress at the interface is equal to.

$$r = a:\ \sigma_{rz} = \mu(\partial u/\partial z+\partial w/\partial r)= \mu(\partial w/\partial r), \tag{3}$$

and hence does not depend on u (it is assumed that the operations of differentiation with respect to z and transition to the boundary for $r = a$ are permutable).

According to the conditions (1) and (2), there is an axial symmetry, i.e., $v = 0$ and the functions u and w are independent of the angular coordinate θ. In this case, the equations of motion of the elastic medium reduce to a system of two Lamé equations for the relative radial u and axial w displacements. Under the assumption that the deformation of the medium $\varepsilon_{zz} = \partial w/\partial z$ in the direction of propagation of the incident wave coinciding with the direction of the pipe axis is dominant, from this system we can extract the equation for the longitudinal displacement w in the following form [6]:

$$\left(\frac{c_2}{c_1}\right)^2\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+\frac{\partial^2 w}{\partial z^2}=\frac{1}{c_1^2}\frac{\partial^2 w}{\partial t^2}, \tag{4}$$

where $c_2 = \sqrt{\mu/\rho}$ is the velocity of propagation of shear waves in the soil.

In the variables r and Z ($Z = c_1t - z$), where Z is the distance from the incident wave front to an arbitrary point, the solution of (4) satisfying boundary conditions (1) and (2), and the shear stress (3) at the interface are given by the relations

$$w(r,Z)=\frac{ka\ln(r/a)}{\mu+ka\ln(R/a)}\left[w_0(Z)H(Z)-U(Z)\right]+\frac{\mu w_0(Z)H(Z)+ka\ln(R/a)U(Z)}{\mu+ka\ln(R/a)},$$

$$\left(\sigma_{rz}\right)_{r=a}=\frac{k\mu}{\mu+ka\ln(R/a)}\left[w_0(Z)H(Z)-U(Z)\right] \tag{5}$$

The equation of forced longitudinal vibrations of the pipeline has the form

$$\rho'\frac{\partial^2 U}{\partial t^2}=E'\frac{\partial^2 U}{\partial z^2}+F, \tag{6}$$

where E' is the Young modulus, ρ' is the density of the pipeline's material, and F is the resultant shear stresses on the surface of the pipeline. According to (5), the body longitudinal force is given by

$$F(U,w_0)=\frac{1}{V}\int_0^{2\pi}\left(\sigma_{rz}\right)_{r=a}a\,d\theta=\frac{2\mu\beta^2}{(a^2-b^2)}\left(w_0-U\right),\ \beta=\sqrt{\frac{ka}{\mu+ka\ln(R/a)}} \tag{7}$$

where V is the volume of a pipe element which has length dz or unit length.

Substituting expression (7) for F into (6), we obtain the equation of coupled pipeline vibrations, which in the case of a given seismic wave (1) has a solution that depends on the variable $Z = c_1t - z$, and after a series of transformations reduces to the equation

$$U''\pm q^2U=\pm q^2 w_0(Z)H(Z). \tag{8}$$

Here the prime denotes differentiation with respect to Z,

$$q = p_0\beta,\ p_0 = \sqrt{2\mu}\left(E'\left|M^2 - 1\right|(a^2 - b^2)\right)^{-1/2} \tag{9}$$

$M = c_1/c_0$ is the Mach number, and $c_0' = \sqrt{E'/\rho'}$ is the velocity of propagation of longitudinal waves in the pipeline.

The upper signs in (8) correspond to the supersonic wave velocity ($M > 1$), and the lower signs to the subsonic velocity ($M < 1$). This equation coincides with the equation of pipeline vibrations for the case of full adhesion for $q = p_0\beta_0$ and $\beta_0 = 1/\sqrt{\ln(R/a)}$. From (7) it follows that $0 < \beta < \beta_0$ (since $0 < k < \infty$) and $\beta \to \beta_0$ for $k \to \infty$.

It should be noted that for $M > 1$ (in the case of supersonic flow), perturbations exist only behind the incident wave front $Z > 0$, so that it is necessary to find a solution of (8) that satisfies $U = dU/dZ = 0$ at the front $Z = 0$. For $M < 1$ (in the case of subsonic flow), the variable Z can take any values in the range of $(-\infty;+\infty)$ and perturbations in the surrounding medium and pipeline exist both behind and ahead of the incident wave front.

In this case, the requirement of limited amplitude of the vibrations at infinity as $Z \to \pm\infty$ is physically justified.

Solutions of (8) satisfying the specified constraints were found in [6]. In particular, if the soil particles behind the wave front move according to the law $w_0 = A_0 \sin(\omega_1 Z)H(Z)$, the stresses in the pipeline are given by the expressions

$$\sigma = -E'\frac{\partial U}{\partial Z} = -\frac{E'A_0\omega_1}{1-\left(\omega_1/(p_0\beta)\right)^2}\left(\cos\omega_1 Z - \cos(p_0\beta Z)\right)H(Z)\ ,\ M > 1 \tag{10}$$

$$\sigma = -\frac{E'A_0\omega_1}{1+\left(\omega_1/(p_0\beta)\right)^2}\left(-\frac{1}{2}\operatorname{sgn}(Z)e^{-p_0\beta|Z|} + H(Z)\cos(\omega_1 Z)\right)\ ,\ M < 1 \tag{11}$$

In the case of adhesion ($\beta = \beta_0$), these expressions coincide with the expressions obtained in [5, 6]. Also from (10), (11) it follows that for $\beta \to 0$ ($k \to 0$), the stress $\sigma \to 0$ by virtue of boundary conditions (1.2) ($k = 0$ means that the pipeline surface is perfectly smooth and is not involved in the motion).

According to [6] for metal (steel or cast iron) and concrete pipelines with various diameters and wall thicknesses in typical soils at a depth of about 1.0–1.5 m, the value of $p_0\beta_0$ is one or two orders of magnitude greater than the value of ω_1, which is inversely proportional to the length of the seismic wave. Therefore, in the case of adhesion ($\beta = \beta_0$), the denominator in formula (10) is close to unity and the stresses are limited. In the case of sliding [$\beta \in (0, \beta_0)$], for $M > 1$, it is possible that $q = p_0\beta \cong \omega_1$ and resonance occurs, i.e., [as follows from formula (8)], the amplitudes of the stresses and pipeline vibrations (with increasing Z) increase linearly.

In the formulation of the external problem for the soil, it was assumed that the radial displacements of the soil u at the interface with the pipeline are zero [condition (1.2)]. It is not difficult to estimate the accuracy of this approximation and the effect of the values $u \neq 0$ for $r = a$. Indeed, the greatest increment in the stresses σ_{rz} is obtained assuming that the change in the pipe wall thickness $h = a - b$ is due to movement of the outer surface of the pipe. In this case, $u|_{r=a} = \Delta h = h\varepsilon_{rr} = -\nu' h\varepsilon_{zz} = -\nu' h\partial U/\partial z$ (ν' is the Poisson's ratio of the pipe's material). Then, according to (3), additional stresses $\Delta\sigma_{rz} = -\nu'\mu h \partial^2 U / \partial z^2$ occur at the interface. Accounting for these stresses, we obtain the equation of pipeline vibrations (8) in which the constants are defined by formulas (9), provided that in the expression for p_0, the quantity $|M2 - 1|$ is replaced by $|M2 + \delta - 1|$, where $\delta = 2\mu\nu'(E')^{-1}(1+b/a)^{-1}$. Thus, for a steel or cast-iron tube (diameter 50 cm, a wall thickness of 1 cm) buried at a depth of 1 m in dense soil (clay), $\delta \approx 0.00082$, and for a concrete pipe with the same dimensions, it varies in the range of 0.002–0.006. Therefore, in these cases, the influence of the transverse compression and tension of the pipeline can be neglected. In the case of flexible pipes (plastic, rubber, etc.), the value of δ cannot be neglected, which leads to a decrease in the value of p_0 in (9). In this case, however, all the above properties of the solution, including those due to the occurrence of resonance, are retained.

3 NUMERICAL VALIDATION OF ANALYTICAL SOLUTION

This section of the paper discusses numerical simulation of the problem for which the closed-form solution is obtained above and its comparison to numerical simulation of a similar problem with stress-free boundary conditions on the outer surface of the soil. The model in latter simulation is called Model A and Model B stands for numerical model in the former simulation that utilizes the boundary condition presented in (1).

3.1 Material Properties and Finite Element Mesh

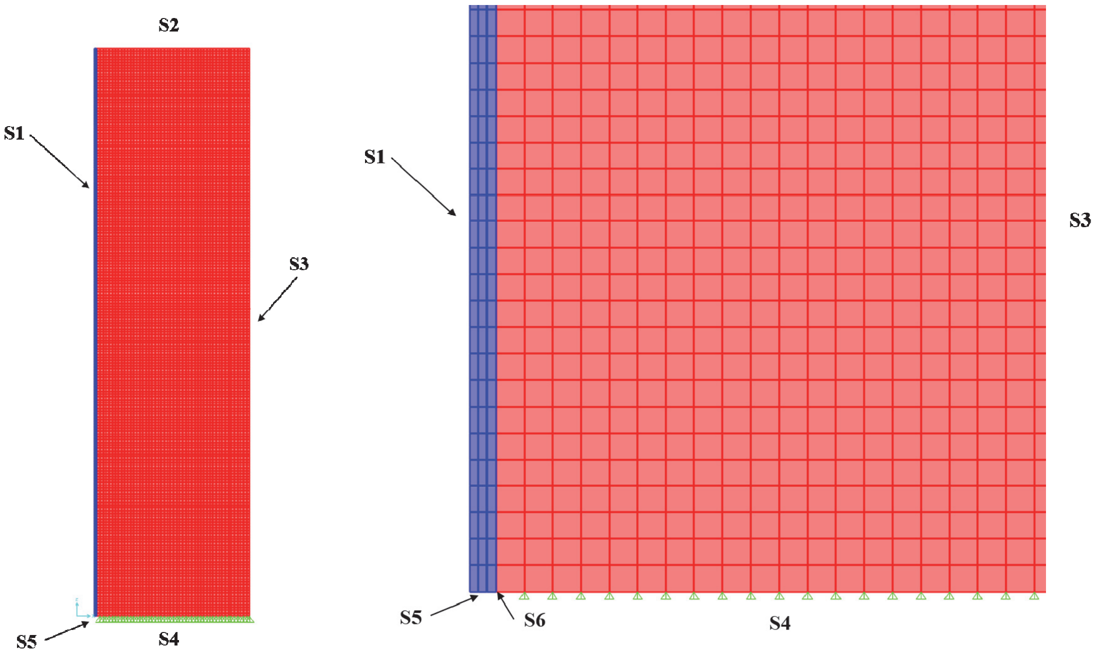
The material properties of the soil and pipe are presented in Table 1. As presented in Table 2, ten finite element (FE) models are created to conduct parametric studies for each major model configuration. FE modeling is based on the following. The inner and outer radii of the pipe do not change from model to model and are selected to be close to the typical dimensions of the new underground tunnel system recently constructed in London, United Kingdom [7]. The models are created with the pipe's depth varying from about 20m to about 40m with about 5m increments. To take advantage of the axial symmetricity of the problem, all mesh elements for both pipe and soil are assigned to ASolid elements in SAP2000 [8]. The pipe consists of three layers of elements about 0.5m by 0.17m. The soil elements are very close to 0.5m by 0.5m. The global and zoomed views of the finite element mesh (example of Model 2A is shown) are presented in Figure 2.

Material	E, GPa	ν	ρ, kg/m^3	C_P, m/sec	C_S, m/sec
Concrete (pipe)	24.9	0.2	2402.8	3390.6	NA
Soil	1.5	0.4	1750.6	1355.0	553.2

Table 1: Material properties used in numerical modeling.

Model	Pipe's inner radius, m	Pipe's outer radius or a, m	Length of model, m	Depth or R, m
1A&1B	3.0	3.5	100.0	41.13
2A&2B	3.0	3.5	100.0	35.75
3A&3B	3.0	3.5	100.0	30.38
4A&4B	3.0	3.5	100.0	25.0
5A&5B	3.0	3.5	100.0	20.16

Table 2: Numerical models used in simulations.



a) Global view of mesh b) Zoomed view

Figure 2: Finite element mesh and boundary surfaces.

3.2 Boundary Conditions

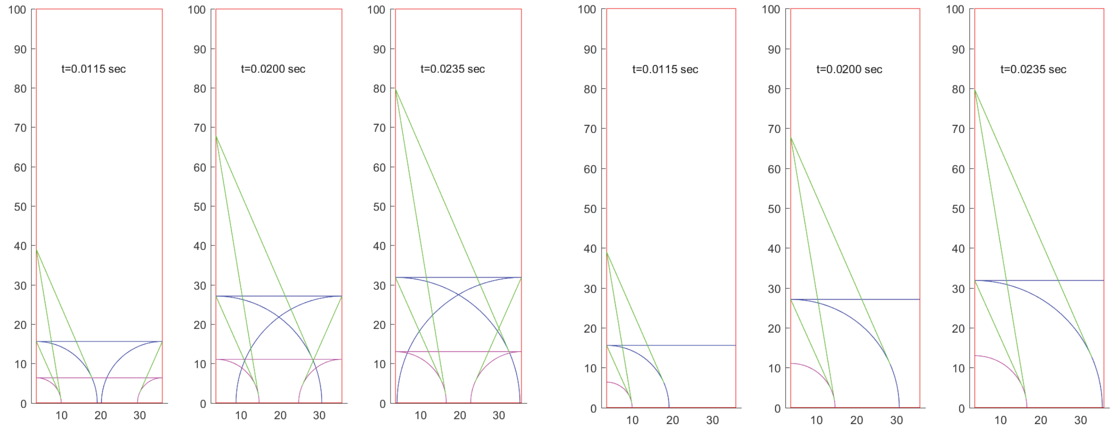
As discussed earlier, the finite element models 1A though 5A are based on stress free assumption on the outer surface of the soil that is noted as S3 in Figure 2 and Models 1B through 5B are based on the boundary condition (1) on the same surface. The incident wave with a sinusoidal acceleration $A_0 = \sin(\omega_1 t)$, is applied at the boundary surface S4. Surfaces S1, S2, S5 are assumed stress free. A rigid contact between pipe and soil elements is assumed at the interface notes as surface S6. The simulations are conducted until the fastest wave front reaches surface S2.

3.3 Results of Parametric Numerical Simulations

The results of numerical simulations for all ten models are discussed in this section. It worthy to note, that the complexity of wave fronts' geometry is not the same for the two major model cases as presented in Figure 3. This plot shows an example of Models 2A and 2B which is typical for all other models. The main difference comes from the boundary condition on the outer surface of the soil and is related to the fact that the incident wave satisfies the boundary condition of Model B and requires introduction of additional waves to satisfy stress-free boundary condition of Model A.

The results of numerical simulations confirm this prediction. Figure 4 shows Von Mises stresses for Model 1B through 5B. The distribution of the stress is much more even than that for Model 1A trough Model 5A. The latter is shown in Figure 5. The Von-Mises stress in vicinity of S3 and S4 surfaces intersection is changes along the inclined wave fronts as presented in Figure 5. It leads to more complex wave dynamics. The waves are diffracting from the corners and reflecting from the pipe and surface S3 multiple times.

As a result, the acceleration of the pipe becomes close to stationery much faster for Model B as presented in Figure 6. This is not the case for Model, A which produces few more waves interacting with each other. This results in acceleration that does not have a stable harmoniclike shape in opposite to the acceleration in Model B. Nevertheless, the maximum acceleration does not exceed 0.25g in both cases. Therefore, for the practical applications, it can be concluded that this difference in the boundary conditions on the outer surface of the soil have insignificant effect on the maximum acceleration of the pipe.



a) Model 2A b) Model 2B

Figure 3: Typical anticipated geometry of wave fronts.

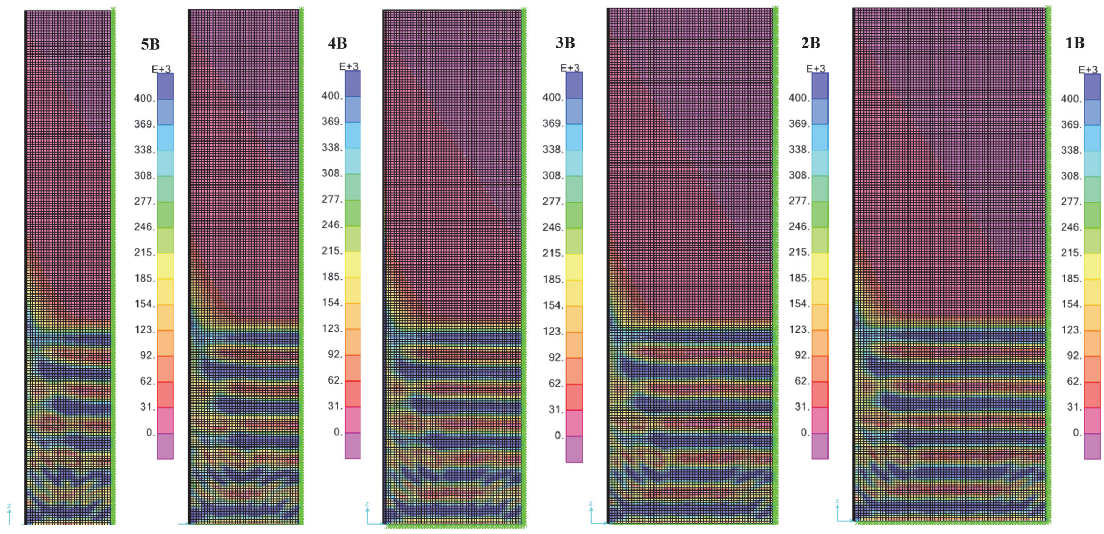


Figure 4: Von Mises stresses at t=0.0295 sec in Model B.

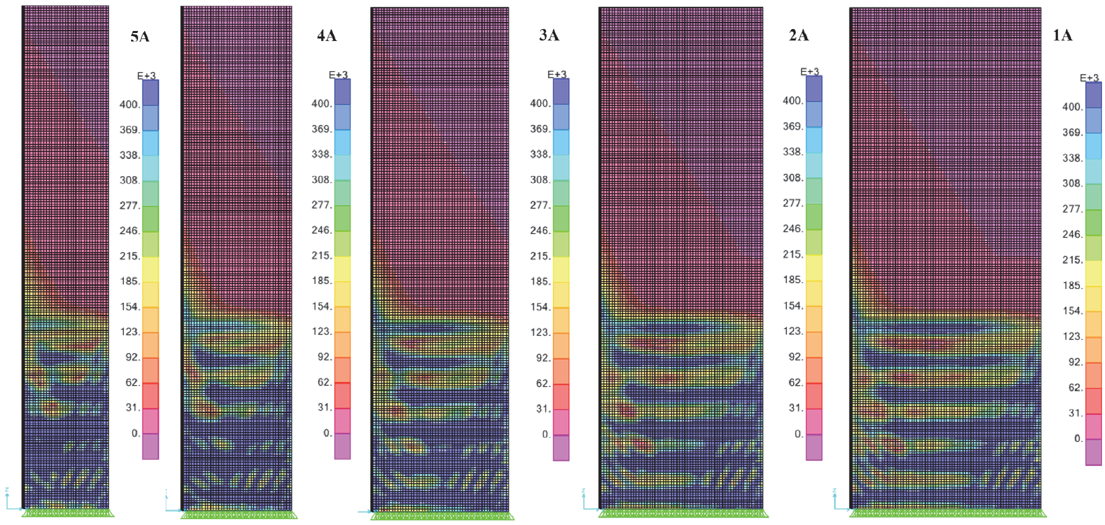
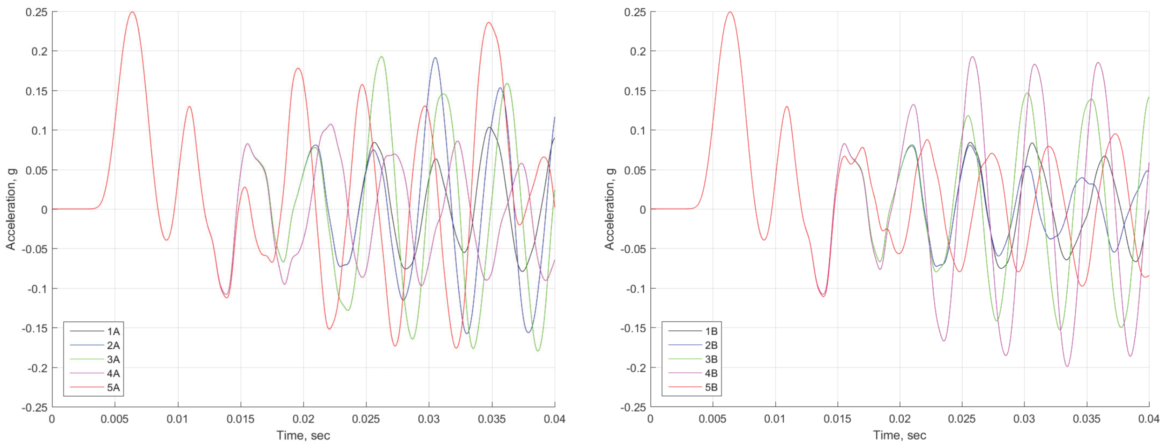


Figure 5: Von Mises stresses at t=0.0295 sec in Model A.



a) Model A b) Model B

Figure 6: Acceleration versus time at pipe-soil interface at 10.5m away from excitation surface.

4 CONCLUSIONS

- Analytical solution of the problem of coupled seismic movements of an underground pipeline and soil is obtained for the case of possible sliding between a pipeline and the surrounding medium.
- For pipelines made of different materials and different types of soils, the obtained solutions can be used to determine the ranges of the parameters in the laws of interaction at the interface in which the maximum tension and compression stresses in the pipeline are substantially less than the corresponding values in the case of adhesion and the regions in which this difference is insignificant. The results can be used to clarify the code requirements for the strength of pipelines under seismic impacts and in development of new provisions.

- In supersonic case, when the friction force is proportional to the relative displacement, the possibility of resonance with decreasing friction force at the interface is predicted. This fact should be taken into account in calculations of the stresses on polymer and composite pipelines under seismic impact, as well as the stresses on segmented pipelines with complex damping joints, when the wave velocity in the soil is higher than the average velocity of propagation of longitudinal perturbations in the pipe.
- Parametric numerical simulations show that the change of stress free boundary conditions to the conditions (1) on the outer surface of the soil has insignificant effect on the maximum acceleration of the pipe. Therefore, this approximated approach can be used in practical applications with acceptable degree of confidence.

REFERENCES

[1] N. M. Newmark, "Problems in Wave Propagation in Soil and Rock," in Proc. of the Int. Symp. on Wave Propagation and the Dynamic Properties of Earth Materials, New Mexico, August 23–25, (Univ. New Mexico Press, New Mexico, 1967), pp. 7–26, 1967.

[2] A. Sakurai and T. Takahashi, "Dynamic Stresses of Underground Pipe Lines during Earthquakes," in Proc. of the 4th World Conf. on Earthquake Eng., Santiago (Chile), January 13–18, (Univ. Chile, Santiago, 1969), Vol. 2, pp. 81–95, 1969.

[3] A. A. Il'yushin and T. P. Rashidov, "On the Action of Seismic Waves on an Underground Pipeline," Izv. Akad. Nauk UzSSR, Ser. Tekh. Nauk, No. 1, 37–42, 1971.

[4] T. Rashidov, The Dynamic Theory of the Seismic Stability of Complex Systems of Underground Structures (Fan, Tashkent, 1973) [in Russian].

[5] D. V. Georgievskii and M. Sh. Israilov, "Seismodynamics of Extended Underground Structures and Soils: Problem Statements and Self-Similar Solutions," Izv. Ross. Akad. Nauk, Mekh. Tverd. Tela, No. 4, 138–151, 2015.

[6] M. Sh. Israilov "Seismodynamics of an Underground Pipeline," Probl. Mekh. (Akad. Nauk Resp. Uzbekistan), No. 3, 18–24, 2012.

[7] http://www.crossrail.co.uk/construction/

[8] Computers and Structures, Inc. (CSI), 2014. SAP2000 Ultimate Version 16.1.1. Structural Analysis Program.